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ON HARMONIOUS CHROMATIC NUMBER OF TRIPLE STAR GRAPH

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Abstract. A Harmonious coloring of a graph G is a proper vertex coloring of G, in which every pair of colors appears on at most one pair of adjacent vertices and the harmonious chromatic number of graph G is the minimum number of colors needed for the harmonious coloring of G and it is denoted byX_H(G).The purpose of this paper is to extend the double star graph [12] and to discuss harmonious coloring for central graph, middle graph and total graph of extended double star graph i.e. triple star graph.

Key Words: Central graph, Middle graph, Total graph, Harmonious coloring and Harmonious chromatic number.

2010 Mathematics Subject Classification: 05C15, 05C75.

1. Introduction

Let G be a finite undirected graph with vertex set $V(G)$ and edge set $E(G)$ having no loops and multiple edges.

All graphs considered here are undirected. In the whole paper, the term coloring will be used to define vertex coloring of graphs. A proper coloring of a graph G is the coloring of the vertices of G such that no two neighbors in G are assigned the same color.

A Harmonious coloring of a graph G is a proper vertex coloring of G, in which every pair of colors appears on at most one pair of adjacent vertices and the harmonious chromatic number of graph G is the minimum number of colors needed for the harmonious coloring of G and it

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is denoted byX_H(G).The purpose of this paper is introduce the triple star graph and to discuss the harmonious coloring of triple star graph families.

The first paper on harmonious graph coloring was published in 1982 by Frank Harary and M. J. Plantholt [6]. However, the proper definition of this notion is due to J.E. Hopcroft and M. S. Krishnamoorthy [5] in1983. A collection of articles in harmonious coloring can be found in the bibliography [3].

In 2012 M. Venkatachalam, J. Vernold Vivin and K. Kaliraj [12] discussed Harmonious Coloring on double star Graph Families. In this paper we extended the double star graph [12] which is known as triple star graph and discuss harmonious coloring for this graph families.

2. Definitions

Definition 2.1. The central graph [2,3,7,9,11,12,13] $C(G)$ of a graph is obtained by subdividing each edge of G exactly once and joining all the non adjacent vertices of G.

Definition 2.2. The middle graph [2,3,4,9,10,11,12,13] of G, denoted by $M(G)$ is dened as follows: The vertex set of $M(G)$ is $V(G)E(G)$. Two vertices x, y in the vertex set of$M(G)$ are adjacent in $M(G)$ in case one the following holds:

(a) x, y are in $E(G)$ andx, y are adjacent in G.
(b) x is in $V(G)$, y is in $E(G)$, and x, y are incident in G.

Definition 2.3. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The total graph [2,3,7,11,12,13] of G is denoted by $T(G)$ and is defined as follows.

The vertex set of $T(G)$ is $V(G)UE(G)$. Two vertices x, y in the vertex set of $T(G)$ is adjacent in $T(G)$, if one of the following holds:

(a) x, y are in$V(G)$ and x is adjacent to y in G.
(b) x, y are in $E(G)$ and x, y are adjacent in G.
(c) x is in $V(G)$, y is in $E(G)$ and x, y are adjacent in G.

Definition 2.4. Triple star $K_{1,n,n,n}$ is a tree obtained from the double star [12] $K_{1,n,n}$ by adding a new pendant edge of the existing n pendant vertices. It has $3n + 1$ vertices and $3n$ edges.

Let$V(K_{1,n,n,n})$= $\{v\} \cup \{v_1, v_2, ...v_n\} \cup \{w_1, w_2, ..., w_n\} \cup \{u_1, u_2, , u_n\}$ and $E(K_{1,n,n,n})$=$\{e_1, e_2, ..., e_n\} \cup \{e_1^{'}, e_2^{'}, ..., e_n^{'}\} \cup \{e_1^{''}, e_2^{''}, ..., e_n^{''}\}$.

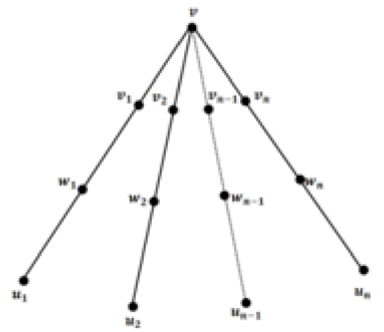

Fig 1.Triple Star Graph

3. Harmonious Coloring of Triple Star Graph Families

Theorem 3.1. *For any triple star graph $K_{1,n,n,n}$ the harmonious chromatic number,* $X_H(C(K_{1,n,n,n})) = 4n+3$.

Proof First we apply the definition of central graph on $K_{1,n,n,n}$. Let the edgevv_i, v_iw_i and w_iu_i $(1 \leq i \leq n)$ of $K_{1,n,n,n}$ be subdivided by the vertices e_i $(1 \leq i \leq n)$, $e_i^{'}$ $(1 \leq i \leq n)$ and $e_i^{''}$ $(1 \leq i \leq n)$.

It is clear that

$V(C(K_{1,n,n,n})) = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{w_i : 1 \leq i \leq n\}$

$\cup \{u_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n\} \cup \{e_i^{'} : 1 \leq i \leq n\} \cup$

$\{e_i^{''} : 1 \leq i \leq n\}$.

The vertices v_i, $(1 \leq i \leq n)$ induce a clique (largest complete subgraph) of order n (sayK_n) and the vertices v, u_i $(1 \leq i \leq n)$ induce a clique (largest complete subgraph) of order$n+1$ (say K_{n+1}) in $C(K_{1,n,n,n})$ respectively (see figure 2). Also we observe that the number of edges in $C(K_{1,n,n,n})$ is $(9n^2 + 9n)/2$.

Thus we have $X_H(C(K_{1,n,n,n})) \geq$ 4n +3.

Now we apply the colors to the vertices of $C(K_{1,n,n,n})$ as follows: Taking color class $C = \{c_1, c_2, c_3, ..., c_{4n+3}\}$.

(i) For $(1 \leq i \leq n)$, assign the color c_i to u_i.

(ii) For $(1 \leq i \leq n)$, assign the color c_{n+i} to w_i.
(iii) For $((1 \leq i \leq n)$, assign the color c_{2n+i} to v_i.
(iv) For$(1 \leq i \leq n)$, assign the color c_{3n+i} to e_i.
(v) For$(1 \leq i \leq n)$, assign the color c_{4n+1} to $e_i^{'}$ and color c_{4n+2} to$e_i^{''}$ and at last assign the color c_{4n+3} to v.

Therefore $X_H(C(K_{1,n,n,n})) \leq 4n + 3$. Hence $X_H(C(K_{1,n,n,n})) = 4n + 3$.

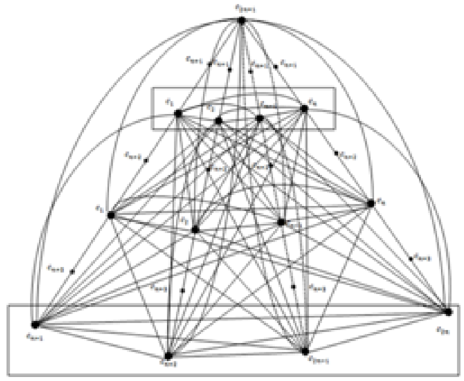

Fig 2. $C(K_{1,n,n,n})$ with coloring

Theorem 3.2. *For any triple star graph $K_{1,n,n,n}$ the harmonious chromatic number, $X_H(M(K_{1,n,n,n})) = 3n + 3$ for $n > 1$.*

Proof First we apply the definition of middle graph. Let the edge vv_i, v_iw_i and w_iu_i $(1 \leq i \leq n)$ of $K_{1,n,n,n}$ be subdivided by the vertices e_i $(1 \leq i \leq n)$, $e_i^{'}(1 \leq i \leq n)$ and $e_i^{''}$ $(1 \leq i \leq n)$.

It is clear that

$V(M(K_{1,n,n,n})) = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{w_i : 1 \leq i \leq n\}$

$\cup \{u_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n\} \cup \{e_i^{'} : 1 \leq i \leq n\} \cup$

$\{e_i^{''} : 1 \leq i \leq n\}$.

The vertices $v, e_i(1 \leq i \leq n)$ induce a clique of order $n+1$ (say K_{n+1}) in $M(K_{1,n,n,n})$ (see figure 3). Also we observe that the number of edges in $M(K_{1,n,n,n})$ is $(n^2 + 15n)/2$.

Thus we have $X_H(M(K_{1,n,n,n})) \geq 3n + 3$.

Now we apply the colors to the vertices of $M(K_{1,n,n,n})$ as follows: Taking color class $C = \{c_1, c_2, c_3, ..., c_{3n+3})\}$.

(i) For $((1 \leq i \leq n)$, assign the color c_1 to u_i and v .
(ii) For $(1 \leq i \leq n)$, assign the colo r c_{1+i} to e_i.
(iii) For $(1 \leq i \leq n)$, assign the color c_{n+1+i} to $e_i^{'}$.
(iv) For $((1 \leq i \leq n)$, assign the color c_{2n+1+i} to $e^{''}i$.
(v) At last for $(1 \leq i \leq n)$, assign the color c_{3n+2} to v_i, color c_{3n+3} to w_i.

Therefore $X_H(M(K_{1,n,n,n})) \leq 3n$ +3. Hence $X_H(M(K_{1,n,n,n})) = 3n$ +3.

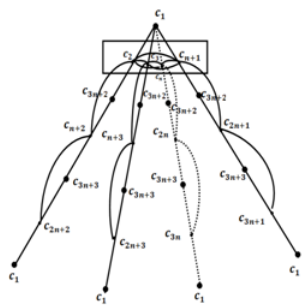

Fig 3. $M(K_{1,n,n,n})$ with coloring

Theorem 3.3. *For any triple star graph $K_{1,n,n,n}$ the harmonious chromatic number $X_H(T(K_{1,n,n,n})) = 4n + 2$.*

Proof First we apply the definition of total graph. Let the edge vv_i, v_iw_i and w_iu_i $(1 \leq i \leq n)$ of $K_{1,n,n,n}$ be subdivided by the vertices e_i $(1 \leq i \leq n)$, $e_i^{'}(1 \leq i \leq n)$ and $e_i^{''}$ $(1 \leq i \leq n)$.
It is clear that

$$V(T(K_{1,n,n,n})) = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{w_i : 1 \leq i \leq n\}$$

$$\cup \{u_i : 1 \leq i \leq n\} \cup \{e_i : 1 \leq i \leq n\} \cup \{e_i^{'} : 1 \leq i \leq n\} \cup$$

$$\{e_i^{''} : 1 \leq i \leq n\}.$$

The vertices $v, e_i(1 \leq i \leq n)$ induce a clique of order $n+1$ (sayK_{n+1}) in $T(K_{1,n,n,n})$ (see figure 4) . also we observe that the number of edges in $T(K_{1,n,n,n})$ is $(n^2 + 21n)/2$.
Thus we have $X_H(T(K_{1,n,n,n})) \geq 4n + 2$.

Now we apply the colors to the vertices of $T(K_{1,n,n,n})$ as follows: Taking color class $C = \{c_1, c_2, c_3, ..., c_{4n+2})\}$.

(i) For $((1 \leq i \leq n)$, assign the color c_i to e_i.
(ii) For $(1 \leq i \leq n)$, assign the colo r c_{n+i} to v_i.
(iii) For $(1 \leq i \leq n)$, assign the color c_{2n+i} to w_i.
(iv) For $((1 \leq i \leq n)$, assign the color c_{3n+1} to $e^{'}i$.
(v) For $((1 \leq i \leq n)$, assign the color c_{3n+1+i} to $e^{''}i$.
(vi) At last for $(1 \leq i \leq n)$, assign the color c_{4n+2} to u_i, v.

Therefore $X_H(T(K_{1,n,n,n})) \leq 4n + 2$. Hence $X_H(M(K_{1,n,n,n})) = 4n + 2$.

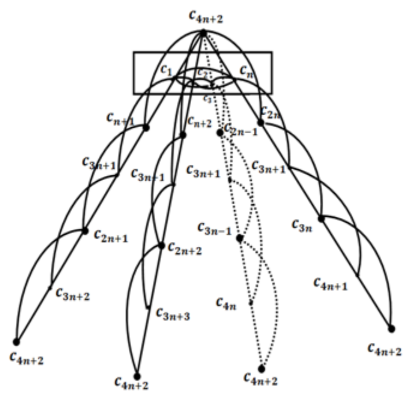

Fig 4. $T(K_{1,n,n,n})$ with coloring

4. Conclusion

In this paper, we introduce triple star graph and discuss the harmonious coloring and find the harmonious chromatic number for central graph, middle graph and total graph of triple star graph $4n + 3, 3n + 3$ and $4n + 2$ respectively.

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