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THE STABILITY OF PEXIDER TYPE FUNCTIONAL EQUATION IN INTUITIONISTIC FUZZY BANACH SPACES VIA FIXED POINT TECHNIQUE

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Abstract. The object of the present paper is to appraise generalization of the Hyers-Ulam-Rassias stability theorem for Pexider type functional equation

$$f(2x+y) - f(x+2y) = 3g(x) - 3h(y) \qquad (1)$$

in intuitionistic fuzzy Banach spaces and stability results have been obtained by a fixed point method . This method shows that the stability is related to some fixed point of a suitable operator .

Key Words: Intuitionistic fuzzy norm, Hyers-Ulam stability, Pexider type functional equation, Intuitionistic fuzzy normed spaces, fixed point alternative theorem.
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1. Introduction

Stability of functional equation was first posed by Ulam [15] in 1940 concerning the stability of group homomorphisms and answered in the next year by Hyers [2] for Cauchy functional equation in Banach spaces and then generalized by T. Aoki [17] and Th. M. Rassias [21] for additive mappings and linear mappings by considering an unbounded Cauchy difference respectively . In the spirit of Rassias's approach , Gavruta [11] replaced the unbounded Cauchy difference by a general control function to generalize Rassias's theorem . The Hyers-Ulam stability theorem was

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generalized by F. Skof [4] for the function $f : X \to Y$, where X is a normed space and Y is a Banach space and then the result of Skof was extended by P. W. Cholewa [12] and S. Czerwik [14]. Like this way several stability problems for various functional equations have been investigated. Recently, fuzzy version of different functional equations is discussed in several papers [1, 3, 16] etc..

Fuzzy set theory is a powerful handset for modeling uncertainty and vagueness. Out of fuzzy sets, intuitionistic fuzzy set [8] have been found to be highly useful to deal with vagueness. Atanassov [8] is ascribed be the introducer of the concept of intuitionistic fuzzy set, as a generalized perspective of fuzzy set [9]. One of the most important problems in intuitionistic fuzzy topology is to obtain an appropriate concept of intuitionistic fuzzy metric spaces and intuitionistic fuzzy normed spaces. J. H. Park [7], Saadati and Park [13] and Samanta et.al.[18] introduced and studied a few notions of intuitionistic fuzzy metric spaces and intuitionistic fuzzy normed spaces.

Several results concerning the Hyers-Ulam-Rassias stability of different functional equations have been established by several researchers [1, 3, 16, 19, 20] in intuitionistic fuzzy Banach spaces. Afterwards, a new version on the notion of the intuitionistic fuzzy set under the perusal of stability theorem for generalized Hyers-Ulam-Rassians stability of quadratic functional equation in intuitionistic fuzzy normed space was examined by Shakeri [16].

In 1996, Isac and Rassias [6] were the first to provide fixed point theorem for establishing stability of functional equations. In 2003, Radu [22] proposed the fixed point alternative method for obtaining the existence of exact solutions and error estimations. Subsequently, Mihet [3] applied the fixed alternative method to study the fuzzy stability of the Jensen functional equation on the fuzzy Banach space. Recently several authors are trying to establish the stability of several functional equations by using the fixed point alternative method.

Our goal is to determine the generalization of the Hyers-Ulam-Rassias stability theorem of the Pexider type functional equation (1) in intuitionistic fuzzy Banach spaces. The stability results would be obtained by a fixed point method. This method would show that the stability is related to some fixed point of a suitable operator.

2. Preliminaries

A new concept of intuitionistic fuzzy normed linear space was marked out by the author S. Shakeri [16]. In this section, first the definition of intuitionistic fuzzy norm according Shakeri [16] and subsequently a few results have been enfolded, that would be applied in the sequel.

Lemma 2.1. [5] Consider the set L^* and the order relation $\leq_{L^*}$ define by

$$Ł^* = \{ (x_1, x_2) : (x_1, x_2) \in [0,1]^2 \text{ and } x_1 + x_2 \leq 1 \},$$

$$(x_1, x_2) \leq_{L^*} (y_1, y_2)$$

$$\Leftrightarrow x_1 \leq y_1, x_2 \geq y_2, \forall (x_1, x_2), (y_1, y_2) \in L^*.$$

Then $(L^*, \leq_{L^*})$ is a complete lattice.
We denote its units by $0_{L^*} = (0, 1)$ and $1_{L^*} = (1, 0)$.

Definition 2.2. [8] Let E be any nonempty set. An intuitionistic fuzzy set A of E is an object of the form $A = \{ (x, \mu_A(x), \nu_A(x)) : x \in E \}$, where the functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ denote the degree of membership and the degree of non-membership of the element $x \in E$ respectively and for every $x \in E$,
$0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.3. [5] A triangular norm (t-norm) on L^* is a mapping $\Gamma : (L^*)^2 \to L^*$ satisfying the following conditions :

(a) $(\forall x \in L^*)(\Gamma(x, 1_{L^*}) = x)$ (boundary condition),

(b) $(\forall (x, y) \in (L^*)^2)(\Gamma(x, y) = \Gamma(y, x))$ (commutativity),

(c) $(\forall (x, y, z) \in (L^*)^3)(\Gamma(x, \Gamma(y, z)) = \Gamma(\Gamma(x, y), z))$ (associativity),

(d) $(\forall (x, x', y, y') \in (L^*)^4)(x \leq_{L^*} x'$ and $y \leq_{L^*} y' \Rightarrow \Gamma(x, y) \leq_{L^*} \Gamma(x', y'))$ (monotonicity).
If $(L^*, \leq_{L^*}, \Gamma)$ is an Abelian topological monoid with unit 1_{L^*}, then Γ is said to be a continuous t-norm.

Definition 2.4. [5] A continuous t-norm Γ on L^* is said to be continuous t-representable if there exists a continuous t-norm $*$ and there exists a continuous t-conorm $\diamond$ on $[0, 1]$ such that for all $x = (x_1, x_2)$, $y = (y_1, y_2) \in L^*$, $\Gamma(x, y) = (x_1 * y_1, x_2 \diamond y_2)$

We now define a sequence Γ^n recursively by $\Gamma^1 = \Gamma$ and
$\Gamma^n(x^{(1)}, x^{(2)}, \cdots, x^{(n+1)})$

$$= \Gamma(\Gamma^{(n-1)}(x^{(1)}, x^{(2)} \cdots, x^{(n)}), x^{(n+1)}),$$

$\forall\, n \geq 2,\, x^{(i)} \in L^*$.

Definition 2.5. Let μ and ν be membership and non-membership degree of an intuitionistic fuzzy set from $X \times (0, \infty)$ to $[0, 1]$ such that $0 \leq \mu_x(t) + \nu_x(t) \leq 1$ for all $x \in X$ and $t > 0$. The triple $(X, P_{\mu,\nu}, T)$ is said to be an intuitionistic fuzzy normed space (briefly IFN-space) if X is a vector space, T is a continuous t-representable and $P_{\mu,\nu}$ is a mapping $X \times (0, \infty) \to L^*$ satisfying the following conditions :
for all $x, y \in X$ and $t, s > 0$,

(i) $P_{\mu,\nu}(x, 0) = 0_{L^*}$;

(ii) $P_{\mu,\nu}(x, t) = 1_{L^*}$ if and only if $x = 0$;

(iii) $P_{\mu,\nu}(\alpha x, t) = P_{\mu,\nu}\left(x, \frac{t}{|\alpha|}\right)$ for all $\alpha \neq 0$;

(iv) $P_{\mu,\nu}(x + y, t + s) \geq_{L^*} \Gamma(P_{\mu,\nu}(x, t), P_{\mu,\nu}(y, s))$.

In this case, $P_{\mu,\nu}$ is called an intuitionistic fuzzy norm. Here,

$$P_{\mu,\nu}(x, t) = (\mu_x(t), \nu_x(t)) = (\mu(x, t), \nu(x, t)).$$

Example 2.6. Let $(X, \|.\|)$ be a normed linear space. Let

$$M(a, b) = (\min\{a_1, b_1\}, \max\{a_2, b_2\})$$

for all $a = (a_1, a_2)$, $b = (b_1, b_2) \in L^*$ and for $a, b \in [0, 1]$ and μ, ν be the membership and the non-membership degree of intuitionistic fuzzy set define by
$P_{\mu,\nu}(x, t) = (\mu_x(t), \nu_x(t)) = \left(\frac{t}{t + k\|x\|}, \frac{k\|x\|}{t + k\|x\|}\right)$, where $k > 0$, and for all $x \in \mathbb{R}^+$. Then $(X, P_{\mu,\nu}, M)$ is an IFN-space.

Definition 2.7. (1) A sequence $\{x_n\}$ in an IFN-space $(X, P_{\mu,\nu}, M)$ is called a **Cauchy sequence** if for any $\varepsilon > 0$ and $t > 0$, there exists $n_0 \in N$ such that

$$P_{\mu,\nu}(x_n - x_m, t) >_{L^*} (1 - \varepsilon, \varepsilon), \forall n, m \geq n_0$$

(2) The sequence $\{x_n\}$ is said to be **convergent** to a point $x \in X$ if

$$P_{\mu,\nu}(x_n - x, t) \to 1_{L^*} \ as\ n \to \infty \ \ for\ every\ \ t > 0.$$

(3) An IFN-space $(X, P_{\mu,\nu}, M)$ is said to be **complete** if every Cauchy sequence in X is convergent to a point $x \in X$.

Let X be a nonempty set. A function $d : X \times X \to [0, \infty]$ is called a **generalized metric** on X if d satisfies
(i) $d(x, y) = 0$ if and only if $x = y$;
(ii) $d(x, y) = d(x, y)$ for all $x, y \in X$;
(iii) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Theorem 2.8. *(The fixed point alternative theorem,* [3] *and* [10]*)* *Let (X, d) be a complete generalized metric space and let $J : X \to X$ be a strictly contractive mapping with Lipschitz constant $0 < L < 1$, that is,*

$$d(Jx, Jy) \leq L\, d(x, y),$$

for all $x, y \in X$.
Then for each $x \in X$, either

$$d(J^n x, J^{n+1} x) = \infty$$

or,

$$d(J^n x, J^{n+1} x) < \infty \ \ \forall\, n \geq n_o$$

for some non-negative integers n_0. Moreover, if the second alternative holds then
(1) the sequence $\{J^n x\}$ converges to a fixed point $y^\star$ of J;
(2) $y^\star$ is the unique fixed point of J in the set

$$Y = \{y \in X : d(J^{n_0} x, y) < \infty\};$$

(3) $d(y, y^\star) \leq (\frac{1}{1-L})\, d(y, Jy)$ *for all $y \in Y$.*

Lemma 2.9. Let X, Y be vector spaces and let $H : X \to Y$ be such that

$$H(2x + y) - H(x + 2y) = H(x) - H(y) \tag{2.1}$$

for all $x, y \in X$. It is easy to show that $H(x) = A(x) + b$ satisfies (2.1), where A is an additive mapping.

3. The Stability Result

Theorem 3.1. *Let X be a linear space, $(Z, P'_{\mu,\nu}, M)$ be a IFN-space, $\phi : X \times X \to Z$ be a function such that for some $0 < \alpha < 2$,*

$$P'_{\mu,\nu}(\phi(2x, 2x), t) \geq_{L^*} P'_{\mu,\nu}(\alpha\phi(x, x), t) \tag{3.1}$$

$(x \in X, t > 0)$ and

$$\lim_{n\to\infty} P'_{\mu,\nu}(\phi(2^n x, 2^n x), 2^n t) = 1_{L^*}$$

for all $x, y \in X$ and $t > 0$. Let $(Y, P_{\mu,\nu}, M)$ be a complete IFN-space. If $f, g, h : X \to Y$ is odd mappings such that

$$P_{\mu,\nu}(f(2x+y) - f(x+2y) - 3g(x) + 3h(y), t)$$
$$\geq_{L^*} P'_{\mu,\nu}(\phi(x, y), t) \tag{3.2}$$

$(x \in X, t > 0)$. Then there exists a unique mapping $H : X \to Y$ define by $H(x) := P_{\mu,\nu} - \lim_{n\to\infty}\left(\frac{f(2^n x)}{2^n}\right)$ for all $x \in X$ satisfying

$$P_{\mu,\nu}(f(x) - H(x), t) \geq_{L^*} M_1(x, t(2-\alpha)) \tag{3.3}$$

and

$$P_{\mu,\nu}(3g(x) + 3h(x) - 2H(x), t) \geq_{L^*} M_1\left(x, \frac{t \times 3(2-\alpha)}{8-\alpha}\right). \tag{3.4}$$

Also,

$$H(2x+y) - H(x+2y) = H(x) - H(y). \tag{3.5}$$

Where

$$M_1(x, t)$$
$$= M^3\left\{P'_{\mu,\nu}\left(\phi(x, 0), \frac{t}{3}\right), P'_{\mu,\nu}\left(\phi(x, -x), \frac{t}{3}\right),\right.$$
$$\left. P'_{\mu,\nu}\left(\phi(x, -x), \frac{t}{3}\right), P'_{\mu,\nu}\left(\phi(x, x), \frac{t}{3}\right)\right\}$$

Proof. Putting $y = x$ in (3.2) we get

$$P_{\mu,\nu}(-3g(x) + 3h(x), t) \geq_{L^*} P'_{\mu,\nu}(\phi(x, x), t) \tag{3.6}$$

Again putting $y = -x$ in (3.2) we get

$$P_{\mu,\nu}(2f(x) - 3g(x) - 3h(x), t) \geq_{L^*} P'_{\mu,\nu}(\phi(x, -x), t) \tag{3.7}$$

Also putting $y = 0$ in (3.2) we get

(3.8)
$$P_{\mu,\nu}(f(2x) - f(x) - 3g(x), t) \geq_{L^*} P'_{\mu,\nu}(\phi(x, 0), t)$$

Let

$$M_1(x, t) = M^3\left\{P'_{\mu,\nu}\left(\phi(x, 0), \frac{t}{3}\right), P'_{\mu,\nu}\left(\phi(x, -x), \frac{t}{3}\right),\right.$$
$$\left. P'_{\mu,\nu}\left(\phi(x, -x), \frac{t}{3}\right), P'_{\mu,\nu}\left(\phi(x, x), \frac{t}{3}\right)\right\}$$

Therefore by using (3.6) and (3.7) we have

$$P_{\mu,\nu}(f(x) - 3h(x), t) = P_{\mu,\nu}(2f(x) - 6h(x), 2t)$$

(3.9) $$\geq_{L^*} M\left(P'_{\mu,\nu}(\phi(x, -x), t), P'_{\mu,\nu}(\phi(x, x), t)\right)$$

Now using (3.7), (3.8) and (3.9, we have)

$$P_{\mu,\nu}(f(2x) - 2f(x), 3t)$$
$$= P_{\mu,\nu}(f(2x) - f(x) - 3g(x) - 2f(x)$$
$$+ 3g(x) + 3h(x) + f(x) - 3h(x), t + t + t)$$
$$\geq_{L^*} M^2(P_{\mu,\nu}(f(2x) - f(x) - 3g(x), t),$$
$$P_{\mu,\nu}(2f(x) - 3g(x) - 3h(x), t), P_{\mu,\nu}(f(x) - 3h(x), t))$$
$$\geq_{L^*} M^3(P'_{\mu,\nu}(\phi(x, 0), t), P'_{\mu,\nu}(\phi(x, -x), t),$$
$$P'_{\mu,\nu}(\phi(x, -x), t), P'_{\mu,\nu}(\phi(x, x), t))$$

Therefore

$$P_{\mu,\nu}(f(2x) - 2f(x), t)$$
$$\geq_{L^*} M^3\left\{P'_{\mu,\nu}\left(\phi(x, 0), \frac{t}{3}\right), P'_{\mu,\nu}\left(\phi(x, -x), \frac{t}{3}\right),\right.$$
$$\left. P'_{\mu,\nu}\left(\phi(x, -x), \frac{t}{3}\right), P'_{\mu,\nu}\left(\phi(x, x), \frac{t}{3}\right)\right\}$$

(3.10)
$$= M_1(x, t)$$

Consider the set $E = \{f : X \to Y\}$ and introduce a complete generalized metric on E and define it on E by

$$d(g, h) = \inf\{k \in R^+, P_{\mu,\nu}(g(x) - h(x), kt) \geq_{L^*} M_1(x, t)\}$$

for all $x \in X$ and $t > 0$.
We now define a function $J : E \to E$

$$J\,g(x) = \frac{1}{2}\,g(2\,x), \text{ for all } x \in X$$

Now we want to show that J is a strictly contractive mapping of E with the Lipschitz constant $\frac{\alpha}{2}$.
Let $g, h \in E$ be given such that $d(g, h) < K$. Then $P_{\mu,\nu}(J\,g(x) - J\,h(x), k\,t) = P_{\mu,\nu}\left(\frac{1}{2}\,g(2\,x) - \frac{1}{2}\,h(2\,x), k\,t\right)$

$$= P_{\mu,\nu}(g(2\,x) - h(2\,x), 2\,k\,t) \geq_{L^*} M_1(2\,x, 2\,t) \geq_{L^*} M_1\left(x, \frac{2\,t}{\alpha}\right)$$

for all $x \in X$ and $t > 0$. Therefore

$$P_{\mu,\nu}\left(J\,g(x) - J\,h(x), \frac{\alpha\,k\,t}{2}\right) \geq_{L^*} M_1(x, t)$$

By the definition $d(J\,g, J\,h) \leq \frac{\alpha\,k}{2}$

$$\text{Therefore } d(J\,g, J\,h) \leq \frac{\alpha}{2}\,d(g, h)$$

for all g, h $\in E$.
Thus from (3.10) $d(f, J\,f) \leq \frac{1}{2}$

It follows from Theorem 2.8, there exists a mapping $H : X \to Y$ satisfying
1. H is a fixed point of J i.e,

$H(2\,x) = 2\,H(x)$ for all $x \in X$.

The mapping H is a unique fixed point of J in the set $E_1 = \{g \in E : d(g, f) < \infty\}$
This implies that H is a unique mapping satisfying $H(2\,x) = 2\,H(x)$ such that there exists $k \in (0, \infty)$ satisfying

$$P_{\mu,\nu}(f(x) - H(x), k\,t) \geq_{L^*} M_1(x, t)$$

for all $x \in X$, $t > 0$;

2. $d(J^n f, H) \to 0$ as $n \to \infty$. This implies the equality

$$H(x) = \lim_{n\to\infty} J^n f(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n} \tag{3.11}$$

for all $x \in X$;

3. $d(f, H) \leqslant \frac{1}{1-L} d(f, Jf)$ with $f \in E_1$ which implies the inequality

$$d(f, H) \leq \frac{1}{1-\frac{\alpha}{2}} \times \frac{1}{2} = \frac{1}{2-\alpha}$$

then it follows that

$$P_{\mu,\nu}(H(x) - f(x), \frac{1}{2-\alpha} t) \geq_{L^*} M_1(x, t)$$

It implies that

$$P_{\mu,\nu}(H(x) - f(x), t) \geq_{L^*} M_1(x, (2-\alpha)t) \tag{3.12}$$

for all $x \in X; t > 0$.
Replacing x and y by $2^n x$ and $2^n y$ in (3.2)

$$P_{\mu,\nu}(f(2^n(2x+y)) - f(2^n(x+2y)) - 3g(2^n(x)) + 3h(2^n(y)), t)$$
$$\geq_{L^*} P'_{\mu,\nu}(\phi(2^n(x, y)), t)$$

or,

$$P_{\mu,\nu}\left(\frac{f(2^n(2x+y))}{2^n} - \frac{f(2^n(x+2y))}{2^n} - 3\frac{g(2^n(x))}{2^n} + 3\frac{h(2^n(y))}{2^n}, t\right)$$

$$\geq_{L^*} P'_{\mu,\nu}(\phi(2^n x, 2^n y), 2^n t)$$

for all $x, y \in X; t > 0$.
Taking the limit as $n \to \infty$ we get

$$H(2x + y) - H(x + 2y) = H(x) - H(y) \tag{3.13}$$

Also

$$P_{\mu,\nu}\left(3g(x) + 3h(x2H(x), \tfrac{8-\alpha}{3}t\right)$$
$$\geq_{L^*} M\left(P_{\mu,\nu}(2f(x) - 2H(x), 2t), P_{\mu,\nu}\left(3g(x) + 3h(x) - 2f(x), \tfrac{t(2-\alpha)}{3}\right)\right)$$
$$\geq_{L^*} M\left(M_1(x, (2-\alpha)t), P'_{\mu,\nu}\left(\phi(x, -x), \tfrac{t\times(2-\alpha)}{3}\right)\right)$$
$$\geq_{L^*} M_1(x, (2-\alpha)t)$$

Hence
(3.14)

$$P_{\mu,\nu}(3g(x) + 3h(x) - 2H(x), t) \geq_{L^*} M_1\left(x, \frac{t \times 3(2-\alpha)}{8-\alpha}\right)$$

The uniqueness of H follows from the fact that H is the unique fixed point of J with the following property that there exists $u \in (0, \infty)$ such that

$$P_{\mu,\nu}(f(x) - H(x), ut) \geq_{L^*} M_1(x, t)$$

for all $x \in X$ and $t > 0$. This completes the proof of the theorem. □

Corollary 1. Let $\delta > 0$ and X be linear space with $p < 2$, $(Z, P'_{\mu,\nu}, M)$ be an IFN-space, $(Y, P'_{\mu,\nu}, M)$ be a complete IFS-space. If $f, g, h : X \to Y$ are odd mappings such that

$P_{\mu,\nu}(f(2x + y) - (x + 2y) - 3g(x) + 3h(y), t)$

then $H(x) := P_{\mu,\nu} - \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$ exists for each $x \in X$ and define a unique mapping $H : X \to Y$ such that

$P_{\mu,\nu}(f(x) - H(x), t) \geq_{L^*} P'_{\mu,\nu}\left(\delta, \frac{t}{3}(2 - 2^p)\right)$

and $P_{\mu,\nu}(3g(x) + 3h(x) - 2H(x), t) \geq_{L^*} P'_{\mu,\nu}\left(\delta, \frac{2-2^p}{8-2^p} t\right)$
for all $x \in X$ and $t > 0$.

Proof : Let $\phi : X \times X \to Z$ be define by $\phi(x, y) = \delta$, then the proof is followed by Theorem (3.1).

Corollary 2. Let $\theta \geq 0$ and $p < 2$ be a non-negative real number and X be linear space, $(Z, P'_{\mu,\nu}, M)$ be an IFN-space, $(Y, P'_{\mu,\nu}, M)$ be a complete IFS-space. If $f, g, h : X \to Y$ are odd mappings such that

$P_{\mu,\nu}(f(2x + y) - (x + 2y) - 3g(x) + 3h(y), t)$

$\geq_{L*} P'_{\mu,\nu}(\theta(\|x\|^p + \|y\|^p), t)$ where $(x, y \in X, t > 0, \theta \in Z)$

then $H(x) := P_{\mu,\nu} - \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$ exists for each $x \in X$ and define a unique mapping $H : X \to Y$ such that

$P_{\mu,\nu}(f(x) - H(x), t) \geq_{L^*} P'_{\mu,\nu}\left(\theta\|x\|^p, \frac{t}{6}(2 - 2^p)\right)$

and $P_{\mu,\nu}(3g(x) + 3h(x) - 2H(x), t) \geq_{L^*} P'_{\mu,\nu}\left(\theta\|x\|^p, \frac{1-2^{p-1}}{8-2^p} t\right)$
for all $x \in X$ and $t > 0, \theta \in Z$.

Proof : Define $\phi(x, y) = \theta(\|x\|^p + \|y\|^p)$ and it can be proved by similar way as theorem (3.1) by $\alpha = 2^p$

Theorem 3.2. *Let X be a linear space,* $(Z, P'_{\mu,\nu}, M)$ *be a IFN-space,* $\phi : X \times X \to Z$ *be a function such that for some* $0 < \alpha < 2$,

(3.15) $$P'_{\mu,\nu}(\phi(2x, 2x), t) \geq_{L^*} P'_{\mu,\nu}(\alpha\phi(x, x), t)$$

$(x \in X, t > 0)$ *and*

$$\lim_{n\to\infty} P'_{\mu,\nu}(\phi(2^n x, 2^n), 4^n t) = 1_{L^*}$$

for all $x, y \in X$ *and* $t > 0$. *Let* $(Y, P_{\mu,\nu}, M)$ *be a complete IFN-space. If* $f, g, h : X \to Y$ *are mappings with* $g(x) = f(x) + h(0)$ *such that*

$$P_{\mu,\nu}(f(2x + y) - f(x + 2y) - 3g(x) + 3h(y), t)$$

$$\geq_{L^*} P'_{\mu,\nu}(\phi(x, y), t) \tag{3.16}$$

$(x \in X, t > 0)$. *Then there exists a unique mapping* $H : X \to Y$ *define by* $H(x) := P_{\mu,\nu} - \lim_{n\to\infty} \frac{f(2^n x)}{4^n}$ *for all* $x \in X$ *satisfying*

$$P_{\mu,\nu}(f(x) - H(x), t) \geq_{L^*} \phi(x, t(4 - \alpha)) \tag{3.17}$$

Proof. It can be proved by similar way as theorem (3.1)

□

Example 3.3. Let $(X, \|.\|)$ be a Banach algebra, M a continuous t-norm defined in Example (2.6). Then $(X, P_{\mu,\nu}, M)$ is a complete IFN-space.
we define $f, g, h : X \to X$, by $f(x) = x^2 + A\|x\|x_0$, $g(x) = x^2 + B\|x\|x_0$,
$h(y) = y^2 + C\|y\|x_0$, where x_0 is unit vectors in X and $A, B, C \in X$.
Then $\|f(2x + y) - f(x + 2y) - 3g(x) + 3h(y)\|$
$\leq 3(A + B)\|x\| + 3(A + C)\|y\|$
for all $x, y \in X$.
Thus $P_{\mu,\nu}(f(2x + y) - f(x + 2y) - 3g(x) + 3h(y), t)$
$\geq_{L^*} P_{\mu,\nu}(3(A + B)\|x\| x_0 + 3(A + C)\|y\| x_0, t)$
for all $x, y \in X$ and $t > 0$.
Let $\phi(x, y) = 3(A + B)\|x\| x_0 + 3(A + C)\|y\| x_0$ for all $x, y \in X$.
Also $P_{\mu,\nu}(\phi(2x, 2y), t) \geq P_{\mu,\nu}(2\phi(x, y), t)$ for all $x, y \in X$ and $t > 0$. Hence all the conditions of theorem 3.1 holds. Therefore f can be approximated by a mapping $H : X \to X$ such that $P_{\mu,\nu}(f(x) - H(x), t) \geq_{L^*} P_{\mu,\nu}\left(\|x\| x_0, \frac{t}{9\min\{(A+B), (2A+B+C)\}}\right)$ for all $x, y \in X$ and $t > 0$.

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