

VAGUE SOFT R-SUBGROUPS AND VAGUE SOFT QUOTIENT NEAR-RING

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ABSTRACT. The purpose of the present paper is to throw light on the study of vague soft R-subgroup over near-ring R . We have defined vague soft R-subgroup over a near-ring. By using quotient near-ring we have defined vague soft quotient near-ring. Also, we have investigated operations of vague soft R-subgroup over a near-ring and vague soft quotient near-ring.

Key Words: Vague set, Soft set, Soft R-subgroup, Vague Soft Set, Vague soft R-subgroup, Vague soft quotient near-ring,

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1. INTRODUCTION

The soft set theory is rapidly growing the branch of Mathematics. Firstly, in 1999 Molodtsov ([3]) initiated the concept of a soft set. A soft set over U represents a parametrized family of subsets of the universe U . In addition, many other researchers ([15], [5], [1]) started working on this. At initial stage, some basic algebraic properties like union, intersection and complementation are stated along with its corresponding properties are discussed in detail. L. A. Zadeh ([13]) introduced fuzzy sets, as a modern mathematical tool for dealing with uncertainties. Later on, in 2001 Maji *et.al.* ([15]) combined fuzzy sets and soft

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sets and defined fuzzy soft set. This work is put forward by ([9]) defining fuzzy soft group, fuzzy soft ring and fuzzy soft ideals.

Gau and Baehrer ([19]) introduced a new concept called vague sets. Vague sets are involved two membership functions one is truth value-function and another is a false-value function. Hence fuzzy set attains a particular case of a vague set. Xu *et.al.* ([18]) discussed properties of it. G. Pilz ([7]) introduced near-rings from a ring with any one of the distributive property. So earlier research done for ring theory applied for near-rings. Fuzzy soft near-rings and vague soft near-rings defined and its properties are discussed in ([8],[16]). By using the concept of R-subgroup of a near ring Kyung Ho Kim and Young Bae Jun ([11]) carried out research on fuzzy R-subgroups of near-ring. Same authors ([12]) achieved other results. Same author introduced normal fuzzy R-subgroups and briefly investigated corresponding properties. Kim and Jun ([10]) generalized the concept of fuzzy R-subgroup and defined vague R-subgroup of near-ring. It has a huge applications in many areas of mathematics. Many researchers has contributed to operations of vague set, vague group ([4]), vague ideals of a ring, vague field and vector spaces ([17]) and vague R-subgroups ([10]) etc. Along with this soft set theory can be applied to vague set theory. Hence we can define vague soft set. Initially the properties of vague soft set are studied by Wei Xu, Jain Ma, Shouyang Wang, Gang Hao in ([18]). In ([16]) vague soft near-ring and its ideals are introduced.

So we continued with a vague soft set and near-ring. In the present paper, we define vague soft R-subgroup over a near-ring with the example. As well, some of its algebraic properties are stated.

The paper is organized in following way, section 2 present required preliminaries, results are discussed in section 3 and section 4. Section 5 concludes the paper.

2. PRELIMINARIES

Some of the basic definitions are listed below for further discussion.

Definition 2.1. ([7]) By a near-ring we mean a non-empty set R with two binary operations ‘+’ and ‘.’ satisfying the following axioms:

- (i) $(R, +)$ is a group,
- (ii) (R, \cdot) is a semi group,
- (iii) $x \cdot (y + z) = x \cdot y + x \cdot z \quad \forall x, y, z \in R.$

In other words, near-ring means it has same structure like a ring with exactly one distributive law. We will use the word near-ring instead of left near-ring. We replace $x \cdot y$ by xy . Note that $x0 = 0$ and $x(-y) = -xy$, but $0x \neq 0$ for $x, y \in R$.

Definition 2.2. A two sided R-subgroup over a near-ring R is a subset H of R such that

- (i) $(H, +)$ is a subgroup of $(R, +)$,
- (ii) $RH \subset H$,
- (iii) $HR \subset H$.

If H satisfies (i) and (ii) then it is called a left R-subgroup over R . If H satisfies (i) and (iii) then it is called a right R-subgroup over R .

Definition 2.3. ([3], [5]) Let U be an initial universe set and E be the set of parameters. Let A be a subset of E . Let $P(U)$ denote the power set of U . A pair (F, A) is called a soft set over U , where F is a mapping given by $F : A \rightarrow P(U)$. For each $x \in A$, $F(x)$ is the set of x -approximate elements of the soft set (F, A) . A soft set (F, E) over U is also denoted by a triple (F, E, U) .

Definition 2.4. ([5]) Let (F, A) and (G, B) be a two soft sets over a universe U . Then restricted intersection of two soft sets is denoted as $(F, A) \cap (G, B) = (H, C)$, where $C = A \cap B$ and defined as,

$$H(c) = F(c) \cap G(c) \quad \forall c \in C$$

Definition 2.5. (F, A) is called a soft R-subgroup of R if F_a is R-subgroup of R . i.e

- (i) $x - y \in F_a$,
- (ii) $rx \in F_a$,
- (iii) $xr \in F_a$. for all $x, y \in F_a$ and $r \in R$.

Definition 2.6. ([19]) Let U be universal set, $U = \{u_1, u_2, u_3, \dots, u_n, \dots\}$. A vague set over U is characterized by a truth-membership function t_v and a false-membership function f_v , $t_v : U \rightarrow [0, 1]$, $f_v : U \rightarrow [0, 1]$, where $t_v(u_i)$ is lower bound on a grade of membership function u_i derived from evidence of u_i , $f_v(u_i)$ is upper bound on the negation of u_i derived from evidence against u_i and $t_v(u_i) + f_v(u_i) \leq 1$. The grade membership of u_i in the vague set is bounded to a subinterval $[t_v(u_i), 1 - f_v(u_i)]$ of $[0, 1]$. The vague value $[t_v(u_i), 1 - f_v(u_i)]$ indicates that the exact grade of membership $\mu_v(u_i)$ of u_i may be unknown, but it is bounded by $t_v(u_i) \leq \mu_v(u_i) \leq 1 - f_v(u_i)$.

Definition 2.7. ([18]) The intersection of two vague soft sets (\hat{F}, A) and (\hat{G}, B) over a universe U is a vague soft set (\hat{H}, C) which is denoted by $(\hat{F}, A) \tilde{\cap} (\hat{G}, B) = (\hat{H}, C)$ where $C = A \cup B$ and for all $c \in C, x \in U$,

$$\begin{aligned}\hat{H}_c(x) &= \hat{F}_c(x) && \text{if } c \in A - B \\ &= \hat{G}_c(x) && \text{if } c \in B - A \\ &= \hat{F}_c(x) \cap \hat{G}_c(x) && \text{if } c \in A \cap B\end{aligned}$$

Definition 2.8. ([18]) If (\hat{F}, A) and (\hat{G}, B) be two vague soft sets over the universe X then $(\hat{F}, A) \text{AND} (\hat{G}, B)$ is a vague soft set denoted by $(\hat{F}, A) \wedge (\hat{G}, B)$ and is defined by,
 $(\hat{F}, A) \wedge (\hat{G}, B) = (\hat{H}, A \times B)$
 where

$$t_{\hat{H}(a,b)}(x) = \min(t_{\hat{F}_a}(x), t_{\hat{G}_b}(x))$$

and

$$1 - f_{\hat{H}(a,b)}(x) = \min(1 - f_{\hat{F}_a}(x), 1 - f_{\hat{G}_b}(x))$$

for all $(a, b) \in A \times B$ and $x \in X$.

Definition 2.9. ([18]) If (\hat{F}, A) and (\hat{G}, B) be two vague soft sets over the universe U then restricted union of these two sets is denoted by $(\hat{H}, C) = (\hat{F}, A) \cap_R (\hat{G}, B)$ and defined by

$$t_{\hat{H}_c}(x) \geq \min(t_{\hat{F}_c}(x), t_{\hat{G}_c}(x)) \text{ and } 1 - f_{\hat{H}_c}(x) \geq \min(1 - f_{\hat{F}_c}(x), 1 - f_{\hat{G}_c}(x))$$

where $c \in C = A \cap B \neq \phi$ and $x \in U$.

That is, $\hat{H}_c(x) = \hat{F}_c(x) \cap \hat{G}_c(x)$.

Definition 2.10. ([6]) Let (\hat{F}, A) be a vague soft set over R . Let $\phi : R \rightarrow R$ be a map. Then we define,

$$\hat{F}^\phi(x) = \hat{F}(\phi(x)).$$

That is, $t_a^\phi(x) = t_a(\phi(x))$ and $1 - f_a^\phi(x) = 1 - f_a(\phi(x))$ for all $a \in A$.

(\hat{F}^ϕ, A) is also vague soft set over R .

Definition 2.11. ([6]) Let (\hat{F}, A) be a vague soft set over R . Then for every $\alpha, \beta \in [0, 1]$, where $\alpha \leq \beta$, (α, β) -cut or the vague soft cut of (\hat{F}, A) is a subset of R which is defined as follows:

$$(\hat{F}, A)_{(\alpha, \beta)} = \left\{ x \in R \mid t_{\hat{F}_a} \geq \alpha, 1 - f_{\hat{F}_a} \geq \beta, e_{\hat{F}_a}(x) \geq [\alpha, \beta] \right\}$$

For every $a \in A$.

Definition 2.12. [16] Let R be a near-ring and (\hat{F}, A) be a non-null vague soft set over R . Then (\hat{F}, A) is called vague soft near-ring if and only if for $a \in A$ and $x, y \in R$

$$(i) \hat{F}_a(x + y) \geq \min \left\{ \hat{F}_a(x), \hat{F}_a(y) \right\},$$

$$(ii) \hat{F}_a(-x) \geq (\hat{F}_a(x)),$$

$$(iii) \hat{F}_a(xy) \geq \min \left\{ \hat{F}_a(x), \hat{F}_a(y) \right\}$$

Definition 2.13. [16] Let R be a near ring. Let (\hat{I}, A) be a non-null vague soft set over R . Then (\hat{I}, A) is called vague soft ideal over R if and only if for each $a \in A$ and $x, y, i \in R$ the following conditions hold:

(i) (\hat{I}, A) is vague soft near-ring over R ,

$$(ii) \hat{I}_a[(x + i)y - xy] \geq \hat{I}_a(i),$$

$$(iii) \hat{I}_a(xy) \geq \hat{I}_a(y).$$

If \hat{I}_a satisfies (i), (ii) then it is called vague soft right ideal over R and if it satisfies (i), (iii) then it is called vague soft left ideal over R .

3. VAGUE SOFT R-SUBGROUP

Definition 3.1. Let $(R, +, \cdot)$ be a near-ring, E be the set of parameters and $A \subseteq E$. let (\hat{F}, A) be a non-null vague soft set over R . Then (\hat{F}, A) is called a vague soft R-subgroup over R if and only if for each $a \in A$, we have,

$$(i) \hat{F}_a(x - y) \geq \min \left\{ \hat{F}_a(x), \hat{F}_a(y) \right\},$$

$$\text{i.e } t_{\hat{F}_a}(x - y) \geq \min \left(t_{\hat{F}_a}(x), t_{\hat{F}_a}(y) \right) \text{ and}$$

$$1 - f_{\hat{F}_a}(x - y) \geq \min \left(1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y) \right)$$

$$(ii) \hat{F}_a(xr) \geq \hat{F}_a(x),$$

$$\text{i.e } t_{\hat{F}_a}(xr) \geq t_{\hat{F}_a}(x) \text{ and } 1 - f_{\hat{F}_a}(xr) \geq 1 - f_{\hat{F}_a}(x)$$

$$(iii) \hat{F}_a(rx) \geq \hat{F}_a(x).$$

$$\text{i.e } t_{\hat{F}_a}(rx) \geq t_{\hat{F}_a}(x) \text{ and } 1 - f_{\hat{F}_a}(rx) \geq 1 - f_{\hat{F}_a}(x)$$

If \hat{F}_a satisfies (i) and (ii) then it is called a vague soft right R-subgroup over R and if it satisfies (i) and (iii) then it is called a vague soft left R-subgroup of R . Here \hat{F}_a is a vague subset of R corresponding to the parameter $a \in A$.

Example 3.2. Let $R = \{a, b, c, d\}$ be a near-ring defined as follows:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	b	c	d

Let $A = \{e_1, e_2\}$ be a subset of set of parameters.
 Consider a vague soft set (\hat{F}, A) over a near-ring R given by,
 $\hat{F}(e_1) = \{[0.6, 0.4]/a, [0.4, 0.4]/b, [0.2, 0.3]/c, [0.2, 0.3]/d\}$
 $\hat{F}(e_2) = \{[0.4, 1]/a, [0.2, 0.9]/b, [0.3, 0.5]/c, [0.3, 0.5]/d\}$.
 Then (\hat{F}, A) is a vague soft R-subgroup over a near-ring R .

Theorem 3.3. *If (\hat{F}, A) is a vague soft R-subgroup over a near-ring R , then $\hat{F}_a(0) \geq \hat{F}_a(x)$ for each $a \in A$ and $x \in R$.*

Proof. Let (\hat{F}, A) be a vague soft R-subgroup over a near-ring R . Let $a \in A$ and $x, y \in R$.
 Then,

$$\begin{aligned} \hat{F}_a(0) &= \hat{F}_a(x - x) \\ &\geq \min(\hat{F}_a(x), \hat{F}_a(x)) \\ &= \hat{F}_a(x) \end{aligned}$$

Thus $\hat{F}_a(0) \geq \hat{F}_a(x)$ □

Theorem 3.4. *Let (\hat{F}, A) and (\hat{G}, B) be two vague soft R-subgroup over a near-ring R . Then $(\hat{F}, A) \wedge (\hat{G}, B)$ is vague soft R-subgroup over R .*

Proof. Let $(\hat{F}, A) \wedge (\hat{G}, B) = (\hat{H}, A \times B)$
 Then for all $(a, b) \in A \times B$ and $x, y, r \in R$ we have,

$$\begin{aligned} \hat{H}_{(a,b)}(x - y) &= \hat{F}_a(x - y) \cap \hat{G}_b(x - y) \\ &= \min \left[\hat{F}_a(x - y), \hat{G}_b(x - y) \right] \\ &\geq \min \left[\min \left\{ \hat{F}_a(x), \hat{F}_a(y) \right\}, \min \left\{ \hat{G}_b(x), \hat{G}_b(y) \right\} \right] \\ &\geq \min \left[\min \left\{ \hat{F}_a(x), \hat{G}_b(x) \right\}, \min \left\{ \hat{F}_a(y), \hat{G}_b(y) \right\} \right] \end{aligned}$$

Hence $\hat{H}_{(a,b)}(x - y) \geq \min \left(\hat{H}_{(a,b)}(x), \hat{H}_{(a,b)}(y) \right)$ for all $x, y \in R$

$$\begin{aligned} H_{(a,b)}(\hat{r}x) &= \hat{F}_a(\hat{r}x) \wedge \hat{G}_b(\hat{r}x) \\ &= \min \left[\hat{F}_a(\hat{r}x), \hat{G}_b(\hat{r}x) \right] \\ &\geq \min \left[\hat{F}_a(x), \hat{G}_b(x) \right] \\ &= \hat{H}_{(a,b)}(x) \end{aligned}$$

Hence $\hat{H}_{(a,b)}(\hat{r}x) \geq \hat{H}_{(a,b)}(x)$ for all $x, r \in R$.

$$\begin{aligned} H_{(a,b)}(\hat{r}x) &= \hat{F}_a(\hat{r}x) \wedge \hat{G}_b(\hat{r}x) \\ &= \min \left[\hat{F}_a(\hat{r}x), \hat{G}_b(\hat{r}x) \right] \\ &\geq \min \left[\hat{F}_a(x), \hat{G}_b(x) \right] \\ &= \hat{H}_{(a,b)}(x) \end{aligned}$$

Hence $\hat{H}_{(a,b)}(\hat{r}x) \geq \hat{H}_{(a,b)}(x)$ for all $x, r \in R$.

Hence $(\hat{F}, A) \wedge (\hat{G}, B)$ is a vague soft R-subgroup over R . \square

Theorem 3.5. Let (\hat{F}, A) and (\hat{G}, B) be two vague soft ideals over a near-ring R . Then $(\hat{F}, A) \tilde{\cap} (\hat{G}, B)$ is vague soft R-subgroup over R .

Proof. Let $(\hat{F}, A) \tilde{\cap} (\hat{G}, B) = (\hat{H}, C)$ where $C = A \in B$ and for all $c \in C, x \in R$,

$$\begin{aligned} \hat{H}_c(x) &= \hat{F}_c(x) && \text{if } c \in A - B \\ &= \hat{G}_c(x) && \text{if } c \in B - A \\ &= \hat{F}_c(x) \cap \hat{G}_c(x) && \text{if } c \in A \cap B \end{aligned}$$

Case1: If $c \in A - B$ then $\hat{H}_c(x) = \hat{F}_c(x)$ and since (\hat{F}, A) is a vague soft R-subgroup over R , (\hat{H}, C) is a vague soft R-subgroup over R .

Case2: If $c \in B - A$ then $\hat{H}_c(x) = \hat{G}_c(x)$ and since (\hat{G}, B) is a vague soft R-subgroup over R , (\hat{H}, C) is a vague soft R-subgroup over R .

Case3: If $c \in A \cap B$ then $\hat{H}_c(x) = \hat{F}_c(x) \cap \hat{G}_c(x)$ we have,

$$\begin{aligned}\hat{H}_c(x-y) &= \hat{F}_c(x-y) \cap \hat{G}_c(x-y) \\ &= \min \left[\hat{F}_c(x-y), \hat{G}_c(x-y) \right] \\ &\geq \min \left[\min \left\{ \hat{F}_c(x), \hat{F}_c(y) \right\}, \min \left\{ \hat{G}_c(x), \hat{G}_c(y) \right\} \right] \\ &\geq \min \left[\min \left\{ \hat{F}_c(x), \hat{G}_c(x) \right\}, \min \left\{ \hat{F}_c(y), \hat{G}_c(y) \right\} \right]\end{aligned}$$

Hence $\hat{H}_c(x-y) \geq \min \left(\hat{H}_c(x), \hat{H}_c(y) \right)$ for all $x, y \in R$

$$\begin{aligned}\hat{H}_c(rx) &= \hat{F}_c(rx) \cap \hat{G}_c(rx) \\ &= \min \left[\hat{F}_c(rx), \hat{G}_c(rx) \right] \\ &\geq \min \left[\hat{F}_c(x), \hat{G}_c(x) \right] \\ &= \hat{H}_c(x)\end{aligned}$$

Hence $\hat{H}_c(rx) \geq \min \left(\hat{H}_c(x), \hat{H}_c(x) \right)$ for all $x, r \in R$

$$\begin{aligned}\hat{H}_c(xr) &= \hat{F}_c(xr) \cap \hat{G}_c(xr) \\ &= \min \left[\hat{F}_c(xr), \hat{G}_c(xr) \right] \\ &\geq \min \left[\hat{F}_c(x), \hat{G}_c(x) \right] \\ &= \hat{H}_c(x)\end{aligned}$$

Hence $\hat{H}_c(xr) \geq \min \left(\hat{H}_c(x), \hat{H}_c(x) \right)$ for all $x, r \in R$.

Hence $(\hat{F}, A) \tilde{\cap} (\hat{G}, B)$ is a vague soft R-subgroup over R . \square

Theorem 3.6. Let (\hat{F}, A) and (\hat{G}, B) be two vague soft R-subgroups over R with $A \cap B \neq \phi$. Then the restricted intersection $(\hat{F}, A) \cap_R (\hat{G}, B)$ of (\hat{F}, A) and (\hat{G}, B) is a vague soft R-subgroup over R .

Proof. Let (\hat{F}, A) and (\hat{G}, B) be two vague soft R-subgroups over R .

Let $(\hat{H}, C) = (\hat{F}, A) \cap_R (\hat{G}, B)$ and $C = A \cap B \neq \phi$.

Let any $x, y, r \in R$ and $c \in C$,

Consider,

$$\begin{aligned}\hat{H}_c(x-y) &= \hat{F}_c(x-y) \cap \hat{G}_c(x-y) \\ &= \min \left[\hat{F}_c(x-y), \hat{G}_c(x-y) \right] \\ &\geq \min \left[\min \left\{ \hat{F}_c(x), \hat{F}_c(y) \right\}, \min \left\{ \hat{G}_c(x), \hat{G}_c(y) \right\} \right] \\ &\geq \min \left[\min \left\{ \hat{F}_c(x), \hat{G}_c(x) \right\}, \min \left\{ \hat{F}_c(y), \hat{G}_c(y) \right\} \right]\end{aligned}$$

Hence $\hat{H}_c(x-y) \geq \min \left(\hat{H}_c(x), \hat{H}_c(y) \right)$ for all $x, y \in R$.

$$\begin{aligned}\hat{H}_c(rx) &= \hat{F}_c(rx) \cap \hat{G}_c(rx) \\ &= \min \left[\hat{F}_c(rx), \hat{G}_c(rx) \right] \\ &\geq \min \left[\hat{F}_c(x), \hat{G}_c(x) \right] \\ &= \hat{H}_c(x)\end{aligned}$$

Hence $\hat{H}_c(rx) \geq \hat{H}_c(x)$ for all $x, r \in R$.

$$\begin{aligned}\hat{H}_c(xr) &= \hat{F}_c(xr) \cap \hat{G}_c(xr) \\ &= \min \left[\hat{F}_c(xr), \hat{G}_c(xr) \right] \\ &\geq \min \left[\hat{F}_c(x), \hat{G}_c(x) \right] \\ &= \hat{H}_c(x)\end{aligned}$$

Hence $\hat{H}_c(xr) \geq \hat{H}_c(x)$ for all $x, r \in R$.

Hence $(\hat{F}, A) \cap_R (\hat{G}, B)$ is a vague soft R-subgroup over R . \square

Theorem 3.7. *Let (\hat{F}, A) be a vague soft R-subgroup over R . Then for any $\alpha, \beta \in [0, 1]$, the vague soft cut of (\hat{F}, A) is a soft R-subgroup over R .*

Proof. Let $a \in A$ and $x, y \in \hat{F}_{a(\alpha, \beta)}$. Then $t_{\hat{F}_a}(x) \geq \alpha, t_{\hat{F}_a}(y) \geq \alpha, 1 - f_a(x) \geq \beta$ and $1 - f_a(y) \geq \beta$.

$$\begin{aligned}t_{\hat{F}_a}(x-y) &\geq \min \left\{ t_{\hat{F}_a}(x), t_{\hat{F}_a}(y) \right\} \geq \min \{ \alpha, \alpha \} = \alpha \text{ and } 1 - f_{\hat{F}_a} \geq \\ &\min \left\{ 1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y) \right\} \geq \min \{ \beta, \beta \} = \beta.\end{aligned}$$

Therefore $x-y \in \hat{F}_{a(\alpha, \beta)}$.

Now let $r \in R$. Thus we have, $t_{\hat{F}_a}(rx) \geq t_{\hat{F}_a}(x) \geq \alpha$ and $1 - f_{\hat{F}_a}(rx) \geq 1 - f_{\hat{F}_a}(x) \geq \beta$.

Therefore $rx \in \hat{F}_{a(\alpha, \beta)}$.

Similarly $xr \in \hat{F}_{a(\alpha, \beta)}$.

Hence $(\hat{F}, A)_{(\alpha, \beta)}$ is a soft R-subgroup over R . □

Theorem 3.8. *Let (\hat{F}, A) be a vague soft R-subgroup over R and ϕ be a homomorphism of R . Then (\hat{F}^ϕ, A) is a vague soft R-subgroup over R .*

Proof. Let (\hat{F}, A) be a vague soft R-subgroup over R . Let $a \in A$ and $x, y \in R$.

$$\begin{aligned} \hat{F}_a^\phi(x - y) &= \hat{F}_a(\phi(x - y)) \\ &= \hat{F}_a[\phi(x) - \phi(y)] \\ &\geq \min [\hat{F}_a(\phi(x)), \hat{F}_a(\phi(y))] \\ &= \min [\hat{F}_a^\phi(x), \hat{F}_a^\phi(y)] \end{aligned}$$

Also we have,

$$\begin{aligned} \hat{F}_a^\phi(xr) &= \hat{F}_a(\phi(xr)) \\ &= \hat{F}_a[\phi(x)\phi(r)] \\ &\geq \hat{F}_a(\phi(x)) \\ &= \hat{F}_a^\phi(x) \end{aligned}$$

Similarly, $\hat{F}_a^\phi(rx) \geq \hat{F}_a^\phi(x)$.

Hence (\hat{F}^ϕ, A) is a vague soft R-subgroup over R . □

4. VAGUE SOFT QUOTIENT NEAR-RING

Definition 4.1. Let I be an ideal of a near-ring R and (\hat{F}, A) be a vague soft set over a near-ring R . Then we define for each $a \in A, n \in R$ vague soft set $(\hat{F}, A)|_I$ as follows:

(i) $\hat{t}_a : R/I \rightarrow [0, 1]$ such that $\hat{t}_a(n + I) = \inf_{i \in I} \hat{t}_a(n + i)$

(ii) $\hat{f}_a : R/I \rightarrow [0, 1]$ such that $1 - \hat{f}_a(n + I) = \inf_{i \in I} 1 - \hat{f}_a(n + i)$
 Condition (i) and (ii) together we denote as $\hat{F}_a(n + I) = \inf_{i \in I} \hat{F}_a(n + i)$

4.1. **Example.** Let $R = \{0, a, b, c\}$ be a non-empty set with two binary operations '+' and '.' defined as follows:

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	b
c	a	a	a	c

Then R is a near-ring.

Let $A = \{e_1, e_2\}$ be a subset of set of parameters E .

Let $I = \{a, 0\}$ be an ideal of R .

Therefore $R/I = \{I, b + I, c + I\}$

$$\begin{aligned}\hat{t}_{e_1}(a + I) &= \hat{t}_{e_1}(I) \\ &= \inf \{ \hat{t}_{e_1}(a), \hat{t}_{e_1}(0) \} \\ &= \inf \{ 0.2, 0.2 \} \\ &= 0.2\end{aligned}$$

$$\begin{aligned}1 - \hat{f}_{e_1}(a + I) &= 1 - \hat{f}_{e_1}(I) \\ &= \inf \{ 1 - \hat{f}_{e_1}(a), 1 - \hat{f}_{e_1}(c) \} \\ &= \inf \{ 0.9, 0.9 \} \\ &= 0.9\end{aligned}$$

Similarly, $\hat{t}_{e_2}(a + I) = 0.4$ and $1 - \hat{f}_{e_2}(a + I) = 1$ Again,

$$\begin{aligned}\hat{t}_{e_1}(b + I) &= \inf \{ \hat{t}_{e_1}(b + 0), \hat{t}_{e_1}(b + a) \} \\ &= \inf \{ \hat{t}_{e_1}(b), \hat{t}_{e_1}(c) \} \\ &= \inf \{ 0.1, 0.1 \} \\ &= 0.1\end{aligned}$$

$$\begin{aligned}1 - \hat{f}_{e_1}(b + I) &= \inf \{ 1 - \hat{f}_{e_1}(b + 0), 1 - \hat{f}_{e_1}(b + a) \} \\ &= \inf \{ 1 - \hat{f}_{e_1}(b), 1 - \hat{f}_{e_1}(c) \} \\ &= \inf \{ 0.8, 0.8 \} \\ &= 0.8\end{aligned}$$

Similarly, $\hat{t}_{e_2}(b + I)=0.1$ and $1 - \hat{f}_{e_1}(b + I) = 0.8$
 Again,

$$\begin{aligned} \hat{t}_{e_1}(c + I) &= \inf \{ \hat{t}_{e_1}(c + 0), \hat{t}_{e_1}(c + a) \} \\ &= \inf \{ \hat{t}_{e_1}(c), \hat{t}_{e_1}(b) \} \\ &= \inf \{ 0.1, 0.1 \} \\ &= 0.1 \end{aligned}$$

$$\begin{aligned} 1 - \hat{f}_{e_1}(c + I) &= \inf \{ 1 - \hat{f}_{e_1}(c + 0), 1 - \hat{f}_{e_1}(c + a) \} \\ &= \inf \{ 1 - \hat{f}_{e_1}(c), 1 - \hat{f}_{e_1}(b) \} \\ &= \inf \{ 0.8, 0.8 \} \\ &= 0.8 \end{aligned}$$

Similarly, $\hat{t}_{e_2}(c + I)=0.3$ and $1 - \hat{f}_{e_1}(c + I) = 0.5$
 Hence vague soft quotient near-ring is given by,

\hat{F}	e_1	e_2
I	[0.2, 0.]	[0.4, 1]
a+I	[0.1, 0.8]	[0.1, 0.8]
b+I	[0.1, 0.8]	[0.3, 0.5]

4.2. Example. Let $R = \{0, 1, 2, 3\}$ be a non-empty set with two binary operations ‘+’ and ‘.’ defined as follows:

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

.	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	0	0	0	2
3	1	1	1	3

Then R is a near-ring.

Let $A = \{e_1, e_2, e_3, e_4\}$ be a subset of set of parameters E.

Define a vague soft set (\hat{F}, A) over a near-ring R as follows:

\hat{F}	e_1	e_2	e_3	e_4
0	[0.8, 0.9]	[0.7, 0.8]	[0.9, 1]	[0.5, 0.4]
1	[0.8, 0.9]	[0.7, 0.8]	[0.9, 1]	[0.5, 0.4]
2	[0.6, 0.7]	[0.5, 0.5]	[0, 0.4]	[0.3, 0.4]
3	[0.6, 0.7]	[0.5, 0.5]	[0, 0.4]	[0.3, 0.4]

Then (\hat{F}, A) is a vague soft near-ring.

Let $I = \{0, 2\}$ be an ideal of R . Therefore $R/I = \{I, 1 + I, 3 + I\}$

$$\begin{aligned}\hat{t}_{e_1}(I) &= \inf \{\hat{t}_{e_1}(0), \hat{t}_{e_1}(2)\} \\ &= \inf \{0.8, 0.6\} \\ &= 0.6\end{aligned}$$

$$\begin{aligned}1 - \hat{f}_{e_1}(I) &= \inf \{1 - \hat{f}_{e_1}(0), 1 - \hat{f}_{e_1}(2)\} \\ &= \inf \{0.9, 0.7\} \\ &= 0.7\end{aligned}$$

Similarly,

$$\begin{aligned}\hat{t}_{e_1}(1 + I) &= \inf \{\hat{t}_{e_1}(1 + 0), \hat{t}_{e_1}(1 + 2)\} \\ &= \inf \{\hat{t}_{e_1}(1), \hat{t}_{e_1}(3)\} \\ &= \inf \{0.8, 0.6\} \\ &= 0.6\end{aligned}$$

$$\begin{aligned}1 - \hat{f}_{e_1}(1 + I) &= \inf \{1 - \hat{f}_{e_1}(1 + 0), 1 - \hat{f}_{e_1}(1 + 2)\} \\ &= \inf \{1 - \hat{f}_{e_1}(1), 1 - \hat{f}_{e_1}(3)\} \\ &= \inf \{0.9, 0.7\} \\ &= 0.7\end{aligned}$$

$$\begin{aligned}\hat{t}_{e_1}(3 + I) &= \inf \{\hat{t}_{e_1}(3 + 0), \hat{t}_{e_1}(3 + 2)\} \\ &= \inf \{\hat{t}_{e_1}(3), \hat{t}_{e_1}(1)\} \\ &= \inf \{0.6, 0.8\} \\ &= 0.6\end{aligned}$$

$$\begin{aligned}1 - \hat{f}_{e_1}(3 + I) &= \inf \{1 - \hat{f}_{e_1}(3 + 0), 1 - \hat{f}_{e_1}(3 + 2)\} \\ &= \inf \{1 - \hat{f}_{e_1}(3), 1 - \hat{f}_{e_1}(1)\} \\ &= \inf \{0.7, 0.9\} \\ &= 0.7\end{aligned}$$

Hence vague soft quotient near-ring is given by,

\hat{F}	e_1	e_2	e_3	e_4
I	[0.6, 0.7]	[0.5, 0.5]	[0, 0.4]	[0.3, 0.4]
1+I	[0.6, 0.7]	[0.5, 0.5]	[0, 0.4]	[0.3, 0.4]
3+I	[0.6, 0.7]	[0.5, 0.5]	[0, 0.4]	[0.3, 0.4]

Theorem 4.2. *If (\hat{F}, A) is a vague soft near-ring then $(\hat{F}, A)|_I$ is also a vague soft near-ring over R/I .*

Proof. (\hat{F}, A) is a vague soft near-ring over R . Consider,

$$\begin{aligned}
 \hat{t}_a[(n_1 + I) - (n_2 + I)] &= \hat{t}_a(n_1 - n_2 + I) \\
 &= \inf_{i \in I} \hat{t}_a(n_1 - n_2 + i) \\
 &= \inf_{i=x-y \in I} \hat{t}_a\{(n_1 - n_2) + (x - y)\} \\
 &\geq \inf_{x,y \in I} \min\{\hat{t}_a(n_1 - y), \hat{t}_a(n_2 - x)\} \\
 &= \min\left\{\inf_{y \in I} \hat{t}_a(n_1 - y), \inf_{x \in I} \hat{t}_a(n_2 - x)\right\} \\
 &= \min\{\hat{t}_a(n_1 + I), \hat{t}_a(n_2 + I)\}
 \end{aligned}$$

$$\begin{aligned}
 \hat{t}_a[(n_1 + I)(n_2 + I)] &= \hat{t}_a(n_1 n_2 + I) \\
 &= \inf_{i \in I} \hat{t}_a(n_1 n_2 + i) \\
 &= \inf_{i=xy \in I} \hat{t}_a\{(n_1 n_2) + (xy)\} \\
 &\geq \inf_{x,y \in I} \min\{\hat{t}_a(n_1 + x), \hat{t}_a(n_2 + y)\} \\
 &= \min\left\{\inf_{y \in I} \hat{t}_a(n_1 + x), \inf_{x \in I} \hat{t}_a(n_2 + y)\right\} \\
 &= \min\{\hat{t}_a(n_1 + I), \hat{t}_a(n_2 + I)\}
 \end{aligned}$$

Similarly we can show that,

$$\begin{aligned}
 1 - \hat{f}_a[(n_1 + I) - (n_2 + I)] &\geq \min\{1 - \hat{f}_a(n_1 + I), 1 - \hat{f}_a(n_2 + I)\} \\
 1 - \hat{f}_a[(n_1 + I)(n_2 + I)] &\geq \min\{1 - \hat{f}_a(n_1 + I), 1 - \hat{f}_a(n_2 + I)\}
 \end{aligned}$$

□

Theorem 4.3. *If (\hat{F}, A) is a vague soft ideal of R then $(\hat{F}, A)|_I$ is a vague soft ideal over R/I .*

Proof. Let (\hat{F}, A) is a vague soft ideal of R. So, (\hat{F}, A) is a vague soft near-ring over R. Therefore $(\hat{F}, A)|_I$ is a vague soft near-ring over R/I .

$$\begin{aligned}
 \hat{t}_a [(n_1 + I) - (x + I) + (n_2 + I)] &= \hat{t}_a(n_1 - x + n_2 + I) \\
 &= \inf_{i \in I} \hat{t}_a(n_1 - x + n_2 + i) \\
 &= \inf_{i=p+q-r \in I} \hat{t}_a(n_1 - x + n_2 + p + q - r) \\
 &= \inf_{p,q,r \in I} \hat{t}_a \{n_1 + p - (x + r) + n_2 + q\} \\
 &\geq \inf_{r \in I} \hat{t}_a \{(x + r)\}
 \end{aligned}$$

Similarly,

$$1 - \hat{f}_a [(n_1 + I) - (x + I) + (n_2 + I)] \geq 1 - \hat{f}_a(x + I)$$

$$\begin{aligned}
 \hat{t}_a [(n_1 + I)(n_2 + I)] &= \hat{t}_a(n_1 n_2 + I) \\
 &= \inf_{i \in I} \hat{t}_a(n_1 - n_2 + i) \\
 &= \inf_{i=xy \in I} \hat{t}_a \{(n_1 n_2) + (xy)\} \\
 &\geq \inf_{x,y \in I} \min \{ \hat{t}_a(n_1 + x), \hat{t}_a(n_2 + y) \} \\
 &= \min \left\{ \inf_{y \in I} \hat{t}_a(n_1 + x), \inf_{x \in I} \hat{t}_a(n_2 + y) \right\} \\
 &= \min \{ \hat{t}_a(n_1 + I), \hat{t}_a(n_2 + I) \}
 \end{aligned}$$

Similarly

$$1 - \hat{f}_a [(n_1 + I)(n_2 + I)] \geq 1 - \hat{f}_a(n_1 + I)$$

$$\begin{aligned}
& \hat{t}_a \{[(x + I) + (y + I)](n_1 + I) - (x + I)(n_1 + I)\} \\
&= \hat{t}_a \{[(x + y + I)](n_1 + I) - (xn_1 + I)\} \\
&= \hat{t}_a \{[(x + y)n_1] - (xn_1 + I)\} \\
&= \hat{t}_a \{[(x + y)n_1] + I - (xn_1 + I)\} \\
&= \hat{t}_a \{[(x + y)n_1] - xn_1 + I\} \\
&= \inf_{i \in I} \hat{t}_a \{(x + y)n_1 - xn_1 + i\} \\
&= \inf_{i=(p+q)r-pr \in I} \{\hat{t}_a[(x + y)n_1 - xn_1 + (p + q)r - pr]\} \\
&= \inf_{p,q,r \in I} \{\hat{t}_a[(x + p) + (y + q)](n_1 + r) - (x + p)(n_1 + r)\} \\
&\geq \inf_{q \in I} \hat{t}_a(y + q) \\
&= \hat{t}_a(y + I)
\end{aligned}$$

Similarly

$$1 - \hat{f}_a \{[(x + I) + (y + I)](n_1 + I) - (x + I)(n_1 + I)\} \geq 1 - \hat{f}_a(y + I)$$

This shows that $(\hat{F}, A)|_I$ is a vague soft ideal over R/I . \square

Theorem 4.4. *If (\hat{F}, A) is a vague soft R-subgroup over a near-ring R then $(\hat{F}, A)|_I$ is a vague soft R-subgroup over R/I .*

Proof. Let (\hat{F}, A) is a vague soft R-subgroup over R .

$$\begin{aligned}
\hat{t}_a[(n_1 + I) - (n_2 + I)] &= \hat{t}_a(n_1 - n_2 + I) \\
&= \inf_{i \in I} \hat{t}_a(n_1 - n_2 + i) \\
&= \inf_{i=x-y \in I} \hat{t}_a \{(n_1 - n_2) + (x - y)\} \\
&\geq \inf_{x,y \in I} \min \{\hat{t}_a(n_1 - y), \hat{t}_a(n_2 - x)\} \\
&= \min \left\{ \inf_{y \in I} \hat{t}_a(n_1 - y), \inf_{x \in I} \hat{t}_a(n_2 - x) \right\} \\
&= \min \{\hat{t}_a(n_1 + I), \hat{t}_a(n_2 + I)\}
\end{aligned}$$

Similarly

$$1 - \hat{f}_a[(n_1 + I) - (x + I) + (n_2 + I)] \geq 1 - \hat{f}_a(x + I)$$

$$\begin{aligned}
\hat{t}_a[(n_1 + I)(n_2 + I)] &= \hat{t}_a(n_1n_2 + I) \\
&= \inf_{i \in I} \hat{t}_a(n_1 - n_2 + i) \\
&= \inf_{i=xy \in I} \hat{t}_a\{(n_1n_2) + (xy)\} \\
&\geq \inf_{x,y \in I} \min \{\hat{t}_a(n_1 + x), \hat{t}_a(n_2 + y)\} \\
&= \min \left\{ \inf_{y \in I} \hat{t}_a(n_1 + x) \right\} \\
&= \min \{\hat{t}_a(n_1 + I), \hat{t}_a(n_2 + I)\}
\end{aligned}$$

Similarly

$$1 - \hat{f}_a[(n_1 + I)(n_2 + I)] \geq 1 - \hat{f}_a(n_1 + I)$$

This shows that $(\hat{F}, A)|_I$ is a vague soft R-subgroup over R/I . \square

Theorem 4.5. *Let I be an ideal of near-ring R . Then there exists a one-to-one correspondence between vague soft ideals (\hat{F}, A) over R such that $\hat{F}_a(i) = \hat{F}_a(0)$ for all $i \in I$ and the vague soft quotient ideals $(\hat{F}, A)|_I$ over R/I .*

Proof. Let I be an ideal of near-ring R and (\hat{F}, A) is a vague soft ideal. Then by theorem $(\hat{F}, A)|_I$ vague soft quotient ideal over R/I .

We have, $\hat{F}_a(x + I) = \inf_{p \in I} \hat{F}_a(x + p)$. Since $\hat{F}_a(i) = \hat{F}_a(0)$ for all $i \in I$.

Consider $\hat{F}_a(x + i) \geq \min \{\hat{F}_a(x), \hat{F}_a(i)\} = \min \{\hat{F}_a(x), \hat{F}_a(0)\} = \hat{F}_a(x)$

Again we have,

$$\begin{aligned}
\hat{F}_a(x) &= \hat{F}_a(x + i - i) \\
&\geq \min \{\hat{F}_a(x + i), \hat{F}_a(i)\} \\
&= \min \{\hat{F}_a(x + i), \hat{F}_a(0)\} \\
&= \hat{F}_a(x + i)
\end{aligned}$$

This shows that $\hat{F}_a(x) = \hat{F}_a(x + i)$ for all $i \in I$. Hence here exists a one-to-one correspondence between vague soft ideals (\hat{F}, A) over R such that $\hat{F}_a(i) = \hat{F}_a(0)$ for all $i \in I$ and the vague soft quotient ideals $(\hat{F}, A)|_I$ over R/I . \square

5. CONCLUSION

In this paper, we have defined vague soft R-subgroup over a near-ring. We have examined and discussed structural properties of vague soft R-subgroup. We have defined vague soft quotient near-ring using a quotient near-ring. We have provided detail example of vague soft quotient near-ring. At the end some of its properties are mentioned with proof.

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