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HYPER UP-ALGEBRAS

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Abstract. In this paper, the concept of hyper UP-algebras is introduced and some related properties are investigated. Also, the concepts of hyper UP-ideals have been introduced and analyzed. In addition, the concept of homomorphisms between hyper UP-algebras is also considered.

Key Words: UP-algebra, Hyper UP-algebra, Hyper UP-ideal of type 1 (2,3,4 res.), Hyper UP-homomorphism.

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1. Introduction

The hyper structures theory (called also multialgebras) is introduced in 1934 by F. Marty [10] at the 8th congress of Scandinavian Mathematicians. Hyper structures have many applications to several sectors of both pure and applied parts of mathematics ([3]. A good reference for the theory of hyper-structures and its applications to Mathematics and Computer Science can be found in [11, 12]. In [19], Y. B. Jun, M. M. Zahedi, X. L. Xin, and R. A. Borzooei applied the hyper structures to BCK-algebras, and introduced the concept of hyper BCK-algebras which is a generalization of BCK-algebras and investigated some related properties. They also introduced the concept of hyper BCK-ideals. For more about hyper BCK- algebra we refer the reader to [15, 2]. Hyper BCC-algebras were introduced and analyzed by R. A. Borzooei, W. A. Dudek and N. Koohestani. in the article [16]. The properties of the hyper BCC-algebra were analyzed by D. S. Uzay and A. Firat in the text

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[9] also. Hyper BE algebras were investigated by A. Radfar, A. Rezaei and A. B. Saeid in [1] and hyper BCI-algebras were analyzed by X. L. Xin in [18]. In [17], S. M. Mostafa, F. F. Kareem and B. Davvaz applied the hyper structures to KU-algebras.

Iampan [4] introduced a new algebraic structure which is called UP-algebras as a generalization of KU-alebras. He studied ideals and congruences in UP-algebras. He also introduced the concept of homomorphism of UP-algebras and investigated some related properties. Moreover, he derived some straightforward consequences of the relations between quotient UP-algebras and isomorphism. In the study of these algebraic structures, this author took part also ([6, 7, 8]). In this paper we introduced the concept of hyper UP-algebras and some types properties of hyper UP-algebras are studied. Also, homomorphisms between hyper UP-algebras are analyzed.

2. Preliminaries

2.1. **UP-algebras.** In this subsection we will describe some elements of UP-algebras and their substructures from the literature [4, 5, 14] necessary for our intentions in this text.

Definition 2.1. ([4]) An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a UP-algebra where A is a nonempty set, $'\cdot'$ is a binary operation on A, and 0 is a fixed element of A (i.e. a nullary operation) if it satisfies the following axioms:

(UP-1) $(\forall x, y \in A)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$,
(UP-2) $(\forall x \in A)(0 \cdot x = x)$,
(UP-3) $(\forall x \in A)(x \cdot 0 = 0)$, and
(UP-4) $(\forall x, y \in A)((x \cdot y = 0 \ \wedge \ y \cdot x = 0) \implies x = y)$.

On a UP-algebra $A = (A, \cdot, 0)$, we define the UP-ordering $\leqslant$ on A as follows:

$$(\forall x, y \in A)(x \leqslant y \iff x \cdot y = 0).$$

Proposition 2.2 ([4])**.** *In a UP-algebra A, the following properties hold:*

(1) $(\forall x \in A)(x \leqslant x)$,
(2) $(\forall x, y \in A)((x \leqslant y \wedge y \leqslant x), \implies x = y)$,
(3) $(\forall x, y, z \in A)((x \leqslant y \wedge y \leqslant z) \implies x \leqslant z)$,
(4) $(\forall x, y, z \in A)(x \leqslant y \implies z \cdot x \leqslant z \cdot y)$,
(5) $(\forall x, y, z \in A)(x \leqslant y \implies y \cdot z \leqslant x \cdot z)$,
(6) $(\forall x, y \in A)(x \leqslant y \cdot x)$ *and*
(7) $(\forall x \in A)(x \leqslant 0)$.

Definition 2.3. A nonempty subset S of a UP-algebra $(A, \cdot, 0)$ is called
(a) ([4]) a UP-subalgebra of A if $(\forall x, y \in S)(x \cdot y \in S)$.
(b) ([4]) a UP-ideal of A if
(i) $0 \in S$; and
(ii) $(\forall x, y, z \in A)((x \cdot (y \cdot z) \in S \wedge y \in S) \implies x \cdot z \in S)$;
(c) ([6]) a proper UP-filter if
(iii) $\neg(0 \in S)$ and
(iv) $(\forall x, y, z \in A)((\neg(x \cdot (y \cdot z) \in S) \wedge x \cdot z \in S) \implies y \in S)$.

The set $\{0\}$ is a trivial UP-subalgebra (trivial UP-ideal) of A. The set $A \setminus \{0\}$ is a trivial proper UP-filter in A.

In the article [8], Theorem 3.3, it has been shown that the conditions (i) and (ii) in the point (b) of the preceding definition are equivalent to the following conditions:
(i') $(\forall x, y \in A)((x \cdot y \in S \wedge x \in S) \implies y \in S)$,
(ii') $(\forall x, y \in A)(y \in S \implies x \cdot y \in S)$.
In the article [7], Theorem 3.1, it has been shown that the conditions (iii) and (iv) in the point (c) of the preceding definition are equivalent to the following conditions:
(iii') $(\forall x, y \in A)(\neg(x \cdot y \in S) \wedge y \in S) \implies x \in S)$,
(iv') $(\forall x, y \in A)(x \cdot y \in S \implies y \in S)$.

Definition 2.4. ([4]) Let $(A, \cdot, 0_A)$ and $(B, \cdot', 0_B)$ be two UP-algebras. A mapping $f : A \longrightarrow B$ is called a UP-homomorphism if

$$(\forall x, y \in A)(f(x \cdot y) = f(x) \cdot' f(y)).$$

A UP-homomorphism $f : A \longrightarrow B$ is called
- a UP-epimorphism if f is surjective,
- a UP-monomorphism if f is injective, and
- a UP-isomorphism if f is bijective.

Let f be a mapping form UP-algebra A to UP-algebra B, and let C and D be nonempty subsets of A and of B, respectively. The set $\{f(x)|x \in C\}$ which denoted by $f(C)$ is called the image of C under f. In particular, $f(A)$ which denoted by $Im(f)$ is called the image of f. The dually set $\{x \in A|f(x) \in D\}$ which denoted by $f^{-1}(D)$ is called the inverse image of D under f. Especially, the set $f^{-1}(\{0_B\})$ which written by $Ker(f)$ is called the kernel of f.

Proposition 2.5 ([4]). *Let $(A, \cdot, 0_A)$ and $(B, \cdot', 0_B)$ be UP-algebras and let $f : A \longrightarrow B$ be a UP-homomorphism. Then the following statements hold:*

(8) *$f(0_A) = 0_B$;*

(9) *$(\forall x, y \in A)(x \leqslant_A y \implies f(x) \leqslant_B f(y))$;*

(10) *if C is a UP-subalgebra of A, then the image $f(C)$ is a UP-subalgebra of B. In particular, $Im(f)$ is a UP-subalgebra of B;*

(11) *if D is a UP-subalgebra of B, then the inverse image $f^{-1}(D)$ is a UP-subalgebra of A. In particular, $Ker(f)$ is a UP-subalgebra of A;*

(12) *if D is a UP-ideal of B, then the inverse image $f^{-1}(D)$ is a UP-ideal of A. In particular, $Ker(f)$ is a UP-ideal of A;*

(13) *if C is a UP-ideal of A such that $Ker(f) \subseteq C$, then the image $f(C)$ is a UP-ideal of $Im(f)$; and*

(14) *$Ker(f) = \{0_A\}$ if and only if f is an injective mapping.*

2.2. **Designing of hyper-structures.** Hyper-structures (also known as hyperalgebras, or non-deterministic algebras) has been studied from different standpoints over the last several decades. A survey which covers a part of the historical development of the theory of hyper-algebras, presenting the main approaches of important concepts on the hyper-structure theory from the point of view of universal algebra, it can be found in [13, 3]. In ordinary algebras the concept of internal binary operations is a fundamental. If it generalized to multi-operation, these leads to the concepts of multi-algebras. An operation in a set X is a total function $w : X \times X \longrightarrow X$ that manipulate elements of a set $X \times X$ and returns the unique value in set X. A multi-operation (or hyper-operation) is a generalization of an operation when it returns a set of values instead of a single value. The class of structures composed by a set and at least one multi-operation is what we call of algebraic hyper-structure ([3], pp. 2).

Let X be a nonempty set and $P^*(X) = P(X) \setminus \{\emptyset\}$ the family of all nonempty subsets of X. A multi-valued operation (said also hyper operation) $' \circ '$ on X is a function, which associates with every pair $(x, y) \in X \times X$ a nonempty subset of X denoted by $x \circ y$. An *algebraic hyper structure* or simply a *hyper structure* is a nonempty set X endowed with one or more hyper operations. Note that if $A, B \subseteq X$, then by $A \circ B$ we mean the subset $\bigcup_{a \in A} \bigcup_{b \in B} a \circ b$ of X.

Let $(X, \circ)$ be a hyper-structure such that $0 \in X$. First, let us determine the relation of hyper order $' \ll '$ in hyper-structure $(X, \circ)$. We define

$(\forall A, B \in P^*(X))(A \ll B \iff (\forall a \in A)(\exists b \in B)(0 \in a \circ b))$.

This relationship is called *hyper-order*. Let $x \ll y$ be instead of $\{x\} \ll \{y\}$. Then

$(\forall x, y \in X)(x \ll y \iff 0 \in x \circ y)$.

3. Hyper UP-algebras

3.1. **The concept of hyper UP-algebras.** In the following definition, the concept of hyper UP-algebras is introduced.

Definition 3.1. Let X be a nonempty set such that $0 \in X$ and $(X, \circ, \ll, 0)$ be a hyper-structure. Then, $(X, \circ, \ll, 0)$ is called a *hyper UP-algebra* if the following formulas are valid:
(HUP1) $(\forall x, y, z \in X)(y \circ z \ll (x \circ y) \circ (x \circ z))$,
(HUP2) $(\forall x \in X)(x \circ 0 = \{0\})$,
(HUP3) $(\forall x \in X)(0 \circ x = \{x\})$, and
(HUP4) $(\forall x, y \in X)((x \ll y \wedge y \ll x) \implies x = y)$.

Example 3.2. Let $X = \{0, a, b\}$ be a set. If we define a hyper operation $\circ$ on X as following

$\circ$	0	a	b
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{0\}$	$\{0, a\}$	$\{a, b\}$
b	$\{0\}$	$\{0, a\}$	$\{0, a, b\}$

,

then $(X, \circ, \ll)$ is a hyper UP-algebra.

Our first proposition shows some of the basic properties of the relationship $\ll$ in any hyper UP algebra $(X, \circ, \ll)$.

Proposition 3.3. *Let $(X, \circ, \ll)$ be a hyper UP-algebra. Then, the following statements hold:*
(15) $(\forall A, B \subseteq X)(A \subseteq B \implies A \ll B)$,
(16) $0 \circ 0 = \{0\}$,
(17) $(\forall x \in X)(x \ll 0)$,
(18) $(\forall x \in X)(x \ll x)$, *i.e.* $(\forall x \in X)(0 \in x \circ x)$,
(19) $(\forall x, z \in X)(z \ll x \circ z)$,
(20) $(\forall A \subseteq X)(A \circ 0 = \{0\})$,
(21) $(\forall B \subseteq X)(0 \circ B = B)$
(22) $(\forall x \in X)((0 \circ 0) \circ x = \{x\})$.

Proof. (16) If we put $x = 0$ in the formula (HUP2), we get $0 \circ 0 = \{0\}$.
(17) Since $x \circ 0 = \{0\}$ for any $x \in X$, according to (HUP2), we have $0 \in x \circ 0$. So, $x \ll 0$ according to the definition of the relation $\ll$.
(15) Let A and B be subsets of X such that $A \subseteq B$. In order for $A \ll B$ to be valid have to be $(Va \in A)(\exists b \in B)(a \ll b)$ valid. Since A

is a subset of B, we can take $b = a$ for a pre-selected a. Then we have $a \ll a = b$ by (17) which is sufficient for $A \ll B$.

(18) If we put $y = x = 0$ and $z = x$, in the formula (HUP1), we get $0 \circ x \ll (0 \circ 0) \circ (0 \circ x)$. Thus $\{x\} \ll \{0\} \circ \{x\}$ by (HUP3) and (16). Then we obtain $\{x\} \ll \{x\}$ by the definition of the operation $\circ$. So, the formula (18) is a valid formula.

(19) If we put $y = 0$ in the formula (HUP1), we get $0 \circ z \ll (x \circ 0) \circ (x \circ z)$. Thus $\{z\} \ll \{0\} \circ (x \circ z)$ by (HUP3) and (HUP2) and $\{z\} \ll x \circ z$ by the definition of the operation $\circ$. Therefore, the formula (19) is proven.

(20) Formula (20) is a direct consequence of the formula (HUP2).

(21) Formula (21) is a direct consequence of the formula (HUP3).

(22) $(0 \circ 0) \circ x = \{0\} \circ x = \{x\}$ by (14), the definition of the operation $\circ$ and (HUP3) for any $x \in X$. □

Definition 3.4. For hyper algebra X, it is said that it is a *diagonal* hyper UP-algebra if

(HUP5) $(\forall x \in X)(x \circ x = \{0\})$

is true.

In the next proposition, we show the left compatibility of the hyper-order relation with a hyper operation in hyper UP-algebras.

Proposition 3.5. *In any hyper UP-algebra* $(X, \circ, \ll, 0))$*, the following formula is valid*

(23) $(\forall x, y, z \in X)(y \ll z \implies (x \circ y) \ll (x \circ z))$.

Proof. Let $x, y, z \in X$ be arbitrary elements such that $y \ll z$. Thus $0 \in y \circ z$. On the other hand, if the axiom (HUP1) is written in the form $0 \in (y \circ z) \circ ((x \circ y) \circ (x \circ z))$, we have $0 \in y \circ z$ and $0 \in (y \circ z) \circ ((x \circ y) \circ (x \circ z))$. From here it follows that $0 \in (x \circ y) \circ (x \circ z)$ according to the definition of the operation $\circ$ and to (HUP3). So, $x \circ y \ll x \circ z$. □

In the next proposition we prove one specificity of hyper UP algebras.

Proposition 3.6. *In any hyper UP-algebra* $(X, \circ, \ll, 0))$*, the following formula is valid*

(24) $(\forall x, y, z \in X)((x \ll y \wedge y \ll z) \implies x \ll z)$.

Proof. Let $x, y, z \in X$ artbitrary elements such that $x \ll y$ and $y \ll z$. This means $0 \in x \circ y$ and $0 \in y \circ z$. From (HUP1) in the form $0 \in (y \circ z \circ z)((x \circ y) \circ (x \circ z))$ follows $0 \in (x \circ y) \circ (z \circ z)$ by a hypothesis $0 \in y \circ z$. Now, from $0 \in (x \circ y) \circ (z \circ z)$ follows $0 \in x \circ z$ by a hypothesis $0 \in x \circ y$. So, $x \ll z$. □

Theorem 3.7. *The relation* $\ll$ *in a hyper UP-algebra* $(X, \circ, 0, \ll)$ *is a partial order.*

Proof. The relation $\ll$ is a reflexive relation according to (18), an antisymmetric relation by (HUP4), and a transitional relation by (24). Therefore, it is a partial order. $\square$

The following theorem is an important result about connection between hyper KU-algebras ([17], Definition 3.1) and hyper UP-algebras. We draw attention to the reader that the relation $\ll$ in the article [17] is determined as follows

$(\forall a, b \in K)(x \ll y \iff 0 \in y \circ x)$.

Theorem 3.8. *Let* $(X, \circ, \ll, 0)$ *be a hyper KU-algebra. Then* $(X, \circ, \ll^{-1}, 0)$ *is a hyper UP-algebra.*

Proof. Let X be hyper KU-algebra. To show that the $(X, \circ, \ll^{-1}, 0)$ is a hyper UP algebra it is sufficient to show that in the system (HKU1), (HKU2), (HKU3), (HKU4) axiom (HUP1) is a valid formula. Since in every hyper KU algebra $(X, \circ, \ll, 0)$ the following formula

$(\forall x, y, z \in X)(z \circ (x \circ y) = y \circ (z \circ x))$

is valid by Lemma 3.6 in [17], we have the following deduction

$(\forall x, y, z \in X)((y \circ z) \circ (x \circ z) \ll x \circ y)$ ((HKU1) - hypothesis)

$(\forall x, y, z \in X)(0 \in (x \circ y) \circ ((y \circ z) \circ (x \circ z)))$ (by definition of $\ll$ in [17])

$(\forall x, y, z \in X)(0 \in (y \circ z) \circ ((x \circ y) \circ (x \circ z)))$ (by Lemma 3.6 in [17])

$(\forall x, y, z \in X)(y \circ z \ll^{-1} (x \circ y) \circ (x \circ z))$ (by definition of $\ll$ in [17])

which was to be proven. Therefore, $(X, \circ, \ll^{-1}, 0)$ is a hyper UP-algebra. $\square$

3.2. **Concept of hyper UP-subalgebras.**

Definition 3.9. Let $(X, \circ, \ll, 0)$ be a hyper UP-algebra and let S be a subset of X containing 0. If S is a hyper UP-algebra with respect to the hyper operation $'\circ'$ on X, we say that S is a hyper UP-subalgebra of X.

Lemma 3.10. *Let* S *be a non-empty subset of a hyper UP-algebra* $(X, \circ, \ll, 0)$*. If* $x \circ y \subseteq S$ *for all* $x, y \in S$*, then* $0 \in S$.

Proof. Assume that $x \circ y \subseteq S$ for all $x, y \in S$. Since S is non-empty, this means that there is an element $a \in S$. Since $a \ll a$ by (18), we have $0 \in a \circ a \subseteq S$. $\square$

Proposition 3.11. *Let S be a non-empty subset of a hyper UP-algebra $(X, \circ, \ll, 0)$. Then S is a hyper UP-subalgebra of X if and only if $(\forall x, y \in S)(x \circ y \subseteq S)$ holds.*

Proof. ($\Longrightarrow$) If S is a hyper UP-subalgebra of UP-algebra X, then it is clear, by definition, that the given formula is valid.

($\Longleftarrow$) Assume that $(\forall x, y \in S)(x \circ y \subseteq S)$ holds. Then $0 \in S$ by Lemma 3.10. For any $x, y, z \in S$, we have $x \circ z \subseteq S$, $y \circ z \subseteq S$ and $x \circ y \subseteq S$. Hence

$$(x \circ y) \circ (x \circ z) = \bigcup_{a \in x \circ y} \bigcup_{b \in x \circ z} a \circ b \subseteq S.$$

Since (HUP1) is valid in X and since all products $z \circ y$, $x \circ y$ and $x \circ z$ lie in S, this means that (HUP1) is valid in S. Similarly we can prove that the axioms (HUP2), (HUP3) and (HUP4) are true in S. Therefore S is a hyper UP-subalgebra of X. □

3.3. **Concept of hyper UP-ideals.** In this subsection, the concept of hyper UP-ideals in the hyper UP-algebra was introduced in an analogous way, as was done in the article [16], Definition 4.1, by defining the concept of the hyper BCC-ideals in hyper BCC algebras. Analogously, the concept of hyper KU-ideals of type 1 (2, 3, 4) is determined in article [17], Definition 5.1.

Definition 3.12. A subset S of a hyper UP-algebra $(X, \circ, \ll, 0)$ such that $0 \in S$ is called the following:

(i) a *hyper UP-ideal of type 1*, if

(25) $(\forall x, y, z \in X)((x \circ (y \circ z) \ll S \wedge y \in S) \Longrightarrow x \circ z \subseteq S)$;

(ii) a *hyper UP-ideal of type 2*, if

(26) $(\forall x, y, z \in X)((x \circ (y \circ z) \subseteq S \wedge y \in S) \Longrightarrow x \circ z \subseteq S)$;

(iii) a hyper UP-ideal of type 3, if

(27) $(\forall x, y, z \in X)((x \circ (y \circ z) \ll S \wedge y \in S) \Longrightarrow x \circ z \ll S)$;

(iv) a *hyper UP-ideal of type 4*, if

(28) $(\forall x, y, z \in X)((x \circ (y \circ z) \subseteq S \wedge y \in S) \Longrightarrow x \circ z \ll S)$.

The set $\{0\}$ is trivial hyper UP-ideal of type 1 in X.

The following theorems describes the basic properties of these UP-ideals.

Theorem 3.13. *Let S be a hyper UP-ideal of a hyper UP-algebra $(X, \circ, \ll, 0)$. Then, the following are valid:*

- *Any hyper UP-ideals of type 1 is a hyper UP-ideal of type 2, 3 and 4.*
- *Any hyper UP-ideals of type 2 is a hyper UP-ideal of type 4.*
- *Any hyper UP-ideals of type 3 is a hyper UP-ideal of type 4.*

Proof. Each of the assertions in the theorem is proved by relying on (15). □

Theorem 3.14. *Let S be a hyper UP-ideal of type 1 of a hyper UP-algebra $(X, \circ, \ll, 0)$. Then, the following are valid:*
(25a) $(\forall x, y \in X)((x \circ y \ll S \wedge x \in S) \implies y \subseteq S)$.
If X is a diagonal hyper UP-algebra, then holds
(25b) $(\forall x, y \in X)(y \in S \implies x \circ y \subseteq S)$.

Proof. (1) If in (25) we put $x = 0$, $y = x$ and $z = y$, we get
$(\forall x, y \in X)((0 \circ (x \circ y) \ll S \wedge x \in S) \implies 0 \circ y \subseteq S)$.
From here, according to (HYP3) and (21), follows
$(\forall x, y \in X)((x \circ y \ll S \wedge x \in S) \implies y \subseteq S)$.
which needed to be proved. Thus, formula (25a) is a valid formula.
(2) Assume that X is a diagonal hyper UP-algebra. If in (25) we put $z = y$, we get
$(\forall x, y \in X)((x \circ (y \circ y) \ll S \wedge y \in S) \implies x \circ y \subseteq S)$.
Since $y \circ y = \{0\}$ and $x \circ \{0\} = \{0\} \subseteq S$ is true because S is a hyper UP-ideal in the diagonal hyper UP-algebra X, we get
$(\forall x, y \in X)(x \in S \implies x \circ y \subseteq S)$. □

Remark 3.15. As a hyper UP-ideal S of type 1, at the same time is a hyper UP-ideal of type 2, 3 and 4, then the analogues assertions expressed in the previous theorem can apply to the hyper UP-ideals of type 2, 3 and 4 respectively.

3.4. **Concept of hyper homomorphism of hyper UP-algebras.**

Definition 3.16. Let $(X, \circ, \ll, 0)$ and $(Y, \circ', \ll', 0')$ be hyper UP-algebras. A mapping $f : X \longrightarrow Y$ is called a *hyper homomorphism* if
(29) $f(0) = 0'$, and
(30) $(\forall a, b \in X)(f(a \circ b) = f(a) \circ' f(y))$.

Theorem 3.17. *Let $f : X \longrightarrow Y$ be a hyper homomorphism of hyper UP-algebras. Then holds*
(31) $(\forall x, y \in X)(x \ll y \implies f(y) \ll' f(y))$.

Proof. Let $x, y \in X$ arbitrary elements such that $x \ll y$. Then, $0 \in x \circ y$, and by previous definition $0' = f(0) \subseteq f(x \circ y) = f(x) \circ' f(y)$. Thus $f(x) \ll' f(x)$. □

Theorem 3.18. *Let $f : X \longrightarrow Y$ be a hyper homomorphism of hyper UP-algebras. If T is a hyper UP-ideal of type 1 in Y, then $f^{-1}(T)$ is a hyper UP-ideal of type 1 in X.*

Proof. Let T be a hyper UP-ideal of type 1 in Y. Since $f(0) = 0' \in T$ by Definition 3.12 and (29), we have $0 \in f^{-1}(0') \subseteq f^{-1}(T)$.

Let $x, y, z \in X$ be arbitrary elements such that $x \circ (y \circ z) \ll f^{-1}(T)$ and $y \in f^{-1}(T)$. Then there exists an element $a \in f^{-1}(T)$ such that $0 \in (x \circ (y \circ z)) \circ a$ Thus

$$0' = f(0) \in f((x \circ (y \circ z)) \circ a) = f(x \circ (y \circ z)) \circ' f(a).$$

So, $f(x) \circ (f(y) \circ' f(z))) \ll' T$ and $f(y) \in T$. Since T is a hyper UP-ideal of type 1 in Y, follows $f(x \circ z) = f(x) \circ' f(z) \subseteq T$. Thus $x \circ z \subseteq f^{-1}(T)$. Therefore, the subset $f^{-1}(T)$ is a hyper UP-ideal of type 1 in X. □

Corollary 3.19. *Let $f : X \longrightarrow Y$ be a hyper UP-homomorphism between hyper UP-algebras. Then $Ker(0')$ is a hyper UP-ideal in X.*

Proof. Since $\{0'\}$ is a trivial hyper UP-ideal of type 1 in Y, then $Kerf = \{x \in X : f(x) = 0'\} = f^{-1}(\{0'\})$ is a hyper UP-ideal of type 1 in X by previous theorem. □

Remark 3.20. In accordance with the Remark 3.15, corresponding assertion also applies to the hyper UP-ideals of the remaining types, respectively.

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