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CHARACTERIZATION OF JORDAN *-DERIVATIONS BY LOCAL ACTION ON RINGS WITH INVOLUTION

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Abstract. Let $\mathcal{R}$ be a ring with an involution $*$ and a symmetric idempotent e. It is shown that, under some mild conditions on $\mathcal{R}$, an additive map $\delta : \mathcal{R} \to \mathcal{R}$ satisfies $\delta(ab + ba) = \delta(a)b^* + a\delta(b) + \delta(b)a^* + b\delta(a)$ whenever $ab = e$ for $a, b \in \mathcal{R}$ if and only if δ is a Jordan *-derivation.

Key Words: Rings with involution, Jordan *-derivations, Derivations.
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1. Introduction

Let $\mathcal{R}$ be a ring with an involution $*$, which will be called a *-ring. Let $\mathcal{R}' \subseteq \mathcal{R}$ be a subring. An additive map $\delta : \mathcal{R}' \to \mathcal{R}$ is called a Jordan *-derivation if $\delta(a^2) = \delta(a)a^* + a\delta(a)$ for all $a \in \mathcal{R}'$. Note that, if $\mathcal{R}$ is 2-torsion free, then a Jordan *-derivation can be equivalently defined as $\delta(ab + ba) = \delta(a)b^* + a\delta(b) + \delta(b)a^* + b\delta(a)$ for all $a, b \in \mathcal{R}'$. It is easy to verify that, for every $r \in \mathcal{R}$, the map δ defined by $\delta(a) = ar - ra^*$ is a Jordan *-derivation.

The study of Jordan *-derivations has been motivated by the problem of the representability of quasi-quadratic functionals by sesquilinear ones. It turns out that the question of whether each quasi-quadratic functional is generated by some sesquilinear functional is intimately connected with the structure of Jordan *-derivations (see [**?**, **?**, **?**, **?**] and the references therein). In [**?**], Brešar and Vukman proved that, if a unital

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*-ring $\mathcal{R}$ contains $\frac{1}{2}$ and a central invertible element μ with $\mu^* = -\mu$, then every additive Jordan *-derivation of $\mathcal{R}$ is inner, that is, it is of the form $x \mapsto xa - ax^*$ for some $a \in \mathcal{R}$; in particular, every additive Jordan *-derivation of a unital complex *-algebra is inner. Let $\mathcal{B}(H)$ be the algebra of all bounded linear operators on a real or complex Hilbert space H with $\dim H > 1$ and let $\mathcal{A}$ be a standard operator algebra on H. Šemrl in [**?**] proved that every additive Jordan *-derivation $\delta : \mathcal{A} \to \mathcal{B}(H)$ is of the form $\delta(A) = AT - TA^*$ for some $T \in \mathcal{B}(H)$. Let $\mathcal{R}$ be a noncommutative prime *-ring. Lee, Wong and Zhou [**?**, **?**] showed that any additive Jordan *-derivation of $\mathcal{R}$ is of the form $x \mapsto xa - ax^*$ for all $x \in \mathcal{R}$, where a is in the maximal symmetric ring of quotients of $\mathcal{R}$, except when $\mathrm{char}\mathcal{R} = 2$ and $\dim_C \mathcal{R}C = 4$, where C is the extended centroid of $\mathcal{R}$. For other related results, see [**?**, **?**] and the references therein.

Recently, the question of under what conditions an additive map becomes a derivation had attracted much attention of many researchers (for example, see [**?**, **?**] and the references therein). For Jordan *-derivations, Qi and Zhang [**?**] first discussed the properties of Jordan *-derivations by local action. Let $\mathcal{R}$ be a 2-torsion free *-ring with a nontrivial symmetric idempotent. Under some mild conditions on $\mathcal{R}$, Qi and Zhang in [**?**, **?**] proved that an additive map δ on $\mathcal{R}$ satisfies $\delta(ab+ba) = \delta(a)b^* + a\delta(b) + \delta(b)a^* + b\delta(a)$ whenever $ab = 0$ (respectively, $ab = 1$) for $a, b \in \mathcal{R}$ if and only if δ is an additive Jordan *-derivation.

The main purpose of the present paper is to continue to consider the characterization of Jordan *-derivations by acting on a nontrivial symmetric idempotent. Recall that an element $a \in \mathcal{R}$ is symmetric (respectively, skew symmetric) if $a^* = a$ (respectively, if $a^* = -a$) and is idempotent if $a^2 = a$. For more details about *-rings, the reader can see the book [**?**].This is a template article, which you can use as a skeleton for your own article.

2. Main result and its proof

In this section, we will give the main result of this paper and its proof.

Theorem 2.1. *Let $\mathcal{R}$ be a 2-torsion free unital *-ring with a nontrivial symmetric idempotent e_1. Assume that $\mathcal{R}$ satisfies the following two conditions:*

(1) *for $a \in \mathcal{R}$, $a\mathcal{R}e_i = \{0\}$ implies $a = 0$, where $i = 1, 2$ and $e_2 = 1 - e_1$;*

(2) *for all* $a \in \mathcal{R}$, *there exists some integer* n *such that* $ne_1 - e_1ae_1$ *is invertible in* $e_1\mathcal{R}e_1$.

Then an additive map $\delta : \mathcal{R} \to \mathcal{R}$ *satisfies* $\delta(ab + ba) = \delta(a)b^* + a\delta(b) + \delta(b)a^* + b\delta(a)$ *whenever* $ab = e_1$ *for* $a, b \in \mathcal{R}$ *if and only if* δ *is a Jordan*-derivation.*

Put $e_i\mathcal{R}e_j = \mathcal{R}_{ij}$ for any $i, j \in \{1, 2\}$. Then we have the decomposition $\mathcal{R} = \mathcal{R}_{11}+\mathcal{R}_{12}+\mathcal{R}_{21}+\mathcal{R}_{22}$. So any element $a \in \mathcal{R}$ can be expressed as $a = a_{11} + a_{12} + a_{21} + a_{22}$,where $a_{ij} \in \mathcal{R}_{ij}$.

Proof of Theorem 2.1. The "if" part is obvious. For the "only if" part, we will prove it by checking a series of claims.

Claim 1. $\delta(e_1) = e_1\delta(e_1)e_2 + e_2\delta(e_1)e_1$.

By $e_1e_1 = e_1$, we have that

$$\delta(e_1e_1+e_1e_1) = \delta(e_1)e_1^*+e_1\delta(e_1)+\delta(e_1)e_1^*+e_1\delta(e_1) = 2(\delta(e_1)e_1+e_1\delta(e_1)).$$

Since $\mathcal{R}$ is 2-torsion free and δ is additive,one gets $\delta(e_1) = \delta(e_1)e_1 + e_1\delta(e_1)$. Multiplying by e_1 and e_2 from both sides in the equation, respectively, we obtain $e_1\delta(e_1)e_1 = e_2\delta(e_1)e_2 = 0$. So $\delta(e_1) = e_1\delta(e_1)e_2 + e_2\delta(e_1)e_1$.

Now, define $\tau(a) = \delta(a) - (aa_0 - a_0a^*)$ for all $a \in \mathcal{R}$, where $a_0 = e_1\delta(e_1)e_2 - e_2\delta(e_1)e_1$. It is easily checked that $\tau : \mathcal{R} \to \mathcal{R}$ is also an additive map satisfying

$$\tau(ab + ba) = \tau(a)b^* + a\tau(b) + \tau(b)a^* + b\tau(a) \tag{2.1}$$

whenever $ab = e_1$ for $a, b \in \mathcal{R}$ and

$$\tau(e_1) = 0. \tag{2.2}$$

Claim 2. $\tau(\mathcal{R}_{ii}) \subseteq \mathcal{R}_{ii}$, $i = 1, 2$.

For any $a_{22} \in \mathcal{R}_{22}$, since $e_1(e_1+a_{22}) = e_1$, by Eqs.(2.1)-(2.2), we have

$$\begin{aligned} 0 = \ & \tau(e_1(e_1 + a_{22}) + (e_1 + a_{22})e_1) \\ = \ & \tau(e_1)(e_1 + a_{22})^* + e_1\tau(e_1 + a_{22}) + \tau(e_1 + a_{22})e_1^* + (e_1 + a_{22})\tau(e_1) \\ = \ & e_1\tau(a_{22}) + \tau(a_{22})e_1 = 2e_1\tau(a_{22})e_1 + e_1\tau(a_{22})e_2 + e_2\tau(a_{22})e_1. \end{aligned}$$

It follows that $e_1\tau(a_{22})e_2 = e_2\tau(a_{22})e_1 = e_1\tau(a_{22})e_1 = 0$. So

$$\tau(a_{22}) \in \mathcal{R}_{22} \text{ for all } a_{22} \in \mathcal{R}_{22}. \tag{2.3}$$

For any invertible $a_{11} \in \mathcal{R}_{11}$, since $a_{11}^{-1}a_{11} = e_1$, we have

$$\begin{aligned} 0 = \ & \tau(a_{11}^{-1}a_{11} + a_{11}a_{11}^{-1}) \\ = \ & \tau(a_{11}^{-1})a_{11}^* + a_{11}^{-1}\tau(a_{11}) + \tau(a_{11})(a_{11}^{-1})^* + a_{11}\tau(a_{11}^{-1}). \end{aligned} \tag{2.4}$$

Since $(a_{11}^{-1}+a_{22})a_{11}=e_1$, by Eqs.(2.2)-(2.4), one has

$$\begin{aligned}
0= &\ \tau((a_{11}^{-1}+a_{22})a_{11}+a_{11}(a_{11}^{-1}+a_{22}))\\
= &\ \tau(a_{11}^{-1}+a_{22})a_{11}^*+(a_{11}^{-1}+a_{22})\tau(a_{11})\\
&+\tau(a_{11})(a_{11}^{-1}+a_{22})^*+a_{11}\tau(a_{11}^{-1}+a_{22})\\
= &\ \tau(a_{11}^{-1})a_{11}^*+\tau(a_{22})a_{11}^*+a_{11}^{-1}\tau(a_{11})+a_{22}\tau(a_{11})\\
&+\tau(a_{11})(a_{11}^{-1})^*+\tau(a_{11})a_{22}^*+a_{11}\tau(a_{11}^{-1})+a_{11}\tau(a_{22})\\
= &\ a_{22}\tau(a_{11})+\tau(a_{11})a_{22}^*.
\end{aligned}$$

Particularly, by taking $a_{22}=e_2$ in the above equation, one gets $e_2\tau(a_{11})+\tau(a_{11})e_2=0$, which implies $e_2\tau(a_{11})e_1=e_1\tau(a_{11})e_2=e_2\tau(a_{11})e_2=0$ as $\mathcal{R}$ is 2-torsion free. So $\tau(a_{11})\in\mathcal{R}_{11}$ for all invertible elements $a_{11}\in\mathcal{R}_{11}$.

Now for any element $a_{11}\in\mathcal{R}_{11}$, by the assumption (2), there exists some integer n such that ne_1-a_{11} is invertible. Hence, by the additivity of τ, $\tau(a_{11})=\tau(ne_1-a_{11})\in\mathcal{R}_{11}$, completing the proof of the claim.

Claim 3. $\tau(\mathcal{R}_{ij})\subseteq\mathcal{R}_{ij}+\mathcal{R}_{ji}$, $1\leq i\neq j\leq 2$.

Take any $a_{ij}\in\mathcal{R}_{ij}$, $1\leq i\neq j\leq 2$. Since $e_1(a_{21}+e_1)=e_1$ and $(e_1+a_{12})e_1=e_1$, by Eqs.(2.1)-(2.2), we have

$$\begin{aligned}
\tau(a_{21})= &\ \tau(e_1(a_{21}+e_1)+(a_{21}+e_1)e_1)\\
= &\ \tau(e_1)(a_{21}+e_1)^*+e_1\tau(a_{21}+e_1)\\
&+\tau(a_{21}+e_1)e_1^*+(a_{21}+e_1)\tau(e_1)\\
= &\ e_1\tau(a_{21})+\tau(a_{21})e_1
\end{aligned}$$

and

$$\begin{aligned}
\tau(a_{12})= &\ \tau((e_1+a_{12})e_1+e_1(e_1+a_{12}))\\
= &\ \tau(e_1+a_{12})e_1^*+(e_1+a_{12})\tau(e_1)\\
&+\tau(e_1)(e_1+a_{12})^*+e_1\tau(e_1+a_{12})\\
= &\ \tau(a_{12})e_1+e_1\tau(a_{12}).
\end{aligned}$$

Multiplying by e_1 and e_2 from both sides in the above two equations, respectively, one can easily obtain $e_i\tau(a_{ij})e_i=e_j\tau(a_{ij})e_j=0$ for $1\leq i\neq j\leq 2$. So the claim holds.

Claim 4. For any $a_{ii}\in\mathcal{R}_{ii}$ and $a_{ij}\in\mathcal{R}_{ij}$ with $1\leq i\neq j\leq 2$, we have $\tau(a_{ii}a_{ij})=\tau(a_{ij})a_{ii}^*+a_{ii}\tau(a_{ij})$.

For any invertible element $a_{11}\in\mathcal{R}_{11}$ and any $a_{12}\in\mathcal{R}_{12}$, noting that $(a_{11}^{-1}+a_{12})a_{11}=e_1$, by Eq.(2.4) and Claim 2, we have

$$\begin{aligned}
&\tau(a_{11}a_{12})=\tau((a_{11}^{-1}+a_{12})a_{11}+a_{11}(a_{11}^{-1}+a_{12}))\\
=\ &\tau(a_{11}^{-1}+a_{12})a_{11}^*+(a_{11}^{-1}+a_{12})\tau(a_{11})\\
&+\tau(a_{11})(a_{11}^{-1}+a_{12})^*+a_{11}\tau(a_{11}^{-1}+a_{12})\\
=\ &\tau(a_{11}^{-1})a_{11}^*+\tau(a_{12})a_{11}^*+a_{11}^{-1}\tau(a_{11})+a_{12}\tau(a_{11})\\
&+\tau(a_{11})(a_{11}^{-1})^*+\tau(a_{11})a_{12}^*+a_{11}\tau(a_{11}^{-1})+a_{11}\tau(a_{12})\\
=\ &\tau(a_{12})a_{11}^*+a_{11}\tau(a_{12}).
\end{aligned}$$

Thus, for any $a_{11}\in\mathcal{R}_{11}$, by the assumption (2), ne_1-a_{11} is invertible. So the above equation yields $\tau((ne_1-a_{11})a_{12})=\tau(a_{12})(ne_1-a_{11})^*+(ne_1-a_{11})\tau(a_{12})$, which and Claim 3 implies that $\tau(a_{11}a_{12})=\tau(a_{12})a_{11}^*+a_{11}\tau(a_{12})$ holds for all $a_{11}\in\mathcal{R}_{11}$ and $a_{12}\in\mathcal{R}_{12}$.

Now, for any $a_{22}\in\mathcal{R}_{22}$ and $b_{21}\in\mathcal{R}_{21}$, since $(e_1+a_{22}-a_{22}b_{21})(e_1+b_{21})=e_1$, by Eq.(2.2), Claims 2-3, we have

$$\begin{aligned}
&\tau(b_{21})\\
=\ &\tau((e_1+a_{22}-a_{22}b_{21})(e_1+b_{21})+(e_1+b_{21})(e_1+a_{22}-a_{22}b_{21}))\\
=\ &\tau(e_1+a_{22}-a_{22}b_{21})(e_1+b_{21})^*+(e_1+a_{22}-a_{22}b_{21})\tau(e_1+b_{21})\\
&+\tau(e_1+b_{21})(e_1+a_{22}-a_{22}b_{21})^*+(e_1+b_{21})\tau(e_1+a_{22}-a_{22}b_{21})\\
=\ &-\tau(a_{22}b_{21})e_1-\tau(a_{22}b_{21})b_{21}^*+e_1\tau(b_{21})+a_{22}\tau(b_{21})-a_{22}b_{21}\tau(b_{21})\\
&+\tau(b_{21})e_1+\tau(b_{21})a_{22}^*-\tau(b_{21})(a_{22}b_{21})^*-e_1\tau(a_{22}b_{21})-b_{21}\tau(a_{22}b_{21}),
\end{aligned}$$

and so

$$\begin{aligned}
\tau(a_{22}b_{21})=\ &a_{22}\tau(b_{21})+\tau(b_{21})a_{22}^*-\tau(a_{22}b_{21})b_{21}^*\\
&-a_{22}b_{21}\tau(b_{21})-\tau(b_{21})(a_{22}b_{21})^*-b_{21}\tau(a_{22}b_{21}).
\end{aligned}$$

Replacing by $2b_{21}$ by b_{21} in the above equation and noting that $\mathcal{R}$ is 2-torsion free, one has

$$\begin{aligned}
\tau(a_{22}b_{21})=\ &a_{22}\tau(b_{21})+\tau(b_{21})a_{22}^*-2\tau(a_{22}b_{21})b_{21}^*\\
&-2a_{22}b_{21}\tau(b_{21})-2\tau(b_{21})(a_{22}b_{21})^*-2b_{21}\tau(a_{22}b_{21}).
\end{aligned}$$

Combining the above two equations, we obtain $\tau(a_{22}b_{21})=a_{22}\tau(b_{21})+\tau(b_{21})a_{22}^*$ and

$$\tau(a_{22}b_{21})b_{21}^*+a_{22}b_{21}\tau(b_{21})+\tau(b_{21})(a_{22}b_{21})^*+b_{21}\tau(a_{22}b_{21})=0. \quad (2.5)$$

So the claim is true.

Similarly, one can check the following claim.

Claim 5. For any $a_{jj}\in\mathcal{R}_{jj}$ and $a_{ij}\in\mathcal{R}_{ij}$ with $1\le i\ne j\le 2$, we have $\tau(a_{ij}a_{jj})=\tau(a_{ij})a_{jj}^*+a_{ij}\tau(a_{jj})+\tau(a_{jj})a_{ij}^*+a_{jj}\tau(a_{ij})$.

In addition, by a similar argument to that of Eq.(2.5), one can show that

$$\tau(a_{12}a_{22})a_{12}^* + a_{12}a_{22}\tau(a_{12}) + \tau(a_{12})(a_{12}a_{22})^* + a_{12}\tau(a_{12}a_{22}) = 0 \quad (2.6)$$

holds for all $a_{12} \in \mathcal{R}_{12}$ and $a_{22} \in \mathcal{R}_{22}$.

Claim 6. For any $a_{ij} \in \mathcal{R}_{ij}$ with $1 \leq i \neq j \leq 2$, we have $0 = \tau(a_{ij})a_{ij}^* + a_{ij}\tau(a_{ij})$.

By taking $a_{11} = e_1$ and $a_{22} = e_2$ in Eqs.(2.5)-(2.6), the claim is obvious.

Claim 7. For any $a_{ii} \in \mathcal{R}_{ii}$, we have $\tau(a_{ii}^2) = \tau(a_{ii})a_{ii}^* + a_{ii}\tau(a_{ii})$, $1 \leq i \leq 2$.

Take any $a_{ii} \in \mathcal{R}_{ii}$ and $a_{ji} \in \mathcal{R}_{ji}$ $(1 \leq i \neq j \leq 2)$. By Claim 5, we have

$$\begin{aligned} & \tau(a_{ji}a_{ii}a_{ii}) \\ = \ & \tau(a_{ji}a_{ii})a_{ii}^* + a_{ji}a_{ii}\tau(a_{ii}) + \tau(a_{ii})(a_{ji}a_{ii})^* + a_{ii}\tau(a_{ji}a_{ii}) \\ = \ & \tau(a_{ji})(a_{ii}a_{ii})^* + a_{ji}\tau(a_{ii})a_{ii}^* + \tau(a_{ii})a_{ji}^*a_{ii}^* \\ & + a_{ii}\tau(a_{ji})a_{ii}^* + a_{ji}a_{ii}\tau(a_{ii}) + \tau(a_{ii})(a_{ji}a_{ii})^* + a_{ii}\tau(a_{ji})a_{ii}^* \\ & + a_{ii}a_{ji}\tau(a_{ii}) + a_{ii}\tau(a_{ii})a_{ji}^* + a_{ii}a_{ii}\tau(a_{ji}) \end{aligned}$$

and

$$\tau(a_{ji}a_{ii}a_{ii}) = \tau(a_{ji})(a_{ii}a_{ii})^* + a_{ji}\tau(a_{ii}a_{ii}) + \tau(a_{ii}a_{ii})a_{ji}^* + a_{ii}a_{ii}\tau(a_{ji}).$$

Comparing the above two equations gives

$$a_{ji}[\tau(a_{ii}a_{ii}) - \tau(a_{ii})a_{ii}^* - a_{ii}\tau(a_{ii})] + [\tau(a_{ii}a_{ii}) - \tau(a_{ii})a_{ii}^* - a_{ii}\tau(a_{ii})]a_{ji}^* = 0,$$

which and Claim 2 imply

$$a_{ji}[\tau(a_{ii}a_{ii}) - \tau(a_{ii})a_{ii}^* - a_{ii}\tau(a_{ii})] = [\tau(a_{ii}a_{ii}) - \tau(a_{ii})a_{ii}^* - a_{ii}\tau(a_{ii})]a_{ji}^* = 0.$$

That is, $[\tau(a_{ii}a_{ii}) - \tau(a_{ii})a_{ii}^* - a_{ii}\tau(a_{ii})]e_iae_j = 0$ holds for all $a \in \mathcal{R}$. It follows from the assumption (1) that $\tau(a_{ii}a_{ii}) = \tau(a_{ii})a_{ii}^* + a_{ii}\tau(a_{ii})$ holds for all $a_{ii} \in \mathcal{R}_{ii}$.

Claim 8. For any $a_{ij} \in \mathcal{R}_{ij}$ and $a_{ji} \in \mathcal{R}_{ji}$, we have $\tau(a_{ij}a_{ji}) = \tau(a_{ji})a_{ij}^* + a_{ji}\tau(a_{ij})$, $1 \leq i \neq j \leq 2$.

For any $a_{12} \in \mathcal{R}_{12}$ and $b_{21} \in \mathcal{R}_{21}$, we have from $(e_1+a_{12}-a_{12}b_{21})(e_1+b_{21}) = e_1$ that

$$\begin{aligned}
& \tau(a_{12}) - \tau(a_{12}b_{21}) + \tau(b_{21}) + \tau(b_{21}a_{12}) - \tau(b_{21}a_{12}b_{21}) \\
= \; & \tau((e_1 + a_{12} - a_{12}b_{21})(e_1 + b_{21}) + (e_1 + b_{21})(e_1 + a_{12} - a_{12}b_{21})) \\
= \; & \tau(e_1 + a_{12} - a_{12}b_{21})(e_1 + b_{21})^* + (e_1 + a_{12} - a_{12}b_{21})\tau(e_1 + b_{21}) \\
& +\tau(e_1 + b_{21})(e_1 + a_{12} - a_{12}b_{21})^* + (e_1 + b_{21})\tau(e_1 + a_{12} - a_{12}b_{21}) \\
= \; & \tau(a_{12})e_1 + \tau(a_{12})b_{21}^* - \tau(a_{12}b_{21})e_1 - \tau(a_{12}b_{21})b_{21}^* + e_1\tau(b_{21}) \\
& +a_{12}\tau(b_{21}) - a_{12}b_{21}\tau(b_{21}) + \tau(b_{21})e_1 + \tau(b_{21})a_{12}^* - \tau(b_{21})(a_{12}b_{21})^* \\
& +e_1\tau(a_{12}) - e_1\tau(a_{12}b_{21}) + b_{21}\tau(a_{12}) - b_{21}\tau(a_{12}b_{21}).
\end{aligned}$$

By Claims 2-4, the above equation reduces to

$$\tau(b_{21}a_{12}) = \tau(a_{12})b_{21}^* + b_{21}\tau(a_{12})$$

and

$$\tau(a_{12}b_{21}) = a_{12}\tau(b_{21}) + \tau(b_{21})a_{12}^*.$$

The claim holds.

Claim 9. τ is a Jordan *-derivation. Therefore, δ is a Jordan *-derivation.

For any $a = \sum_{i,j=1}^{2} a_{ij} \in \mathcal{R}$, by Claims 4-8 and the additivity of τ, one can easily check $\tau(a^2) = \tau(a)a^* + a\tau(a)$, that is, τ is a Jordan *-derivation. Now by the definition of τ, it is obvious that δ is also a Jordan *-derivation. □

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