

ON G-FUZZY CONE METRIC SPACES

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ABSTRACT. The aim of this paper is to introduce and study the concept of generalized fuzzy cone metric space and to obtain the topology by using G-fuzzy cone metric. Moreover, the existence of common fixed point theorem for contractive map and occasionally weakly compatible map and is investigated in generalized fuzzy cone metric space.

Key Words: \mathcal{M} -fuzzy metric space, fuzzy cone metric space, fixed point.

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1. INTRODUCTION

Huang and Zhang [3] defined the concept of cone metric space, replacing real numbers with an ordered Banach space and proposed some fixed point theorems satisfying different contractive mappings.

The theory of fuzzy sets was initiated by Zadeh [12]. After the introduction of the notion of fuzzy set, many researchers have improved this concept to topology and analysis. Specially, the concept of fuzzy metric space was defined by Kramosil and Michalek in [7]. Then, George and Veeramani in [5] revised this definition. So, they obtained that fuzzy metric induces a Hausdorff topology in this space and later many researchers contributed to the development of the subject. One of these developments is the generalization of fuzzy metric space. Recently, Sedghi et.al [10] introduced \mathcal{M} -fuzzy metric space and proved a common fixed

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point theorem for some mappings satisfying sufficient conditions in this space. In 2015, Oner et.al [9] introduced the notion of fuzzy cone metric space and proved Banach contraction theorem. In 2019, Gregori et. al [6] studied new notion called fuzzy partial metric space.

In this paper, we introduce a new type of generalized fuzzy metric space which we call generalized fuzzy cone metric space, as a generalization of both fuzzy cone metric and \mathcal{M} -fuzzy metric spaces. We give some properties of these generalized metric space and extend the fuzzy Banach contraction theorem and fixed point results in this new framework.

2. PRELIMINARIES

Definition 2.1. [11] A binary operation $*$: $[0,1] \times [0,1] \rightarrow [0,1]$ is a continuous t-norm if $*$ satisfies the following conditions;

- (a) $*$ is associative and commutative,
- (b) $*$ is continuous,
- (c) $a * 1 = a$ for all $a \in [0,1]$,
- (d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, $a, b, c, d \in [0,1]$.

The basic two continuous t-norms are (see [11])

- (i) the minimum t-norm defined by $a * b = \min\{a, b\}$.
- (ii) the product t-norm defined by $a * b = ab$

Definition 2.2. [4] Let X be a set, F and A are self maps of X . A point x in X is called a coincidence point of F and A if and only if $Fx = Ax$. We shall call $w = Fx = Ax$ a point of coincidence of F and A .

Definition 2.3. [1] Let F, A self maps of a set X . F and A are occasionally weakly compatible if and only if there is a point x in X which is a coincidence point of F and A at which F and A commute.

Lemma 2.4. [1] *Let X be a set, F and A are occasionally weakly compatible self maps of X . If F and A have a unique point of coincidence, $w = Fx = Ax$, then w is unique common fixed point of F and A .*

Definition 2.5. [10] A three-tuple $(X, \mathcal{M}, *)$ is said to be \mathcal{M} -fuzzy metric space if X is arbitrary nonempty set, $*$ is a continuous t-norm, and \mathcal{M} is a fuzzy set on $X \times X \times X \times (0, \infty)$ satisfying the following conditions for each $x, y, z, a \in X$ and $t, s > 0$,

$$(GFM1) \quad \mathcal{M}(x, x, y, t) > 0 \text{ if } x \neq y,$$

- (GFM2) $\mathcal{M}(x, y, z, t) = 1 \Leftrightarrow x = y = z$,
 (GFM3) $\mathcal{M}(x, y, z, t) = \mathcal{M}(p(x, z, y, t))$ where p is a permutation function,
 (GFM4) $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, z, s) \leq \mathcal{M}(x, y, z, t + s)$ for all $x, y, z, a \in X$,
 (GFM5) $\mathcal{M}(x, y, z, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous.

Throughout this paper E denotes a real Banach space and θ denotes the zero of E .

Definition 2.6. [3] A subset P of E is called a cone if

- (a) P is closed, non-empty and $P \neq \{\theta\}$
 (b) If $a, b \in R$, $a, b \geq 0$ and $x, y \in P$ then $ax + by \in P$
 (c) If both $x \in P$ and $-x \in P$ then $x = \theta$

For a given cone, a partial ordering \preceq on E via P is defined by $x \preceq y$ if and only if $y - x \in P$. $x \prec y$ will stand for $x \preceq y$ and $x \neq y$ while $x \ll y$ will stand for $y - x \in \text{int}(P)$. Throughout the paper, we assume that all cones have nonempty interior.

Definition 2.7. [9] A three-tuple $(X, M, *)$ is said to be fuzzy cone metric space if P is cone of E . X is arbitrary nonempty set, $*$ is a continuous t-norm, and M is a fuzzy set on $X^2 \times P$ satisfying the following conditions for each $x, y, z \in X$ and $t, s \in \text{int}(P)$,

- (FCM1) $M(x, y, t) > 0$ if $x \neq y$,
 (FCM2) $M(x, y, t) = 1 \Leftrightarrow x = y$,
 (GFM3) $M(x, y, t) = M(y, x, t)$,
 (GFM4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$,
 (GFM5) $M(x, y, \cdot) : \text{int}(P) \rightarrow [0, 1]$ is continuous.

For $t \gg \theta$, the open ball $B(x, r, t)$ with center x radius $r \in (0, 1)$ is defined by $B(x, r, t) = \{y \in X \mid M(x, y, t) > 1 - r\}$

Theorem 2.8. Let $(X, M, *)$ be a fuzzy cone metric space. Define $\tau = \{G \subset X \mid \forall x \in G, t \gg \theta \text{ and } 0 < r < 1 \text{ such that } B(x, r, t) \subset G\}$. Then τ is a topology on X .

3. G-FUZZY CONE METRIC SPACES

Definition 3.1. If P is a cone of E , X is an arbitrary set, \star is a continuous t-norm and G is a fuzzy set on $X \times X \times X \times \text{int}(P)$ satisfies:

(GFCM1) $G(x, x, y, t) > 0$ if $x \neq y$,
 (GFCM2) $G(x, x, y, t) \geq G(x, y, z, t)$ where $y \neq z$,
 (GFCM3) $G(x, y, z, t) = 1 \Leftrightarrow x = y = z$,
 (GFCM4) $G(x, y, z, t) = G(p(x, z, y, t))$ where p is a permutation function,
 (GFCM5) $G(x, y, a, t) * G(a, z, z, s) \leq G(x, y, z, t+s)$ for all $x, y, z, a \in X$,
 (GFCM6) $G(x, y, z, t) : \text{int}(P) \rightarrow [0, 1]$ is continuous.
 Then $(X, G, *)$ is said to be generalized fuzzy cone metric space or more specifically a G-fuzzy cone metric space.

Remark 3.2. If we take $E = \mathbb{IR}$, $P = [0, \infty)$, $a*b = ab$ then every \mathcal{M} -fuzzy metric space becomes a G-fuzzy cone metric space.

Example 3.3. Let (X, \mathcal{M}) be a generalized metric space, P be a cone of E and denote $a*b = a.b$ for all $a, b \in [0, 1]$. If we define $G : X^3 \times \text{Int}(P) \rightarrow [0, 1]$ by

$$G(x, y, z, t) = \frac{t}{t + \mathcal{M}(x, y, z)}$$

for every $x, y, z \in X$, then $(X, G, *)$ is a G-fuzzy cone metric space. (GFCM1-2-3-4-6) are obvious.

(GFCM5) Since \mathcal{M} is generalized metric space, we have $\mathcal{M}(x, y, z) \leq \mathcal{M}(x, a, a) + \mathcal{M}(a, y, z)$ for all $x, y, z, a \in X$. By $t, s \in \text{int}(P)$.

$$\begin{aligned} \mathcal{M}(x, y, z) &\leq \frac{t+s}{t} \mathcal{M}(x, a, a) + \frac{t+s}{s} \mathcal{M}(a, y, z) \\ 1 + \frac{\mathcal{M}(x, y, z)}{t+s} &\leq 1 + \left(\frac{\mathcal{M}(x, a, a)}{t} + \frac{\mathcal{M}(a, y, z)}{s} \right) \\ \left(\frac{t}{t + \mathcal{M}(x, a, a)} \right) \cdot \left(\frac{s}{s + \mathcal{M}(a, y, z)} \right) &\leq \frac{t+s}{t+s + \mathcal{M}(x, y, z)} \end{aligned}$$

Therefore

$$G(x, y, a, t) * G(a, z, z, s) \leq G(x, y, z, t + s).$$

Definition 3.4. G-fuzzy cone metric $(X, G, *)$ is said to be symmetric if $G(x, x, y, t) = G(x, y, y, t)$ for any $x, y \in X$, $t \gg \theta$.

Lemma 3.5. Let $(X, G, *)$ be a G-fuzzy cone metric. $G(x, y, z, \cdot) : \text{int}(P) \rightarrow [0, 1]$ is nondecreasing for all $x, y, z \in X$.

Proof. Let $s, t \in \text{int}(P)$ and $s \gg t$. By GFCM5), we have $G(x, y, z, t) * G(z, z, z, s - t) \leq G(x, y, z, s)$. Since $G(z, z, z, s - t) = 1$, we obtain $G(x, y, z, t) \leq G(x, y, z, s)$. \square

Lemma 3.6. *Let $(X, G, *)$ be a G-fuzzy cone metric. For any $x, y, z \in X$, $t \gg \theta$ and $k \in (0, 1)$ such that $G(x, y, z, kt) \geq G(x, y, z, t)$ then $x=y=z$.*

Proof. It is obvious by previous Lemma 3.1. □

Definition 3.7. Let $(X, G, *)$ be a G-fuzzy cone metric space. For any $t \gg \theta$, $a \in X$ and $r \in (0, 1)$, the G-open ball with a center a and radius r is defined by

$$B_G(a, r, t) = \{x \in X | G(a, x, x, t) > 1 - r\}.$$

A subset A of X is called open set if for each $a \in A$ there exists $t \gg \theta$ and $0 < r < 1$ such that $B_G(a, r, t) \subset A$.

Proposition 3.8. *Let $(X, G, *)$ be a G-fuzzy cone metric space. If $G(a, y, z, t) > 1 - r$ then $y, z \in B_G(a, r, t)$, for any $t \gg \theta$, $a \in X$ and $r \in (0, 1)$.*

Proof. It is obvious from (GFCM3). □

Proposition 3.9. *In a G-fuzzy cone metric space, every G-open ball is an open set.*

Proof. Let $B_G(a, r, t)$ be a G-open ball and $x \in B_G(a, r, t)$. Then $G(a, x, x, t) > 1 - r$. By Proposition, there exists $t_0 \gg t$, for $t \gg \theta$. Since $G(x, y, z, t)$ is non-decreasing and continuous, $G(a, x, x, t_0) > 1 - r$. Take $r_0 = G(a, x, x, t_0)$. Since $r_0 > 1 - r$, there exists $0 < \rho < 1$ such that $r_0 > 1 - \rho > 1 - r$. Now, for a given r_0 and ρ such that $r_0 > 1 - \rho$, we can find $0 < r_1 < 1$ such that $r_0 * r_1 > 1 - \rho$. We shall show $B_G(x, 1 - r_1, t - t_0) \subset B_G(a, r, t)$. Let $y \in B_G(x, 1 - r_1, t - t_0)$. Then we have $G(x, y, y, t - t_0) > r_1$. By GFCMS5), $G(a, y, y, t) \geq G(a, x, x, t_0) * G(x, y, y, t - t_0) > r_0 * r_1 > 1 - r$. So $y \in B_G(a, r, t)$. □

Remark 3.10. Let $(X, G, *)$ be a G-fuzzy cone metric space. Define

$$\tau_G = \{A \subset X | \forall a \in A, t \gg \theta \text{ and } 0 < r < 1 \text{ such that } B_G(a, r, t) \subset A\}.$$

Then τ_G is a topology on X .

Proposition 3.11. *Let $(X, C, *)$ be a fuzzy cone metric, P be a cone of E , and $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$. We define $G : X^3 \times \text{Int}(P) \rightarrow [0, 1]$ by*

$$G(x, y, z, t) = C(x, y, t) * C(y, z, t) * C(z, x, t)$$

for every $x, y, z \in X$. Then

(a) $(X, G, *)$ is a G-fuzzy cone metric space.

(b) The topologies generated by C and G are the same.

Proof. (a) (GFCM1-2-3-4-6) are obvious.

(GFCM5) For all $x, y, z, a \in X$ and $t, s \in \text{int}(P)$,

$$\begin{aligned} G(x, y, z, t + s) &= C(x, y, t + s) * C(y, z, t + s) * C(z, x, t + s) \\ &\geq C(x, y, t) * C(y, y, s) * C(y, a, t) * C(a, z, s) * C(x, a, t) * C(a, z, s) \\ &= C(x, y, t) * C(y, a, t) * C(a, x, t) * C(a, z, s) * C(z, z, s) * C(z, a, s) \\ &= G(x, y, a, t) * G(a, z, z, s) \end{aligned}$$

(b) Assume that $A \in \tau_C$. Then there exists $r > 0$ and $t \gg \theta$ such that $B_C(x, r, t) \subset A$ for every $x \in A$. Since $a * b = \min\{a, b\}$ and by definition of fuzzy cone metric, we obtain

$$G(x, y, y, t) = C(x, y, t) * C(y, y, t) * C(y, x, t) = C(x, y, t) > 1 - r$$

It follows that, $B_G(x, r, t) \subset A$ and hence $A \in \tau_G$. Similarly, we obtain $\tau_G \subset \tau_C$. \square

Remark 3.12. As shown in the following example, $(X, G, *)$ which is given by Proposition 3.3 is not a G-fuzzy cone metric space for all t-norm.

Example 3.13. Let $E = \mathbb{R}^2$. Then $P = \{(m, n) | m, n \geq 0\} \subset E$ is a normal cone constant $K=1$. Let $X = \mathbb{R}$, $a * b = ab$ and $C : X^2 \times \text{Int}(P) \rightarrow [0, 1]$ defined by $C(x, y, t) = \frac{1}{e^{\frac{|x-y|}{\|t\|}}}$ for all $x, y \in X$ and $t \gg \theta$. $(X, C, *)$

be a fuzzy cone metric space [9]. Let $G : X^3 \times \text{Int}(P) \rightarrow [0, 1]$ by $G(x, y, z, t) = C(x, y, t) * C(y, z, t) * C(z, x, t)$ for every $x, y, z \in X$. But $(X, G, *)$ is not a G-fuzzy cone metric space.

Definition 3.14. Let $(X, G, *)$ be a G-fuzzy cone metric space and (x_n) be a sequence in X . The sequence (x_n) is said to be G-convergent at $x \in X$ if for any $t \gg \theta$ and any $0 < r < 1$, there exists a natural number N such that $G(x_n, x_n, x, t) > 1 - r$ whenever $n \geq N$ i.e $n \geq N \Rightarrow x_n \in B_G(x, r, t)$. We denote this by $x_n \rightarrow x$ as $n \rightarrow \infty$ or by $\lim_{n \rightarrow \infty} x_n = x$.

Definition 3.15. Let $(X, G, *)$ be a G-fuzzy cone metric space. For any $t \gg \theta$, $a \in X$ and $r \in (0, 1)$, the G-closed ball with a center a and radius r is defined by

$$B_G[a, r, t] = \{x \in X | G(a, x, x, t) \geq 1 - r\}.$$

Definition 3.16. Let $(X, G, *)$ be a G-fuzzy cone metric space and (x_n) be a sequence in X. The sequence (x_n) is said to be G-cauchy sequence if for any $t \gg \theta$ and any $0 < r < 1$, there exists a natural number N such that $G(x_n, x_n, x_m, t) > 1 - r$ whenever $n, m \geq N$.

Definition 3.17. A generalized fuzzy cone metric space is called complete if every G-cauchy sequence is G-convergent.

Proposition 3.18. Let $(X, G, *)$ be a G-fuzzy cone metric space, $x \in X$ and (x_n) be a sequence in X. Then (x_n) convergence to x if and only if $G(x_n, x_n, x, t) \rightarrow 1$ as $n \rightarrow \infty$ for each $t \gg \theta$.

Proof. (\Rightarrow): Suppose that $x_n \rightarrow x$. Then, for each $t \gg \theta$ and $r \in (0,1)$, there exists a natural number N such that $G(x_n, x_n, x, t) > 1 - r$ whenever $n \geq N$. We have $1 - G(x_n, x_n, x, t) < r$. Hence $G(x_n, x_n, x, t) \rightarrow 1$ as $n \rightarrow \infty$.

(\Leftarrow): Assume that $G(x_n, x_n, x, t) \rightarrow 1$ as $n \rightarrow \infty$. Then, for each $t \gg \theta$ and $r \in (0,1)$, there exists a natural number N such that $1 - G(x_n, x_n, x, t) < r$ whenever $n \geq N$. Then, we obtain $G(x_n, x_n, x, t) > 1 - r$. Hence (x_n) convergence to x . \square

4. FIXED POINT THEOREMS

Definition 4.1. Let $(X, G, *)$ be a G-fuzzy cone metric space and $f : X \rightarrow X$ is a self mapping. Then f is said G-fuzzy cone contractive if there exists α , $0 < \alpha < 1$ such that

$$\frac{1}{G(f(x), f(y), f(z), t)} - 1 \leq \alpha \left(\frac{1}{G(x, y, z, t)} - 1 \right)$$

for each $x, y, z \in X$ and $t \gg \theta$. Moreover, α is called the contractive constant of f .

Theorem 4.2. Let $(X, G, *)$ be a G-fuzzy cone metric space in which G-fuzzy cone contractive mapping is cauchy. Let $T : X \rightarrow X$ be a G-fuzzy cone contractive mapping being α the contractive constant. Then, T has a unique fixed point.

Proof. Fix $x \in X$ and let $x_n = T^n(x)$, $n \in \mathbb{N}$. For $t \gg \theta$, we have

$$\frac{1}{G(T(x), T(x), T^2(x), t)} - 1 \leq \alpha \left(\frac{1}{G(x, x, x_1, t)} - 1 \right)$$

and by induction

$$\frac{1}{G(x_{n+1}, x_{n+1}, x_{n+2}, t)} - 1 \leq \alpha \left(\frac{1}{G(x_n, x_n, x_{n+1}, t)} - 1 \right)$$

Then (x_n) is a G-fuzzy contractive sequence. By hypothesis, it is a Cauchy sequence. Since X is complete, (x_n) is convergence to a , for some $a \in X$. By Theorem, we have

$$\frac{1}{G(T(a), T(x_n), T(x_n), t)} - 1 \leq \alpha \left(\frac{1}{G(a, x_n, x_n, t)} - 1 \right) \rightarrow 0$$

as $n \rightarrow \infty$. Then we obtain

$$\lim_{n \rightarrow \infty} G(T(a), T(x_n), T(x_n), t) = 1 \text{ for each } t \gg \theta.$$

and by Theorem, we get

$$\lim_{n \rightarrow \infty} T(x_n) = a \text{ and } T(a) = a.$$

Now let us prove the uniqueness. Let $b \in X$ be any fixed point of T in X .

$$\begin{aligned} \frac{1}{G(a, b, b, t)} - 1 &= \frac{1}{G(T(a), T(b), T(b), t)} - 1 \\ &\leq \alpha \left(\frac{1}{G(a, b, b, t)} - 1 \right) \\ &\leq \alpha \left(\frac{1}{G(T(a), T(b), T(b), t)} - 1 \right) \\ &\leq \dots \leq \\ &\leq \alpha^n \left(\frac{1}{G(a, b, b, t)} - 1 \right) \rightarrow 0 \end{aligned}$$

as $n \rightarrow \infty$. Hence $G(a, b, b, t) = 1$ and $a = b$. \square

Theorem 4.3. $(X, G, *)$ be a complete G-fuzzy cone metric space and let P, Q, A and B are self mappings of X . Let the pairs $\{P, A\}$ and $\{Q, B\}$ are occasionally weakly compatible. If there exists $\alpha \in (0, 1)$ such that

$$\begin{aligned} G(Px, Qy, Qy, \alpha t) &\geq \min\{G(Px, Ax, Qy, t), G(Ax, By, Qy, t) \\ &\quad G(Px, Ax, By, t), G(Px, By, Qy, t)\} \end{aligned} \quad (1)$$

for all $x, y \in X$ and for all $t \gg \theta$, there exists a unique point $u \in X$ such that $P(u) = A(u) = u$ and a unique point $v \in X$ such that $Q(v) = B(v) = v$. Moreover $u = v$ so that there is a unique common fixed point of P, Q, A and B .

Proof. Let the pairs $\{P, A\}$ and $\{Q, B\}$ are occasionally weakly compatible. Then there are points $x, y \in X$ such that $Px = Ax$ and $Qy = By$. We

argue that $Px=Qy$. By inequality (1) and G is symmetric,

$$\begin{aligned} G(Px, Qy, Qy, \alpha t) &\geq \min\{G(Px, Ax, Qy, t), G(Ax, By, Qy, t) \\ &\quad G(Px, Ax, Qy, t), G(Px, By, Qy, t)\} \\ &\geq \min\{G(Px, Px, Qy, t), G(Px, Qy, Qy, t) \\ &\quad G(Px, Px, Qy, t), G(Px, Qy, Qy, t)\} \\ &\geq G(Px, Qx, Qy, t) \end{aligned}$$

By Lemma 3.2, $Px = Qy$. Suppose that there is another point w such that $Pw=Aw$ then by (1), we have $Pw = Aw = Qy = By$. So, $Px=Pw$ and $u=Pw=Aw$ is the unique common fixed point of P and A . By Lemma 2.1, w is the only common fixed point P and A .

Similarly, there is a unique point $v \in X$ such that $Qv = Bv = v$. Suppose that $u \neq v$.

$$\begin{aligned} G(u, v, v, \alpha t) &= G(Pu, Qv, Qv, \alpha t), \\ &\geq \min\{G(Pu, Au, Qv, t), G(Au, Bv, Qv, t) \\ &\quad G(Pu, Au, Bv, t), G(Pu, Bv, Qv, t)\} \\ &\geq \min\{G(u, u, v, t), G(u, v, v, t) \\ &\quad G(u, u, v, t), G(u, v, v, t)\} \\ &\geq G(u, u, v, t) \end{aligned}$$

So, we have $u = v$ by Lemma 3.2 and u is a unique common fixed point of P, Q, A and B . \square

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