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HYPER STRUCTURE THEORY APPLIED TO KU-ALGEBRAS

SAMY M. MOSTAFA, FATEMA F. KAREEM AND BIJAN DAVVAZ

Abstract. In this paper, the concept of a hyper structure KU-algebra is introduced and some related properties are investigated. Also, some types of hyper KU-algebras are studied and the relationship between them is stated. Then, a hyper KU-ideal of a hyper structure KU-algebra is studied and a few properties are obtained. Furthermore, the notion of a homomorphism is discussed.

Key Words: KU-algebra, hyper KU-algebra, hyper KU-ideal.
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1. Introduction

Prabpayak and Leerawat [6, 7] introduced a new algebraic structure which is called KU-algebras. They studied ideals and congruences in KU-algebras. Also, they introduced the concept of homomorphism of KU-algebra and investigated some related properties. Moreover, they derived some straightforward consequences of the relations between quotient KU-algebras and isomorphism. The hyper structure theory (called also multialgebras) is introduced in 1934 by F. Marty [4] at the 8th congress of Scandinvian Mathematiciens. Around the 40's, several authors worked on hyper groups, especially in France and in the United States, but also in Italy, Russia and Japan. Hyper structures have many applications to several sectors of both pure and applied sciences. In [3], Y. B. Jun et al. applied the hyper structures to BCK-algebras, and introduced the concept of a hyper BCK-algebra which is a generalization of

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∗ Address correspondence to Samy M. Mostafa; E-mail: samymostafa@yahoo.com.

a BCK-algebra and investigated some related properties. They also introduced the notion of a hyper BCK-ideal, a weak hyper BCK-ideal and gave relations between hyper BCK-ideals and weak hyper BCK-ideals. Y. B. Jun et al. [2] gave a condition for a hyper BCK-algebra to be a BCK-algebra and introduced the notion of a strong hyper BCK-ideal, a weak hyper BCK-ideal and a reflexive hyper BCK-ideal. They showed that every strong hyper BCK-ideal is a hyper sub-algebra, a weak hyper BCK-ideal and a hyper BCK-ideal; and every reflexive hyper BCK-ideal is a strong hyper BCK-ideal. In this paper we introduced the notion of hyper KU-algebra and some types of hyper KU-algebras are studied. Also, a homomorphism of hyper KU-algebra is obtained.

2. Preliminaries

In this section, we present some concepts related to KU-algebra and theories from the literature, which are necessary for our discussion.

Definition 2.1. [6, 7] Let X be a set with a binary operation $\star$ and a constant 0, then $(X, \star, 0)$ is called a KU-algebra, if it satisfies the following axioms:

(KU$_1$) $(x \star y) \star [(y \star z) \star (x \star z)] = 0$,
(KU$_2$) $x \star 0 = 0$,
(KU$_3$) $0 \star x = x$,
(KU$_4$) $x \star y = 0$ and $y \star x = 0$ imply $x = y$,

for all $x, y, z \in X$.

A binary relation $\leq$ in X is defined by: $x \leq y$ if and only if $y \star x = 0$, we can prove that$(X, \leq)$ is a poset.

In a KU-algebra $(X, \star, 0)$, the following properties are satisfied:

(KU$_{1'}$) $(y \star z) \star (x \star z) \leq x \star y$,
(KU$_{2'}$) $0 \leq x$,
(KU$_{3'}$) $x \leq y$ and $y \leq x$ imply $x = y$.

Corollary 2.2. [5] *In a KU-algebra X. The following identities are true, for all$x, y, z \in X$.*

(1) $z \star z = 0$,
(2) $z \star (x \star z) = 0$,
(3) *If* $x \leq y$ *implies that* $y \star z \leq x \star z$,
(4) $z \star (y \star x) = y \star (z \star x)$,
(5) $y \star ((y \star x) \star x) = 0$.

Definition 2.3. [9] A subset S of a KU-algebra $(X;\star,0)$ is called a sub algebra of X, if $x\star y\in X$ whenever $x,y\in X$.

Definition 2.4. [9] A nonempty subset A of a KU-algebra $(X;\star,0)$ is called an ideal of X, if for any $x,y\in X$, the following conditions hold:

(1) $0\in A$,
(2) $y\star z\in A$ and $y\in A$ imply that $z\in A$, for all $y,z\in X$.

Definition 2.5. [9] Let A be a nonempty subset of a KU-algebra X. Then, A is said to be a KU-ideal of X, if

(1) $0\in A$,
(2) $x\star(y\star z)\in A$ and $y\in A$ imply that $x\star z\in A$, for all $x,y,z\in X$.

Example 2.6. Let $X=\{0,a,b,c,d,e\}$ with $\star$ is defined by the following table.

$\star$	0	a	b	c	d	e
0	0	a	b	c	d	e
a	0	0	b	b	d	e
b	0	0	0	a	d	e
c	0	0	0	0	d	e
d	0	0	0	a	0	e
e	0	0	0	0	0	0

Then, $(X,\star,0)$ is a KU-algebra and it is easy to show that $\{0,a\}$,$\{0,a,b,c,d\}$ are KU-ideals of X.

Definition 2.7. [7] Let $(X,\star,0)$ and $(Y,\star',0')$ be two KU-algebras, a homomorphism is a map $Q:X\to Y$ satisfying $Q(x\star y)=Q(x)\star' Q(y)$, for all $x,y\in X$.

Definition 2.8. [7] Let Q be a mapping of a KU-algebra X into a KU-algebra Y and $A\subseteq X$, $B\subseteq Y$. Then, the image of A in Y under Q is $Q(A)=\{Q(a):a\in A\}$ and the inverse image of B in X is $Q^{-1}(B)=\{b\in X:Q(b)\in B\}$.

Theorem 2.9. [7] *Let Q be a homomorphism of a KU-algebra X into a KU-algebra Y, then*

(1) *If 0 is the identity in X, then $Q(0)$ is the identity in Y;*
(2) *If S is a KU-subalgebra of X, then $Q(S)$ is a KU-subalgrbra of Y;*
(3) *If A is a KU-ideal of X, then $Q(A)$ is a KU-ideal in Y;*
(4) *If S is a KU-subalgebra of Y, then $Q^{-1}(S)$ is a KU-algebra of X;*

(5) *If B is a KU-ideal in $Q(X)$, then $Q^{-1}(B)$ is a KU-ideal in X;*
(6) *Q is $1-1$ if and only if $kerQ = \{0\}$;*
(7) *$kerQ$ is an KU-ideal of X.*

3. Hyper KU-algebra

Let H be a nonempty set and $P^{\star}(H) = P(H) - \{\phi\}$ the family of the nonempty subsets of H. A multi valued operation (said also hyper operation) "$\circ$" on H is a function, which associates with every pair $(x, y) \in H \times H = H^2$ a nonempty subset of H denoted $x \circ y$. An algebraic hyper structure or simply a hyper structure is a nonempty set H endowed with one or more hyper operations.

Definition 3.1. Let H be a nonempty set and "$\circ$" a hyper operation on H, such that $\circ : H \times H \longrightarrow P^{\star}(H)$. Then, H is called a hyper KU-algebra if it contains a constant "0" and satisfies the following axioms: for all $x, y, z \in H$

(HKU$_1$) $[(y \circ z) \circ (x \circ z)] << x \circ y$,
(HKU$_2$) $x \circ 0 = \{0\}$,
(HKU$_3$) $0 \circ x = \{x\}$,
(HKU$_4$) if $x << y$ and $y << x$ imply $x = y$,

where $x << y$ is defined by $0 \in y \circ x$ and for every $A, B \subseteq H$, $A << B$ is defined by $\forall a \in A, \exists b \in B$ such that $a << b$. In such case, we call $<<$ the hyper order in H.

We shall use the $x \circ y$ instead of $x \circ \{y\}$ or $\{x\} \circ \{y\}$. Note that if $A, B \subseteq H$, then by $A \circ B$ we mean the subset ${}_{a \in A, b \in B} \bigcup a \circ b$ of H.

Example 3.2. Let $H = \{0, a, b\}$ be a set. Define a hyper operation $\circ$ on H as follows:

$\circ$	0	a	b
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{0\}$	$\{0, a\}$	$\{a, b\}$
b	$\{0\}$	$\{0, a\}$	$\{0, a, b\}$

Then, $(H, \circ, 0)$ is a hyper KU-algebra.

Example 3.3. (1) Define the hyper operation $\circ$ on $\mathbb{Z}$, the set of integers, as follows:

$$x \circ y = \begin{cases} \{y\} & \text{if } x = 0 \\ \mathbb{Z} & \text{otherwise.} \end{cases}$$

Then, $(Z, \circ, 0)$ is a hyper KU-algebra.

(2) Let $H = \{0, 1, 2, 3\}$ be a set. Define the hyper operation $\circ_i$ on H as follows:

$\circ_1$	0	1	2	3
0	$\{0\}$	$\{1\}$	$\{2\}$	$\{3\}$
1	$\{0\}$	$\{0\}$	$\{1\}$	$\{3\}$
2	$\{0\}$	$\{0\}$	$\{0\}$	$\{0,3\}$
3	$\{0\}$	$\{0\}$	$\{1\}$	$\{0,3\}$

$\circ_2$	0	1	2	3
0	$\{0\}$	$\{1\}$	$\{2\}$	$\{3\}$
1	$\{0\}$	$\{0\}$	$\{1\}$	$\{3\}$
2	$\{0\}$	$\{0\}$	$\{0,1\}$	$\{0,3\}$
3	$\{0\}$	$\{0\}$	$\{1\}$	$\{0,3\}$

Then, $(H, \circ_i, 0)$, $i = 1, 2$ are hyper KU-algebras.

Proposition 3.4. *Let H be a hyper KU-algebra. Then, for all $x, y, z \in H$, the following statements hold:*

(P_1) *$A \subseteq B$ implies $A << B$, for all nonempty subsets A, B of H;*
(P_2) $0 \circ 0 = \{0\}$*;*
(P_3) $0 << x$*;*
(P_4) $z << z$*;*
(P_5) $x \circ z << z$*;*
(P_6) $A \circ 0 = \{0\}$*;*
(P_7) $0 \circ A = A$*;*
(P_8) $(0 \circ 0) \circ x = \{x\}$.

Proof. (P_1) Clearly by definition of $<<$.
(P_2) In (HKU_3), put $x = 0$. Then, $0 \circ 0 = \{0\}$.
(P_3) By (HKU_2), $0 \in x \circ 0$ and so $0 << x$.
(P_4) By (HKU_1), let $y = x = 0$, then by (P_2) and (HKU_3), z¡¡z.
(P_5) By(HKU_1), let $y = 0$, we get $0 \circ (x \circ z) << 0 \circ z \Rightarrow x \circ z << \{z\}$.
(P_6) The proof follows by (HKU_2).
(P_7) The proof follows by (HKU_3).
(P_8) The proof follows by (P_2), (HKU_3) and (P_4). □

Lemma 3.5. *In hyper KU-algebra $(H, \circ, 0)$, the following hold:*

$$x << y \text{ imply } y \circ z << x \circ z \text{ for all } x, y, z \in H.$$

Proof. Since $x << y$, it follows that $0 \in y \circ x$. By (HKU_1), we obtain $[(x \circ z) \circ (y \circ z)] << y \circ x$. Then $0 \in (y \circ x) \circ [(x \circ z) \circ (y \circ z)]$, but $0 \in y \circ x$. Hence, $0 \in [(x \circ z) \circ (y \circ z)]$, i.e., $(y \circ z) << (x \circ z)$. □

Lemma 3.6. *In hyper KU-algebra $(H, \circ, 0)$, we have*

$$z \circ (y \circ x) = y \circ (z \circ x) \text{ for all } x, y, z \in H.$$

Proof. From (HKU_1), $[(z \circ x) \circ (y \circ x)] << y \circ z$. If $y = 0$, then

$$[(z \circ x) \circ x)] << z. \tag{3.1}$$

Making use of (3.1) and (HKU_1), we get $[(z \circ (y \circ x) << [(z \circ x) \circ x] \circ (y \circ x) << y \circ (z \circ x)$, since x, y, z are arbitrary, interchanging y and z in the above inequality, we obtain $y \circ (z \circ x) << z \circ (y \circ x)$, and by ($HKU_4$), we get $z \circ (y \circ x) = y \circ (z \circ x)$. □

Lemma 3.7. *For all $x, y, z \in H$, the following statements hold:*

1. $x \circ y << z \Leftrightarrow z \circ y << x$,
2. $0 << A \Rightarrow 0 \in A$,
3. $y \in (0 \circ x) \Rightarrow y << x$.

Proof. (1) Let $x, y, z \in H$ be such that $x \circ y << z$. Then, there exists $t \in x \circ y$ such that $t << z$. Thus $0 \in z \circ t \subseteq z \circ (x \circ y) = x \circ (z \circ y)$, we get $0 \in x \circ (z \circ y)$, hence $z \circ y << x$. The proof of the converse is similar.

(2) Let $0 << A$. It means that there is $a \in A$ such that $0 << a$. By using (HKU_4), $a = 0$, and so $0 \in A$.

(3) Let $x \in 0 \circ y$. Then, by (HKU_3), $0 \in (0 \circ y) \circ x = y \circ x$. i.e., $0 \in y \circ x$. Hence $x << y$. □

4. Some types of hyper KU-algebras

In this section we discuss the formalization of Some types of hyper KU-algebras and we prove some related properties.

Definition 4.1. A hyper KU-algebra is said

1. Row hyper KU-algebra (briefly, R-hyper KU-algebra), if $0 \circ x = \{x\}$, for all $x \in H$;
2. Column hyper KU-algebra (briefly, C-hyper KU-algebra), if $x \circ 0 = \{0\}$, for all$x \in H$;
3. Diagonal hyper KU-algebra (briefly, D-hyper KU-algebra), if $x \circ x = \{0\}$, for all $x \in H$;
4. Thin hyper KU-algebra (briefly, T-hyper KU-algebra), if it is a RC-hyper KU-algebra;
5. Very thin hyper KU-algebra (briefly, V-hyper KU-algebra), if it is a RCD-hyper KU-algebra.

Example 4.2. (1) Every KU-algebra is a RCD-hyper KU-algebra.

(2) Let $H = \{0, a, b\}$ be a set. Define a hyper operation $\circ_1$ on H as follows:

$\circ_1$	0	a	b
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{0, b\}$	$\{0, a, b\}$	$\{0, a\}$
b	$\{0, a, b\}$	$\{a\}$	$\{0, a, b\}$

Then, $(H, \circ_1, 0)$ is an R-hyper KU-algebra.

(3)

$\circ_2$	0	a
0	$\{0\}$	$\{a\}$
a	$\{0\}$	$\{0, a\}$

$\circ_3$	0	a
0	$\{0\}$	$\{a\}$
a	$\{0, a\}$	$\{0\}$

$\circ_4$	0	a
0	$\{0\}$	$\{a\}$
a	$\{0, a\}$	$\{0, a\}$

Then, $(H, \circ_2, 0)$ is a T-hyper KU-algebra, $(H, \circ_3, 0)$ is an RD-hyper KU-algebra and $(H, \circ_4, 0)$ is an R-hyper KU-algebra.

Theorem 4.3. *Let H be a D-hyper KU-algebra. Then, for all$a, x, y \in H$*

(1) $a \in 0 \circ x$ *implies* $x \circ a = \{0\}$,
(2) $y \circ (x \circ y) = x \circ 0$,
(3) $0 \circ (x \circ 0) = x \circ 0$.

Proof. (1) By Lemma 3.6 and Definition 4.1,$\{0\} = 0 \circ 0 = 0 \circ (x \circ x) = x \circ (0 \circ x)$. It follows that, for all $a \in 0 \circ x$, $x \circ a = \{0\}$.
(2) $y \circ (x \circ y) = x \circ (y \circ y) = x \circ 0$.
(3) $0 \circ (x \circ 0) = x \circ (0 \circ 0) = x \circ 0$. □

Theorem 4.4. *Let H be a CD-hyper KU-algebra. Then, for all $x, y, z \in H$*

(1) $x \circ (y \circ x) = \{0\}$,
(2) $z \in x \circ y$ *implies* $y \circ z = \{0\}$.

Proof. (1) By Lemma 3.6 and Definition 4.1, $x \circ (y \circ x) = y \circ (x \circ x) = y \circ 0 = \{0\}$.
(2) Let $z \in x \circ y$. Then, by (1), $y \circ z \subseteq y \circ (x \circ y) = \{0\}$. Hence $y \circ z = \{0\}$. □

5. Hyper KU-ideals

Definition 5.1. Let A be a nonempty subset of a hyper KU-algebra H. Then, A is said to be a hyper ideal of H if

(HI_1) $0 \in A$,
(HI_2) $y \circ x << A$ and $y \in A$ imply $x \in A$, for all $x, y \in H$.

Definition 5.2. A nonempty set A of a hyper KU-algebra H is called a distributive hyper ideal if it satisfies (HI_1) and

$$(HI_3)\ (z \circ y) \circ (z \circ (z \circ z)) << A \text{ and } y \in A \text{ imply } x \in A.$$

Example 5.3. Consider a hyper KU-algebra $H = \{0, a, b, c\}$ with the following Cayley table.

$\{0, a\}, \{0, a, b\}$ are the only hyper ideals in H which are also distributive hyper ideals of H.

$\circ$	0	a	b	c
0	$\{0, a\}$	$\{a\}$	$\{0, b\}$	$\{c\}$
a	$\{0, a\}$	$\{0, a\}$	$\{0, b\}$	$\{c\}$
b	$\{0, a\}$	$\{0, a\}$	$\{0, a, b\}$	$\{c\}$
c	$\{0, a\}$	$\{0, a\}$	$\{0, a, b\}$	$\{0, a, c\}$

Theorem 5.4. *Every distributive hyper ideal A is a hyper ideal.*

Proof. Since A is a distributive hyper ideal, it follows that for $(z \circ y) \circ (z \circ (z \circ x)) << A$ and $y \in A$, substituting $z = 0$. We have $(y \circ x) << A$ and $y \in A$, $x \in A$. □

The converse of Theorem 5.4 may not be true as follows from Example 5.5.

Example 5.5. Consider a hyper KU-algebra $H = \{0, a, b, c\}$ with the following Cayley table. It can be easily checked that $A = \{0, b\}$ is a hyper ideal but A is not a distributive hyper ideal of H because $(b \circ 0) \circ (b \circ (b \circ c)) << A$ and $0 \in A$ implies $c \in A$ which is a contradiction.

$\circ$	0	a	b	c
0	$\{0\}$	$\{a\}$	$\{b\}$	$\{c\}$
a	$\{0\}$	$\{0, a\}$	$\{0, b\}$	$\{b, c\}$
b	$\{0\}$	$\{0, b\}$	$\{0\}$	$\{a\}$
c	$\{0\}$	$\{0, b\}$	$\{0\}$	$\{0, a\}$

Definition 5.6. Let I be a nonempty subset of a hyper KU-algebra H and $0 \in I$. Then,

1. (1) I is called a weak hyper ideal of H if $y \circ x \subseteq I$ and $y \in I$ imply that $x \in I$, for all $x, y \in H$;
2. (2) I is called a strong hyper ideal of H if $(y \circ x) \bigcap I \neq \phi$ and $y \in I$ imply that $x \in I$, for all $x, y \in H$.

Definition 5.7. For a hyper KU-algebra H, a nonempty subset $I \subseteq H$, containing 0 is:

(1) A weak hyper KU-ideal of H if $a \circ (b \circ c) \subseteq I$ and $b \in I$ imply $a \circ c \in I$;
(2) A hyper KU-ideal of H if $a \circ (b \circ c) << I$ and $b \in I$ imply $a \circ c \in I$;
(3) A strong hyper KU-ideal of H if $(\forall x, y \in H)$ $((a \circ (b \circ c) \bigcap I \neq \phi)$ and $b \in I$ imply $a \circ c \in I$.

Theorem 5.8. *Every (strong, weak) hyper KU-ideal is a (strong, weak) hyper ideal.*

Proof. Let I be a hyper KU-ideal of H. Then, $\forall a, b, c \in H$ $a \circ (b \circ c) << I$ and $b \in I$ imply $a \circ c \in I$. Putting $a = 0$ we get $0 \circ (b \circ c) << I$ and $b \in I$ imply $0 \circ c \in I$. Hence $(b \circ c) << I$ and $b \in I$ imply $c \in I$. □

Generally, every (strong, weak) hyper ideal is not a (strong, weak) hyper KU-ideal. It can be observed with the help of examples given below:

Example 5.9. Let $H = \{0, a, b, c\}$ be a set with the following Cayley table.

$\circ$	0	a	b	c
0	$\{0\}$	$\{a\}$	$\{b\}$	$\{c\}$
a	$\{0\}$	$\{0, a\}$	$\{0, b\}$	$\{b, c\}$
b	$\{0\}$	$\{0, b\}$	$\{0\}$	$\{a\}$
c	$\{0\}$	$\{0, b\}$	$\{0\}$	$\{0, a\}$

Then, $(H, \circ, 0)$ is a hyper KU-algebra. Take $I = \{0, b\}$, then I is a weak hyper ideal, however, not a weak hyper KU-ideal of H as $b \circ (b \circ c) \subseteq I, b \in I$, but $b \circ c = a \notin I$.

Example 5.10. Let $H = \{0, a, b\}$ be a set with the following Cayley table.

$\circ$	0	a	b
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{0\}$	$\{0, a\}$	$\{b\}$
b	$\{0\}$	$\{b\}$	$\{0, b\}$

Then, $(H, \circ, 0)$ is a hyper KU-algebra. Take $I = \{0, b\}$, then I is a hyper ideal, but not a hyper KU-ideal of H, since $0 \circ (b \circ a) << I, b \in I$, but $a \notin I$.Here $I = \{0, b\}$ is also a strong hyper ideal but it is not a strong hyper KU-ideal of H, since $0 \circ (b \circ a) = \{b\} \cap I \neq \phi$ and $b \in I$ but $a \notin I$.

Definition 5.11. A subset I of a hyper KU-algebra H such that $0 \in I$ is called the following:

(1) A hyper KU-ideal of type1, if $x \circ (y \circ z) << I, y \in I \Rightarrow x \circ z \subseteq I$,

(2) A hyper KU-ideal of type 2, if $x \circ (y \circ z) \subseteq I, y \in I \Rightarrow x \circ z \subseteq I$,
(3) A hyper KU-ideal of type 3, if $x \circ (y \circ z) << I, y \in I \Rightarrow x \circ z << I$,
(4) A hyper KU-ideal of type 4, if $x \circ (y \circ z) \subseteq I, y \in I \Rightarrow x \circ z << I$.

Theorem 5.12. *In any hyper KU-algebra, the following statements are valid.*

(1) *Any hyper KU-ideal of type 1 is a hyper KU-ideal of types 2 and 3.*
(2) *Any hyper KU-ideal of type 2 is a hyper KU-ideal of type 4.*
(3) *Any hyper KU-ideal of type 3 is a hyper KU-ideal of type 4.*
(4) *Any hyper KU-ideal of type 1 is a hyper -ideal.*
(5) *Any hyper KU-ideal of type 2 is a weak hyper -ideal.*

Proof. The statements (1), (2), and (3) are clear. So, we prove only (4) and (v).

(4) Let I be a hyper KU-ideal of type 1,$y \circ x << I$, and $y \in I$. Hence, by Proposition 3.4, (P_7). We obtain $0 \circ (y \circ x) = y \circ x << I$. But $y \in I$, so applying the hypothesis and (HKU_3), we get $\{x\} = 0 \circ x \subseteq I$. This shows that I is a hyper-ideal of H.

(5) The proof of (4) is analogous. □

6. A Homomorphism on hyper KU-algebras

Definition 6.1. Let $(H, \circ, 0), (K, \circ', 0')$ be hyper KU-algebras. A mapping $f : H \longrightarrow K$ is called a hyper homomorphism if

(HH_1) $f(0) = 0'$,
(HH_2) $f(x \circ y) = f(x) \circ' f(y)$, for all $x, y \in H$.

Theorem 6.2. *Let $f : H \longrightarrow K$ be a hyper homomorphism of hyper KU-algebras. If $x << y$ in H, then $f(y) << f(x)$ in K.*

Proof. Let $x, y \in H$ be such that $x << y$. Then, $0 \in y \circ x$, and so $0' = f(0) \in f(x \circ y) = f(x) \circ f(y)$. Hence, $f(y) << f(x)$ in K. □

Theorem 6.3. *Let $f : H \longrightarrow K$ be a hyper homomorphism of hyper KU-algebras. If I is a hyper ideal of K, then $f^{-1}(1)$ is a hyper ideal of H.*

Proof. Clearly $0 \in f^{-1}(1)$. Let $x, y \in H$ be such that $x \circ y << f^{-1}(1)$ and $x \in f^{-1}(1)$. Then, $f(x) \in I$, and for every $z \in x \circ y$ there exists $w \in f^{-1}(1)$ such that $z << w$, that is, $0 \in w \circ z$. It follows that $0' = f(0) \in f(w \circ z) = f(w) \circ f(z) \subseteq I \circ f(x \circ y) = I \circ f(x) \circ f(y)$, so that $f(x) \circ f(y) << I$. Since I is a hyper ideal of K, it follows that $f(y) \in I$, that is, $y \in f^{-1}(1)$ by (HI_1). Hence $f^{-1}(1)$ is a hyper ideal of H. □

Theorem 6.4. *If $f : H \longrightarrow K$ is a hyper homomorphism of hyper KU-algebras, then $ker(f) = \{x \in H : f(x) = 0\}$, called the kernel of f, is a hyper ideal of H.*

Proof. Clearly $0 \in ker(f)$. Let $x, y \in H$ be such that $x \circ y << ker(f)$ and $x \in ker(f)$. Then, $f(x) = 0\acute{}$, and for each $a \in x \circ y$ there exists $b \in ker(f)$ such that $a << b$. It follows from (HH_2) and (HKU_3) that is, $a \in ker(f)$ so that $0\acute{} = f(a) \in f(x \circ y) = f(x) \circ f(y) = 0\acute{} \circ f(y) = f(y)$,that is $y \in ker(f)$. Hence $ker(f)$ is a hyper ideal of H. □

Theorem 6.5. *Let $f : H \longrightarrow K$ be an onto hyper homomorphism of hyper KU-algebras. If I is a hyper ideal of H containing $ker(f)$, then $f(1)$ is a hyper ideal of K.*

Proof. Note that $0\acute{} = f(0) \in f(1)$. Let $x, y \in K$ be such that $x \circ y << f(1)$ and $x \in f(1)$. Since f is onto, it follows that there exist $a, b \in H$ such that $f(a) = x$ and $f(b) = y$. Thus $f(a \circ b) = f(a) \circ f(b) = x \circ y << f(1)$. Let $w \in a \circ b$. And for every $z \in f(a \circ b)$ there exist $w \in f(1)$ such that $z << w$. Then, $f(w) << f(z)$, that is $0 \in f(z) \circ f(w) = f(z \circ w)$. It follows that $z \circ w \subseteq kerf \subseteq I$ so that $z \circ w << I$ by (P_1). Since I is a hyper ideal of H, it follows that $w \in I$ by (HI_2). Hence $a \circ b \subseteq I$ and so $a \circ b << I$. Since $a \in I$, it follows from (HI_2) that $b \in I$ so that $y = f(b) \inf(1)$. Hence $f(1)$ is a hyper ideal of$K$. □

Theorem 6.6. *Let $f : H_1 \longrightarrow H_2$ and $g : H_1 \longrightarrow H_3$ be two homomorphism of hyper KU-algebras such that f is onto and $ker(f) \subseteq ker(g)$. Then, there exists a homomorphism $h : H_2 \longrightarrow H_3$ such that $h \circ f = g$.*

Proof. Let $y \in H_2$. Since f is onto, there exists $x \in H_1$ such that $y = f(x)$. Define $h : H_2 \longrightarrow H_3$ by $h(y) = g(x)$, for all $y \in H_2$. Now, we show that h is well-defined. Let $y_1, y_2 \in H_2$ and $y_1 = y_2$. Since f is onto, then there are $x_1, x_2 \in H_1$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Hence $f(x_1) = f(x_2)$ and $0 \in f(x_1) \circ f(x_2) = f(x_1 \circ x_2)$. It follows that there exists $z \in x_1 \circ x_2$ such that $f(z) = 0$. Thus $z \in ker(f) \subseteq ker(g)$ and $g(z) = 0$. Since $z \in x_1 \circ x_2$, then $0 = g(z) \in g(x_1 \circ x_2) = g(x_1) \circ g(x_2)$ which implies that $g(x_2) << g(x_1)$. On the other hand since $0 \in f(x_1) \circ f(x_2) = f(x_1 \circ x_2)$, similarly we can conclude that $0 \in g(x_2 \circ x_1) = g(x_2) \circ g(x_1)$, then $g(x_1) << g(x_2)$. Thus $g(x_1) = g(x_2)$, which shows that h is well-defined. Clearly, $h \circ f = g$. Finally, we show that h is a homomorphism. Let $y_1, y_2 \in H_2$. Since f is onto, there are $x_1, x_2 \in H_1$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Then, $h(y_1 \circ y_2) = h(f(x_1) \circ f(x_2)) = h(f(x_1 \circ x_2)) = (h \circ f)(x_1 \circ x_2) =$

$g(x_1 \circ x_2) = g(x_1) \circ g(x_2) = (h \circ f)(x_1) \circ (h \circ f)(x_2) = h(f(x_1)) \circ h(f(x_2)) = h(y_1) \circ h(y_2)$. Moreover, since $f(0) = 0$ and $g(0) = 0$, then $h(0) = h(f(0)) = (h \circ f)(0) = g(0) = 0$ Thus h is a homomorphism. □

7. Conclusion

The concept of a hyper structure KU-algebra is introduced and some related properties are investigated. Also, some types of hyper KU-algebras are studied and the relationship between them is stated. Then, a hyper KU-ideal of a hyper structure KU-algebra is studied and a few properties are obtained. Furthermore, the notion of a homomorphism is discussed. Main purpose of our future work is to investigate fuzzy of several types of hyper ideals with special properties such as an intuitionistic bipolar (interval value) fuzzy n-fold of hyper KU-algebras.

Algorithm for hyper KU-algebras
Input (X: set, $\circ$: hyper operation)
Output ("X is a hyper KU-algebra or not")
Begin
If $X = \phi$ then go to (1.);
EndIf
If $0 \notin X$ then go to (1.);
EndIf
Stop: =false;
$i := 1$;
While $i \leq |X|$ and not (Stop) do
If $0 \notin x_i \circ x_i \neq 0$ then
Stop: = true;
EndIf
$j := 1$
While $j \leq |X|$ and not (Stop) do
If $0 \notin ((y_j \circ x_i) \circ x_i)$ or $0 \in (x_i \circ y_j) \neq 0$ and $x_i \neq y_j$ then
Stop: = true;
EndIf

$j := 1$
While $j \leq |X|$ and not (Stop)do
If $0 \notin x_i \circ (y_j \circ x_i)$ or $x_i \circ y_j$ and $0 \in (y_j \circ x_i)$ and $x_i \neq y_j$, then
Stop: = true;

EndIf

EndIf

$k := 1$
While $k \leq |X|$ and not (stop) do
If $0 \notin (x_i \circ y_j) \circ ((y_j \circ z_k) \circ (x_i \circ z_k))$ then
Stop:=true EndIf
End While
End While
End While
If Stop then

(1.) Output ("X is not a hyper KU-algebra")
Else
Output ("X is a hyper KU-algebra")
EndIf
End

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Samy M. Mostafa
Department of Mathematics, Faculty of Education, Ain Shams University, Roxy, Cairo, Egypt
Email:samymostafa@yahoo.com

Fatema F. Kareem
Department of Mathematics , Ibn-Al-Haitham college of Education, University of Baghdad, Iraq
Email:fa_sa20072000@yahoo.com

Bijan Davvaz
Department of Mathematics, Yazd University, Yazd, Iran
Email:davvaz@yazd.ac.ir