Journal of Hyperstructures 11 *(1) (2022), 109-114*
ISSN: 2322-1666 print/2251-8436 online

STUDY OF FIXED POINT THEOREM IN COMPLEX VALUED INTUITIONISTIC FUZZY METRIC SPACE

RAM MILAN SINGH

ABSTRACT. We will show several common fixed point theorems for contraction condition satisfying certain requirements in complex valued intuitionistic fuzzy metric spaces in this study.

Key Words:common fixed point, Intuitionistic fuzzy set, Complex valued, Continuous t-norm.
2010 Mathematics Subject Classification: 47 H10 .

1. Introduction

In 1965, Zadeh [12] proposed the concept of fuzzy sets. Fuzzy set theory is a useful tool for describing situations involving imprecise or ambiguous data. Fuzzy sets deal with situations like these by assigning a degree of belonging to a set to each object. Since then, it has become a burgeoning field of study in engineering, medicine, social science, graph theory, metric space theory, and complex analysis, among other fields. Kramosil and Michalek [6] introduced fuzzy metric spaces in a variety of ways in 1975. With the help of continuous t-norms, George and Veermani [4] improved the concept of fuzzy metric spaces in 1994.

Buckley [3] was the one who originally established the concept of fuzzy complex numbers and fuzzy complex analysis. 1987. Some authors were influenced by Buckley's work. The re-examination of fuzzy complex numbers continues. The year was 2002, and fuzzy sets were extended to complicated fuzzy sets by Ramot et al. [8]. as though it were a blanket statement Ramot et al. claim that a membership function

Received: 20 March 2022, Accepted: 13 September 2022. Communicated by Ahmad Yousefian Darani;
∗Address correspondence to Ram Milan Singh; E-mail: rammilansinghlig@gmail.com.

defines a sophisticated fuzzy set. The complicated plane's unit circle has a function with a range that extends beyond [0, 1]. Singh was born in the year 2016. The concept of "complex valued fuzzy" was introduced by D. Singh, et al. [10].creating metric spaces t-norm and the concept of convergent convergence using complex valued continuous.in a complex valued fuzzy sequence, a Cauchy sequence in complex valued fuzzy metric spaces. By introducing the concept of non-membership grade to fuzzy set theory, Atanassov [1] created a stir in 1983. In this paper, we generalise the results of Jeyaraman and Shakila [13].

In the complex valued intuitionistic fuzzy metric spaces, this work gives some common fixed point theorems for pairs of occasionally weakly compatible mappings satisfying various requirements.

2. Preliminaries

Definition 2.1. A binary operation $* : r_s(\cos\theta + i\sin\theta) \times r_s(\cos\theta + i\sin\theta) \rightarrow r_s(\cos\theta + i\sin\theta)$, where $r_s \in [0,1]$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$, is called complex valued continuous t-norm if it satisfies the followings:

(1) $*$ is associative and commutative,

(2) $*$ is continuous,

(3) $a * e^{i\theta} = a, \forall a \in r_s(\cos\theta + i\sin\theta)$

(4) a$*$ b $\leq c*d$ whenever $a \leq c$ and $b \leq d$, $\forall$a, b, c, d $\in r_s(\cos\theta + i\sin\theta)$.

Definition 2.2. A binary operation : $r_s(\cos\theta + i\sin\theta) \times r_s(\cos\theta + i\sin\theta) \rightarrow r_s(\cos\theta + i\sin\theta)$,where $r_s \in [0,1]$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$, is called complex valued continuous t-co norm if it satisfies the followings:

(1) is associative and commutative,

(2) is continuous,

(3) a $\diamond 0 = a, \forall a \in r_s(\cos\theta + i\sin\theta)$,

(4) a$\diamond$b $\leq$ c$\diamond$d whenever $a \leq c$ and $b \leq d$, $\forall$a, b, c, d $\in r_s(\cos\theta + i\sin\theta$).

Definition 2.3. The following are examples for complex valued continuous t-norm:

(i) $a * b = \min\{a, b\}, \forall a, b \in r_s(\cos\theta + i\sin\theta)$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$

(ii) $a*b = \max(a+b-(\cos\theta+i\sin\theta), 0)$, for all $a, b \in r_s(\cos\theta+i\sin\theta)$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$.

Definition 2.4. The following are examples for complex valued continuous t-conorm:

(i) a $\diamond\, b = \max\{a, b\}, \forall a, b \in r_s(\cos\theta + i\sin\theta)$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$

(ii) a$\diamond b = \min(a + b, 1)$, for all $a, b \in r_s(\cos\theta + i\sin\theta)$ and a fix $\theta \in \left[0, \frac{\pi}{2}\right]$.

Definition 2.5: The 5-triplet (X, M, N, $*, \diamond$) is said to be Complex Valued Intuitionistic Fuzzy Metric Space if X is an arbitrary non empty set, $*$ is a complex valued continuous t-norm, $\diamond$ is a complex valued continuous t-conorm and $M, N : X \times X \times (0, \infty) \to r_s(\cos\theta + i\sin\theta)$ are complex valued fuzzy sets, where $r_s \in [0,1], r_s(\cos\theta + i\sin\theta)$ are complex valued fuzzy sets, where $r_s \in$ and $\theta \in \left[0, \frac{\pi}{2}\right]$, satisfying the following conditions:

for all x, y, z $\in X$; t, s $\in (0, \infty)$; $r_s \in [0,1]$ and $\theta \in \left[0, \frac{\pi}{2}\right]$.

(cf1) $M(a,b,p) + M(a,b,p) \le (\cos\theta + i\sin\theta)$,

(cf2) $M(a,b,p) > 0$,

(cf3) $M(a,b,p) = (\cos\theta + i\sin\theta)$, for all $p \in (0,\infty)$ if and only if a = b,

(cf4) $M(a,b,p) = M(b,a,p)$,

(cff) $M(a,b,p+s) \ge M(a,c,p) * M(c,b,s)$,

(cf6) $M(a,b,p) : (0,\infty) \to r_s(\cos\theta + i\sin\theta)$ is continuous,

(cf7) $N(a,b,p) < (\cos\theta + i\sin\theta)$,

(cf8) $N(a,b,p) = 0$, for all $p \in (0,\infty)$ if and only if a = b,

(cf9) $N(a,b,p) = N(b,a,p)$,

(cf10) $N(a,b,p+s) \le N(a,c,p) \diamond N(c,b,s)$,

(cf11) $N(a,b,p) : (0,\infty) \to r_s(\cos\theta + i\sin\theta)$ is continuous,

The pair (M, N) is called a Complex Valued Intuitionistic Fuzzy Metric Space. The functions $M(a,b,p)$ and $N(a,b,p)$ denotes the degree of nearness and non-nearness between a and b with respect to t. It is noted that if we take $\theta = 0$, then complex valued intuitionistic fuzzy metric simply goes to real valued intuitionistic fuzzy metric.

3. Main Result

Theorem 3.1. Let (X, M, N, $*, \diamond$) be a Complex Valued Intuitionistic Fuzzy Metric Space with $\lim_{t\to\infty} M(a,b,p) = (\cos\theta + i\sin\theta)$ and $\lim_{t\to\infty} N(a,b,p) = 0$, for all $a, b \in X, p > 0$ and let A and B be self mappings on X. If there exists $d \in (0,1)$ such tha $M(Aa, Bb, dp) \ge M(a,b,p)$ and $N(Aa, Bb, dp) \le N(a,b,p)$ for all $a, b \in X$ and for all $p > 0, \ldots\ldots..(3.1)$ then A and B have a unique common fixed point in X.

Proof. Let $a_0 \in X$ be an arbitrary point and we define the sequence $\{a_n\}$ by $a_{2n+1} = Aa_{2n}$ and $a_{2n+2} = Ba_{2n+1}; n = 0, 1, 2, \ldots$ Now, for $d \in (0,1)$ and for all $p > 0$, then from (3.1) we have

$$M\left(a_{2n+1}, a_{2n+2}, dp\right) = M\left(Aa_{2n}, Ba_{2n+1}, dp\right) \ge M\left(a_{2n}, a_{2n+1}, p\right),$$

$M\left(a_{2n}, a_{2n+1}, dp\right)=M\left(Aa_{2n-1}, Ba_{2n}, dp\right) \geq M\left(a_{2n-1}, a_{2n}, p\right)$,
and
$N\left(a_{2n+1}, a_{2n+2}, dp\right)=N\left(Aa_{2n}, Ba_{2n+1}, dp\right) \leq N\left(a_{2n}, a_{2n+1}, p\right)$,
$N\left(a_{2n}, a_{2n+1}, dp\right)=N\left(Aa_{2n-1}, Ba_{2n}, dp\right) \leq N\left(a_{2n-1}, a_{2n}, p\right)$.

In general, we have $M\left(a_{n+1}, a_{n+2}, dp\right) \geq M\left(a_{n}, a_{n+1}, p\right)$ and
$N\left(a_{n+1}, a_{n+2}, dp\right) \leq N\left(a_{n}, a_{n+1}, p\right)$ for for all $p>0$ and $d \in(0,1) ; n=0,1,2, \ldots$...but $\{a_{n}\}$ be a sequence in a complex valued intuitionistic fuzzy metric space ($X, \mathrm{M}, \mathrm{N}, *, \diamond)$, with $\lim_{p \rightarrow \infty} M(a, b, p)=\cos \theta+\mathrm{i} \sin \theta$ and $\lim_{p \rightarrow \infty} N(a, b, p)=0, \forall a, b \in X$. If $\lim_{p \rightarrow 0} N(a, b, p)=0$, there exists $d \in(0,1)$ such that $M\left(a_{n+1}, a_{n+2}, dp\right) \geqslant M\left(a_{n}, a_{n+1}, p\right)$ and $N\left(a_{n+1}, a_{n+2}, dp\right) \leqslant\left(a_{n}, a_{n+1} p\right)$, for all $p>0$, then $\{a_{n}\}$ is a cauchy sequence in X.Since X is Complete then there exist $V \in X$ such that $a_{n} \rightarrow v$ as $n \rightarrow \infty$ and $\{a_{2n}\}$ and $\langle a_{2n+1}\}$ are subsequences of the same point $v \in X$, i.e. $a_{2n} \rightarrow v, a_{2n+1} \rightarrow v$, as $n \rightarrow \infty$. Now from eq (1) we have, $M(Av, v, dp)=M\left(Av, v, \frac{dp}{2}+\frac{dp}{2}\right)$

$$
\begin{aligned}
& \geqslant M\left(Au, a_{2n+2} \frac{dp}{2}\right) * M\left(a_{2n+2}, v, \frac{dp}{2}\right) \\
& =M\left(Au, Ba_{2n+1}, \frac{dp}{2}\right) * M\left(a_{2n+2}, v, \frac{dp}{2}\right) \\
& \geqslant M\left(v, a_{2n+1}, \frac{p}{2}\right) * M\left(a_{2n+2}, v, \frac{dp}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
& N(Av, v, dp)=N\left(Av, v, \frac{dp}{2}+\frac{dp}{2}\right) \\
& \leq N\left(Av, a_{2n+2}, \frac{dp}{2}\right) \diamond N\left(a_{2n+2}, v, \frac{dp}{2}\right) \\
& =N\left(Av, Ba_{2n+1}, \frac{dp}{2}\right) \diamond \mathrm{N}\left(q_{2n+2}, v, \frac{dp}{2}\right) \\
& \leq N\left(v, a_{2n+1}, \frac{p}{2}\right) \diamond\left(a_{2n+2}, v, \frac{dp}{2}\right)
\end{aligned}
$$

On taking limit $n \rightarrow \infty$

$$
\begin{aligned}
M(Av, v, dp) & \geqslant(\cos \theta+i \sin \theta) *(\cos \theta+i \sin \theta) \\
& =\cos \theta+i \sin \theta
\end{aligned}
$$

$$N(Av, v, dp) \quad \leqslant 0 \diamond 0 = 0$$

so $Av = v$; Again,

$$M(Av, v, dp) = M\left(v,\ \mathrm{B}v, \frac{dp}{2} + \frac{dp}{2}\right)$$

$$\geqslant M\left(v, a_{2n+1}, \frac{dp}{2}\right) * M\left(a_{2n+1},\ \mathrm{B}v, \frac{dp}{2}\right)$$

$$= M\left(v, a_{2n+1}, \frac{dp}{2}\right) * M\left(Aa_{2n},\ \mathrm{B}v, \frac{dp}{2}\right)$$

$$\geqslant M\left(v, a_{2n+1}, \frac{p}{2}\right) * M\left(a_{2n}, v, \frac{p}{2}\right)$$

$$N(Av, v, dp) = N\left(v,\ \mathrm{B}v, \frac{dp}{2} + \frac{dp}{2}\right)$$

$$\leq N\left(v, a_{2n+1}, \frac{dp}{2}\right) \diamond N\left(a_{2n+1},\ \mathrm{B}v, \frac{dp}{2}\right)$$

$$= N\left(v, a_{2n+1}, \frac{dp}{2}\right) \diamond N\left(Aa_{2n},\ \mathrm{B}v, \frac{dp}{2}\right)$$

$$\leq N\left(v, a_{2n+1}, \frac{p}{2}\right) \diamond N\left(a_{2n}, v, \frac{p}{2}\right)$$

On taking limit $n \to \infty$

$$M(Av, v, dp) \geqslant (\cos\theta + i\sin\theta) * (\cos\theta + i\sin\theta)$$
$$= \cos\theta + i\sin\theta$$

$$N(Av, v, dp) \quad \leqslant 0 \diamond 0 = 0$$

so $Bv = v$, and $Av = Bv = v$. Hence v is a common fixed point of A and B. For uniqueness let c be any another fixed point of A and B. Now from (1),

$M(v, c, dp) = M(Av, Bc, dp) \geq M(v, c, p)$ and $N(v, c, dp) = N(Av, Bc, dp) \leq N(v, c, p)$.

we know that when $(X, \mathrm{M}, \mathrm{N}, *, \diamond)$ be a complex valued intuitionistic fuzzy metric space such that $\lim_{p\to\infty} M(a, b, p) = \cos\theta + \mathrm{i}\sin\theta$ and $\lim_{p\to\infty} N(a, b, p) = 0, \forall a, b \in X$. If $M(a, b, dp) \geq M(a, b, p)$ and $N(a, b, dp) \leq N(a, b, p)$ for some $0 < d < 1$, for all $a, b \in X, p \in (0, \infty)$, then $a = b$. Hence $v = c$

References

[1] K. Atanassov, *on Intuitionistic Fuzzy Sets*,Fuzzy sets and systems, **20** (1986), 87-96 .

[2] A. Azam, B. Fisher and M. Khan, *on Common fixed point theorems in complex valued metric space*, Numerical Functional Analysis and Optimization, **32**(2011), 243-253.
[3] J. J. Buckley, *on Fuzzy complex numbers*, Fuzzy Sets and System, **33** (1989), 333-345.
[4] A. George and P. Veeramani, *On some results in fuzzy metric spaces*, Fuzzy sets and systems, **64** (1994), 395-399.
[5] G. Jungck, *Compatible mappings and common fixed points*,Internat.Math. J. Maths. Sci., **9** (1986), 771 - 779.
[6] I. Kramosil and J. Michalek, *On Fuzzy metric and statistical spaces*, Kybernetica, **11**(1975), 336-344.
[7] J. H. Park, *On Intuitionstic fuzzy metric spaces*, Chaos, Solitons Fractals, **22**(2004), 1039-1046.
[8] D. Ramot, R. Milo, M. Friedman, and A. Kandel,*Complex fuzzy sets*, IEEE Transactions of Fuzzy System, **10** (2002) $405-413$
[9] R. Saadati, J. H. Park, *On the intuitionstic fuzzy topological spaces*, Chaos Solitons Fractals, **27** (2006), 331 - 344 .
[10] D. Singh, V. Joshi, M. Imdad, P. Kumar, *A novel framework of complex valued fuzzy metric spaces fixed point theorems* , **30(6)**(2016), 3227 - 3238.
[11] Uday Dolas, *A common fixed point theorem in fuzzy metric spaces using common E.A. Like property* Journal of Research in Applied Mathematics, **2(6)**(2018), 245 250.
[12] L. A. Zadeh, *Fuzzy sets* Inform. and Control, **8**(1965), 338 - 353.
[13] M.Jeyaraman, V.B. Shukila, *Some common fixed point theorems in complex valued Intuitionistic Fuzzy metric Spaces* International Journal of Scientific Research In Mathematical and Statistical Sciences **9**(2022).

Ram Milan Singh
Department of Mathematics, Govt. P. G. college, Tikamgarh , P.O.Box 472001, Tikamgarh, India
Email: `rammilansinghlig@gmail.com`