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ON CLEAN HYPERRINGS

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Abstract. We introduce and study clean hyperrings. A hyperring R is called a clean hyperring if for every element x of R, $x \in u + e$ where u is a unit and e is an idempotent. We also introduce GC-hyperring which is a proper generalization of clean hyperrings and obtain some related results of such hyperrings.

Key Words: Hyperring, Hyperideal, Clean hyperring, GC-hyperring.
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1. Introduction

The hyperstructure theory was introduced by Marty in 1934, at the 8th Congress of Scandinavian Mathematicians [9]. Hyperstructures have many applications to several sectors of both pure and applied mathematics [4]. A hypergroup in the sense of Marty is a nonempty subset of H, endowed by a hyperoperation $* : H \times H \to P^*(H)$, the set of all nonempty subsets of H, which satisfies the associative law and the reproduction axiom. Mittas [10] introduced canonical hypergroups as a special class of the hypergroups. The more general structure that satisfies the ring-like axioms is the hyperring $(R, +, .)$, where $+$ and $.$ are two hyperoperations such that $(R, +)$ is a hypergroup, $(R, .)$ a semihypergroup and $.$ is an associative hyperoperation, which is distributive with respect to $+$. There are different classes of hyperrings. If only the addition $+$ is a hyperoperation and the maltiplication $.$ is a usual operation, then we say that R is an additive hyperring. A special case of this type is the hyperring introduced by Krasner (for more detail see

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[7]). If only . is a hyperoperation, we shall say that R is a multiplicative hyperring. Rota in [15] introduced the multiplicative hyperring; subsequently, many authors worked on this field (Olson and Ward in [13]; Procesi and Rota in [14]; Rota in [16]; Dasgupta in [6]). Irina Cristea and Sanja Jancic-Rasovic in [2] introduced the composition hyperrings, as a quadruple $(R, +, ., \circ)$ such that $(R, +, .)$ is a commutative hyperring in the general sense of Spartalis, and the composition hyperoperation $\circ$ is an associative hyperoperation, distributive to the right side with respect to the addition and multiplication. A comprehensive review of the theory of hyperstructures appears in Corsini [3], Corsini and Leoreanu [4] and Vougiuklis [18]. In this paper, by a hyperring we mean a Krasner hyperring [8], that is, a triple (R,+,.) such that (R,+) is a canonical hypergroup, $(R, .)$ is a semigroup with a zero 0 where 0 is the scalar identity of $(R, +)$ and . distributive over $+$. An element x of a ring R is *clean* (see [12]) if it can be written as a sum of a unit and an idempotent element in R. R is called *clean* if every element of R is clean. Based on the notion of clean rings introduced by Nicholson [12], we define here the concept of clean hyperrings. In section 2 of this paper, we remember some definitions and basic notions of hyperring. In section 3, we define clean hyperring, then some results concerning this concept are proved. In section 4, we define uniquely clean hyperrings and investigate some properties of them.

2. Basic Definitions and Results

In this section we briefly recall some definitions and results of hyperring, which we need to develop our paper.

Definition. A non-empty set H with a hyperoperation $+$ is called a *canonical hypergroup* if the following axioms are satisfied:

(i) for every $x, y, z \in R, x + (y + z) = (x + y) + z$;

(ii) For every $x, y \in R, x + y = y + x$;

(iii) There exists $0 \in R$ such that $0 + x = \{x\}$ for all $x \in R$;

(iv) For every $x \in R$ there exists a unique $x' \in R$ such that $0 \in x + x'$ (we shall write $-x$ for x' and we call it the opposite of x);

(v) $z \in x + y$ implies $y \in -x + z$ and $x \in z - y$ (for a study of canonical hypergroups see Mittas [10]).

Definition. [8] Let $(R, +)$ be a hyperstructure and . be an internal composition on R. Then $(R, +, .)$ is called a *hyperring* if:

(i) $(R, +)$ is a canonical hypergroup;

(ii) $(R, .)$ is a multiplicative semigroup having zero as a bilaterally absorbing element, i.e., $x.0 = 0.x = 0$;

(iii)The multiplication is distributive with respect to the hyperoperation $+$.

A hyperring R is called:

(i) With identity if there exists an element, say $1 \in R$, such that $1x = x1 = x$;

(ii) *Commutative* iff $a.b = b.a, \forall a, b \in R$;

(iii) A *hyperintegral domain* if R is a commutative hyperring with identity and $ab = 0$ implies that $a = 0$ or $b = 0$.

For simplicity of notations we write sometimes xy instead of $x.y$.

If $A, B \subseteq R$ then $A + B = \cup\{a + b \mid a \in A, b \in B\}$. Moreover, for every $x \in R$, $A + x$ is used for $A + \{x\}$ and $x + B$ for $\{x\} + B$. The following elementary facts follow easily from the axiom: $-(-x) = x$ and $-(x+y) = -x-y$. where $-A = \{-a|a \in A\}$. Also, for each $a, b, c, d \in R$ we have $(a+b).(c+d) \subseteq a.c+b.c+a.d+b.d$ whenever R is commutative $(a + b).(c + d) = a.c + b.c + a.d + b.d$.

A nonempty subset A of R is said to be a subhyperring of R if $(A, +, .)$ is itself a hyperring. If $R\backslash\{0\}$ is a multiplicative group then $(R, +, .)$ is a hyperfield.

Remark. In the sequel $(R.+, .)$ is a commutative hyperring with identity in the sense of Krasner.

Recall that a nonempty subset I of a hyperring R is called a *hyperideal* if

$i)$ $a, b \in I$ implies $a - b \subseteq I$;

$ii)$ $a \in I, r \in R$ imply $r.a \in I$.

A hyperideal P of R is called *prime hyperideal* if for each $a, b \in P$, $a.b \in P$ implies $a \in P$ or $b \in P$.

Recall that a proper hyperideal M of R is a *maximal hyperideal* of R if the only hyperideals of R that contain M are M itself and R. A commutative hyperring R with identity is called *local* if it has a unique maximal hyperideal.

Theorem. [19] If R is a commutative hyperring with identity, then the following conditions are equivalent:

(i) R is local

(ii) All non units (=noninvertibles) of R are contained in a hyperideal, say $M \neq R$.

(iii) The non units of R form a hyperideal.

Definition. Let R_1 and R_2 be hyperrings. A map $f : R_1 \to R_2$ is called a *homomorphism* if the following conditions are satisfied:

(i) $f(a+b) \subseteq f(a)+f(b)$ for all $a, b \in R_1$;

(ii) $f(a.b) = f(a).f(b)$ for all $a, b \in R_1$.

If in (i) equality holds, then f is called a *good homomorphism*.

A map f is called an *epimorphism* if f is a surjective homomorphism and also if for every $a_2, b_2 \in R_2$ the following holds:

3) $(\forall y \in a_2 + b_2)(\exists a_1, b_1 \in R_1)(\exists x \in a_1 + b_1), f(a_1) = a_2, f(b_1) = b_2, f(x) = y$.

Finally, a map f is said to be an *isomorphism* if it is a bijective good homomorphism.

Definition. The hyperideal I of R is normal in R if and only if $x + I - x \subseteq I$ for all $x \in R$.

Definition. If I is a normal hyperideal of a hyperring R, then we define the relation $x \equiv y \pmod{I}$ if and only if $x - y \cap I \neq \emptyset$. It is easy to see that this relation is an equivalence relation on R. Let $I^*[x]$ denote the equivalence class of the element $x \in R$. Suppose $y \in x + I$ then there exists $a \in I$ such that $y \in x + a$, which implies that $a \in -x + y$ and so $y - x \cap I \neq \emptyset$ or $x + I \subseteq I^*[x]$. Thus $x + I \subseteq I^*[x]$. Similarly we have $I^*[x] \subseteq x + I$. Therefore $I^*[x] = x + I$. All the cosets $a + I, a \in R$ form the factor hyperring R/I with respect to the hyperoperation $\oplus$ defined by $(a + I) \oplus (b + I) = \{c + I \mid c \in a + b\}$ and the multiplication $(a + I) \odot (b + I) = a.b + I$, and the coset I as the zero element. Note that a hyperring R/I is a hyperdomain, i.e., $x.y = 0$ implies $x = 0$ or $y = 0$ in R/I iff I is a prime hyperideal.

Theorem. [19] Let R be a commutative hyperring and $M \neq R$ be hyperideal of R. Then M is maximal if and only if R/M is a hyperfield.

Proposition. [5] Let R be a hyperring and let I be a proper hyperideal of R. Then there exists a maximal hyperideal of R containing I.

For a hyperring R we define the Jacobson radical $J(R)$ of R as the intersection of all maximal hyperideals of R.

Proposition. [5] Let R be a hyperring and $U(R)$ the set of all invertible elements in R. Then an element $a \in R$ belongs to $J(R)$ if and only if $1 - ba \subseteq U(R)$ for all $b \in R$.

Throughout this paper $U(R)$ will denote the set of all invertible elements in R and $Id(R)$ denotes set of all idempotent elements of R.

Definition. (i) A subset U of a hyperring R is called a *unit set*, if there exists $B \subseteq R$ such that $1 \in U.B$.

(ii) A subset E of a hyperring R is called an *idempotent set* if $E \subseteq E^2$. It is clear that $U(R)$ is a unit set and $Id(R)$ is an idempotent set.

3. Clean Hyperrings

Let R be a commutative hyperring with identity in the sense of Krasner.

Definition. A commutative hyperring R is a *clean hyperring* if for every element $x \in R$ there exist $u \in U(R)$ and $e \in Id(R)$, such that $x \in u + e$.

Example. Let $R = \{0, 1, 2\}$ be a set with the hyperoperation $+$ and the binary operation . defined as follow: $R = \{0, 1, 2\}$

$+$	0	1	2
0	$\{0\}$	$\{1\}$	$\{2\}$
1	$\{1\}$	$\{1\}$	R
2	$\{2\}$	R	$\{2\}$

$\circ$	0	1	2
0	0	0	0
1	0	1	2
2	0	1	2

Then $(R, +, .)$ is a clean hyperring.

Definition. A commutative hyperring R is a *generalized clean hyperring (or GC-hyperring)* if for every element $x \in R$ there exist a unit set U and an idempotent set E such that $x \in U + E$.

It is clear that every clean hyperring is a GC-hyperring.

Proposition. (i) A homomorphic image of a clean hyperring is a clean hyperring.

(ii) A direct product $R = \prod R_\alpha$ of hyperrings $\{R_\alpha\}$ is a clean hyperring, if and only if each R_α is a clean hyperring.

(iii) A local hyperring is a clean hyperring.

Proof. (i) Let R be a clean hyperring. For each $x + A \in R/A$, write $x \in u+e$ where $u \in U(R)$ and $e \in Id(R)$. Then $x+A \in (u+A) \oplus (e+A)$ we have $u + A \in U(R/A)$ and $e + A \in Id(R/A)$.

(ii) $(\Rightarrow)$ This follows from (i).

$(\Leftarrow)$ Suppose that each R_α is a clean hyperring. Let $x = (x_\alpha) \in \prod R_\alpha$. For each α, write $x_\alpha \in u_\alpha + e_\alpha$ where $u_\alpha \in U(R_\alpha)$ and $e_\alpha \in Id(R_\alpha)$. Then $x \in u + e$ where $u = (u_\alpha) \in U(\prod R_\alpha)$ and $e = (e_\alpha) \in Id(\prod R_\alpha)$. So $\prod R_\alpha$ is a clean hyperring.

(iii) Let R be a local hyperring with a maximal hyperideal M. Let $x \in U(R), x \in x + 0$. If $x \in M$, by Proposition 2, $x - 1 \subseteq U(R)$ and so $x \in x+0 \subseteq x-1+1 = \cup\{a+1 \mid a \in x-1\}$. Thus there exists $y \in x-1$ such that $x \in y + 1$, and hence $y \in x - 1 \subseteq U(R)$. □

Proposition. Let I be a normal hyperideal of R, and assume that $I \subseteq J(R)$. Then R is a clean hyperring if and only if R/I is clean and idempotent lift modulo I.

Proof. If R is clean so is R/I, by Proposition 3. Conversely, suppose that $\overline{x}$ denote $x+I$ in the ring R/I. If $r \in R$, $\overline{r} \in \overline{e} \oplus \overline{u}$ where $\overline{e}^2 = \overline{e}$ and $\overline{u}$ is a unit in R/I. By hypothesis we may assume that $e^2 = e$. Since $\overline{r} \in \overline{e} \oplus \overline{u} = \{c+I \mid c \in e+u\}$, then there exists $c \in e+u$ such that $r+I = c+I$. From $c-e \subseteq e-e+u$, we have $r-e+I = c-e+I \subseteq e-e+u+I \subseteq u+I$, as I is normal. Thus $r-e+I = \cup \{t+I \mid t \in r-e\} \subseteq U(R/I)$, then $t+I$ is a unit in R/I, and so there exists $t' \in R$ such that $(t+I) \odot (t'+I) = 1+I$. So $tt' + I = 1 + I$, then $1 - tt' \cap I \neq \emptyset$. Thus there exists $x \in 1 - tt' \cap I$, by Proposition 2, $1 - x \subseteq U(R)$. Then $tt' \in 1 - x \subseteq U(R)$, and so t is a unit in R. Since $t \in r - e$, $r \in t + e$, therefore R is a clean hyperring. □

We next characterize the indecomposable clean hyperrings.

Theorem. Let R be a commutative hyperring. Consider the following conditions.

(1) R is local.

(2) R is an indecomposable clean hyperring.

(3) Every element $x \in R$ has the form $x \in u + e$ where $u \in U(R)$ and $e \in \{0,1\}$. Then (1) $\Rightarrow$ (2) $\Rightarrow$ (3), moreover, if every maximal hyperideal of R is normal, then (3) $\Rightarrow$ (1).

Proof. (1) $\Rightarrow$ (2) If R is local, then R is indecomposable, since if $R = I_1 \oplus I_2$ such that I_1, I_2 are hyperideals of R, then there exists $x_i \in I_i$ $(i = 1, 2)$ such that $1 \in x_1 + x_2$. x_1 or x_2 is invertible otherwise Rx_1 and Rx_2 contained in the unique maximal hyperideal say M, and so $1 \in x_1 + x_2 \subseteq M$, which is a contradiction. Hence R is indecomposable.

(2) $\Rightarrow$ (3) It is clear.

(3) $\Rightarrow$ (1) It suffices to show that every nonunit $x \in R$ is in $J(R)$. For $r \in R, rx$ is a nonunit. Hence $rx \in u + 1$ for some $u \in U(R)$. We show that $1 - rx \subseteq U(R)$. Let $y \in 1 - rx$, if $y \notin U(R)$, then there exist a maximal hyperideal M such that $y \in M$. Thus $u \in -1 + rx \subseteq -1 + 1 - y \subseteq -1 + 1 + M \subseteq M$; that is, $u \in M$, which is a contradiction. Therefore by Proposition 2, $x \in J(R)$. □

A commutative ring R is called a *pm-ring* if each prime ideal of R is contained in a unique maximal ideal of R.

Definition. A commutative hyperring R is called a *pm-hyperring* if each prime hyperideal of R is contained in a unique maximal hyperideal of R, or equivalently, R/P is local for each prime hyperideal P of R.

Corollary. A clean hyperring R is a pm-hyperring.

Proof. Let P be a prime hyperideal of R. By Proposition 3, R/P is an indecomposable clean ring. Hence by Theorem 3, R/P is local. Therefore R is a pm-hyperring. $\square$

A ring R is called *von Neumann regular* if $\forall x \in R$ there exists $y \in R$ such that $x = xyx$. Similarly, for hyperrings we have the following definition:

Definition. A hyperring R is called *von Neumann regular* if $\forall x \in R$ there exists $y \in R$ such that $x = xyx$.

Lemma. A commutative R is von Neumann regular if and only if each element of R can be written as the product of a unit and an idempotent.

Proof. If $x = e.u$ where e is an idempotent and u is a unit, then $x = xu^{-1}x$. Conversely, if $x = xyx$, then $e = xy$ is an idempotent, $U := xe + (1-e)$ is a unit set, and $x \in e.U$. Thus $x = e.u$ for some $u \in U$. $\square$

Theorem. A commutative von Neumann regular hyperring R is a generalized clean hyperring.

Proof. By Lemma 3, we can suppose that every element $x \in R$ can be written as $x = u.e$ where $u \in U(R)$ and $e \in Id(R)$. If we take $V := ue-(1-e)$ and $E := 1-e$, then $x \in V+E$ where E is an idempotent set and V is a unit set. (For $1 \in (ue-(1-e)).(u^{-1}e-(1-e))$). $\square$

Proposition. Let R be a commutative hyperring. Consider the following conditions:

(1) R is a clean hyperring.

(2) For every element $x \in R$, $x \in u-i$ where $u \in U(R)$ and $i \in Id(R)$.

(3) For every element $x \in R$, $x \in u+i$ where $u \in U(R) \cup \{0\}$ and $i \in Id(R)$.

(4) For every element $x \in R$, $x \in u-i$ where $u \in U(R) \cup \{0\}$ and $i \in Id(R)$.

(5) R is a GC-hyperring.

Then (1) $\Leftrightarrow$ (2), (3) $\Leftrightarrow$ (4), (1) $\Rightarrow$ (3) $\Rightarrow$ (5).

Proof. (1) $\Rightarrow$ (2) Let $x \in R$. Then $-x \in u+i$, where $u \in U(R)$ and $i \in Id(R)$. Thus $x \in (-u)-i$, where $-u \in U(R)$ and $i \in Id(R)$.

(2) $\Rightarrow$ (1) Similar to (1) $\Rightarrow$ (2), but we have $-x \in u - i$. (3) $\Leftrightarrow$ (4) Similar to (1) $\Leftrightarrow$ (2). (1) $\Rightarrow$ (3) Clear. (3) $\Rightarrow$ (5) It suffices to show that if $e \in Id(R)$, then $e \in U + E$, where U is a unit set and E is an idempotent set. It is clear that $e \in (e + e - 1) + (1 - e)$ where $1 \in (e + e - 1).(e + e - 1)$ and $(1 - e) \subseteq (1 - e) + 0 \subseteq (1 - e)^2$. □

Proposition. Let R be a commutative hyperring with exactly two maximal hyperideals. If for a maximal hyperideal M of R and $x \in R$, whenever $x + x \subseteq M$, we have $x \in M$, then for $x \in R$, either $x \in u + e$ or $x \in u - e$, where $u \in U(R)$ and $e \in Id(R)$.

Proof. Let M_1 and M_2 be two maximal hyperideals of R. Let $x \in R$. If $x \in U(R)$, then $x \in x + 0$ where $x \in U(R)$ and $0 \in Id(R)$; while if $x \in M_1 \cap M_2 = J(R)$, both $x+1$ and $x-1$ are subsets of $U(R)$; and so $x \in u_1 + 1 = u_2 - 1$ where $u_1, u_2 \in U(R)$ and $1 \in Id(R)$. Next, suppose that $x \in M_1 - M_2$. So $x+1, x-1 \nsubseteq M_1$, for otherwise $1 \in M$ a contradiction. Suppose that $x + 1, x - 1 \subseteq M_2$. Then $x + x \subseteq (x + 1) + (x - 1) \subseteq M_2$. So $x \in M_2$ a contradiction. Hence either $x + 1 \nsubseteq M_2$ or $x - 1 \nsubseteq M_2$ and so $x + 1 \subseteq U(R)$ or $x - 1 \subseteq U(R)$, i.e., $x \in u - 1$ or $x \in u + 1$ for some $u \in U(R)$. The case $x \in M_2 - M_1$ is similar. Hence for $x \in R$, either $x \in u + e$ or $x \in u - e$, where $u \in U(R)$ and $e \in Id(R)$. □

4. Uniquely Clean Hyperring

Definition. A commutative hyperring R is a *uniquely clean hyperring* if for every element $x \in R$ there exist uniquely $u \in U(R)$ and $e \in Id(R)$, such that $x \in u+e$, that is if $x \in u_1+e_1 = u_2+e_2$, then $u_1 = u_2, e_1 = e_2$.

Lemma. Let R be a commutative hyperring. Let $e, f \in R$ be idempotents with $e - f \subseteq J(R)$. Then $e = f$.

Proof. $f(1-e) \subseteq 0+f(1-e) \subseteq (e-f).(e-1) \subseteq J(R)$. But $f(1-e).f(1-e) = f(1-e)+0$, so $0 \in (f(1-e).(f(1-e)-1+0))$. Then there exists $u \in f(1-e)-1+0$ such that $0 \in f(1-e).u$. Since $f(1-e)+0 \subseteq J(R)$, by Proposition 2, $f(1 - e) - 1 + 0 \subseteq U(R)$. Thus u is unit. Then $0 \in f(1 - e) = f - fe$. Hence $f = fe$. Likewise, $f - e \in J(R)$ gives $e = ef$. Hence $0 \in ef - ef = e - f$. So $e = f$. □

Theorem. Let R be a commutative hyperring such that R/M is a hyperfield with two elements, for each maximal normal hyperideal M of R. If $x \in R$ has a representation in the form $x \in u + e$ where $u \in U(R), e \in Id(R)$, then this representation is unique. Hence a clean hyperring R such that R/M is a hyperfield with two element, for each maximal normal hyperideal M of R is a uniquely clean hyperring.

Proof. Suppose that $x \in u_1 + e_1 = u_2 + e_2$ where $u_1, u_2 \in U(R)$ and $e_1, e_2 \in Id(R)$. Let M be a maximal and normal hyperideal of R. Now $u_1 + M, u_2 + M$ are units in hyperfield R/M; so $u_1 + M = u_2 + M$. Thus $u_1 \in u_2 + M$ (Definition 2). Hence $e_1 - e_2 \subseteq x - u_1 - x + u_2 \subseteq x - u_2 + M - x + u_2 \subseteq M$. Thus $e_1 - e_2 \subseteq J(R)$. By Lemma 4, $e_1 = e_2$. Hence $u_1 = u_2$. The last statement of the theorem immediately follows. □

Corollary. A local hyperring R is a uniquely clean hyperring if and only if R/M is a hyperfield with exactly two elements.

Proof. ($\Leftarrow$) By Theorem 4, R is a uniquely clean hyperring. ($\Rightarrow$) Let $u \in U(R)$. Then $u+1 \nsubseteq U(R)$. Otherwise, for all $x \in u+1$, $x \in u+1 \subseteq U(R)$. Then $x \in x + 0$, where x is unit and 0 is idempotent. Thus there exist two representations for x, $x \in u + 1$ and $x \in x + 0$. Hence $u+1 \subseteq M$; so $u+1+M = \cup\{c+M | c \in u+1\} = \cup\{M\}$, then $\overline{0} \in \overline{u+1}$ in $\overline{R} = R/M$. Since each nonzero element of $\overline{R}$ is the image of a unit of R; it follows that $\overline{R}$ is a hyperfield with $0 \in x + 1$ for each $0 \neq x \in \overline{R}$. Hence $\overline{R}$ has exactly two elements. □

Proposition. A direct product $R = \prod R_\alpha$ of commutative hyperrings $\{R_\alpha\}$ is a uniquely clean hyperring, if and only if each R_α is a uniquely clean hyperring.

Proof. Similarly to the proof of Proposition 3. □

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