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AN EFFICIENT TECHNIQUE FOR SOLVING LINEAR AND NONLINEAR WAVE EQUATIONS WITHIN LOCAL FRACTIONAL OPERATORS

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Abstract. In this paper, we utilize reduced differential transform method (RDTM) to obtain approximate solutions for linear and nonlinear wave equations within local fractional differential operators. The operators are taken in the local fractional sense. The efficiency of the considered method is illustrated by some examples. This method reduces significantly the numerical computations compare with local fractional variational iteration method. The results reveal that the suggested algorithm is very effective and simple and can be applied for other linear and nonlinear problems in sciences and engineering.

Key Words: Local fractional wave equation, Local fractional derivative operators, Reduced differential transform method.
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1. Introduction

There are many physical applications in science and engineering can be represented by models using the differential equations, which are very helpful for many physical problems. These equations are represented by linear and nonlinear partial differential equations and solving such differential equations is very important. Our concern in this work is to consider the linear and nonlinear wave equation with local fractional

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differential operator as the following:

$$L_{\kappa\kappa}^{(2\vartheta)}\varphi(\eta,\kappa) - L_{\eta\eta}^{(2\vartheta)}\varphi(\eta,\kappa) + \Lambda(\varphi) = \psi(\eta,\kappa), 0 < \alpha \leq 1, \tag{1.1}$$

$$L_{\kappa\kappa}^{(2\vartheta)}\varphi(\eta,\kappa) - L_{\eta\eta}^{(2\vartheta)}\varphi(\eta,\kappa) + \Upsilon(\varphi) = \psi(\eta,\kappa), 0 < \alpha \leq 1, \tag{1.2}$$

with the initial conditions

$$\varphi(\eta,0) = \omega(\eta), \frac{\partial^{\vartheta}\varphi(\eta,0)}{\partial\kappa^{\vartheta}} = \rho(\eta), \tag{1.3}$$

where $\Lambda(\varphi)$ and $\Upsilon(\varphi)$are linear and nonlinear functions respectively, $\psi(\eta,\kappa)$ is source term of nondifferentiable function, and $\omega(\eta)$ and $\rho(\eta)$ are given functions.
There are many analytical and numerical methods used to solve local fractional partial differential equations such as, local fractional function decomposition method [1, 2, 3], local fractional Adomian decomposition method [3, 4], local fractional series expansion method [5, 6], local fractional Laplace transform method [7, 8], local fractional Fourier series method [9], local fractional homotopy perturbation method [10], local fractional varitional iteration method [11, 12, 13], local fractional differential transform method [14, 15], local fractional Laplace decomposition method [16], local fractional Laplace variational iteration method [17, 18, 19].The local fractional reduced differential transform technique is an iterative procedure for obtaining Taylor series solution of partial differential equations. This method reduces the size of computational work and easily applicable to many physical problems. Our aim is to extend the applications of the proposed method to obtain the analytical approximate solutions to wave equations with local fractional derivative operators. The paper has been organized as follows. In Section 2, we give analysis of the method used. In Section 3, we consider several illustrative examples. Finally, in Section 4, we present our conclusions.

2. Analysis of the Method

As in [20], the basic definition of reduced differential transform with local fractional operator is proposrd as follows:

Definition 2.1. If $\varphi(\eta,\kappa)$ is a local fractional analytical function in the domain of interest, then the local fractional spectrum function

$$\Phi_{\xi}(\eta) = \frac{1}{\Gamma(1+\xi\vartheta)}\left[\frac{\partial^{\xi\vartheta}\varphi(\eta,\kappa)}{\partial\kappa^{\xi\vartheta}}\right]_{\kappa=\kappa_0}, \tag{2.1}$$

is reduce differential transformed of the function $\varphi(\eta,\kappa)$ via local fractional operator, where $\xi = 0, 1, \ldots, n$ and $0 < \vartheta \leq 1$.

Definition 2.2. The inverse reduced differential transform of $\Phi_\xi(\eta)$ via local fractional operator is defined as follows:

$$\varphi(\eta,\kappa) = \sum_{\xi=0}^{\infty} \Phi_\xi(\eta)(\kappa-\kappa_0)^{\xi\vartheta}. \tag{2.2}$$

From (2.1) and (2.2) we get

$$\varphi(\eta,\kappa) = \sum_{\xi=0}^{\infty} \frac{(\kappa-\kappa_0)^{\xi\vartheta}}{\Gamma(1+\xi\vartheta)} \left[\frac{\partial^{\xi\vartheta}\varphi(\eta,\kappa)}{\partial\kappa^{\xi\vartheta}}\right]_{\kappa=\kappa_0}. \tag{2.3}$$

From (2.3), it is obvious that the local fractional reduced differential transform is derived from the local fractional Taylor theorems.

Whenever $\kappa_0 = 0$, then (2.1) and (2.2) become

$$\Phi_\xi(\eta) = \frac{1}{\Gamma(1+\xi\vartheta)} \left[\frac{\partial^{\xi\vartheta}\varphi(\eta,\kappa)}{\partial\kappa^{\xi\vartheta}}\right]_{\kappa=0}, \tag{2.4}$$

$$\varphi(\eta,\kappa) = \sum_{\xi=0}^{\infty} \Phi_\xi(\eta)\kappa^{\xi\vartheta}. \tag{2.5}$$

The following theorems that can be deduced from (2.1) and (2.2) are presented below:

Theorem 2.3. If $\varphi(\eta,\kappa) = \psi(\eta,\kappa) + \theta(\eta,\kappa)$ then

$$\Phi_\xi(\eta) = \Psi_\xi(\eta) + \Theta_\xi(\eta). \tag{2.6}$$

Theorem 2.4. If $\varphi(\eta,\kappa) = \psi(\eta,\kappa)\theta(\eta,\kappa)$ then

$$\Phi_\xi(\eta) = \sum_{l=0}^{\xi} \Psi_l(\eta)\Theta_{\xi-l}(\eta). \tag{2.7}$$

Theorem 2.5. If $\varphi(\eta,\kappa) = a\psi(\eta,\kappa)$, where a is a constant, then

$$\Phi_\xi(\eta) = a\Psi_\xi(\eta). \tag{2.8}$$

Theorem 2.6. .If $\varphi(\eta,\kappa) = \dfrac{\partial^{n\vartheta}\psi(\eta,\kappa)}{\partial\kappa^{n\vartheta}}$ then

$$\Phi_\xi(\eta) = \frac{\Gamma(1+(\xi+n)\vartheta)}{\Gamma(1+\xi\vartheta)}\Psi_{\xi+n}(\eta). \tag{2.9}$$

Theorem 2.7. If $\varphi(\eta,\kappa)=\frac{\eta^{n\vartheta}}{\Gamma(1+n\vartheta)}\frac{\kappa^{m\vartheta}}{\Gamma(1+m\vartheta)}$ then

$$\Phi_\xi(\eta)=\frac{\eta^{n\vartheta}}{\Gamma(1+n\vartheta)}\frac{\delta_\vartheta(\xi-m)}{\Gamma(1+m\vartheta)}, \tag{2.10}$$

where the local fractional Dirac-delta function is given by

$$\delta_\vartheta(\xi-m)=\begin{cases}1, & \xi=m,\\ 0, & \xi\neq m.\end{cases}$$

Theorem 2.8. .If $\varphi(\eta,\kappa)=\frac{\partial^{n\vartheta}\psi(\eta,\kappa)}{\partial\eta^{n\vartheta}}$ then

$$\Phi_\xi(\eta)=\frac{\partial^{n\vartheta}\Psi_\xi(\eta)}{\partial\eta^{n\vartheta}}. \tag{2.11}$$

For illustration of the methodology of the presented method, we write the partial differential equation within local fractional operator as:

$$\begin{aligned}&L_\vartheta\left[\varphi(\eta,\kappa)\right]+R_\vartheta\left[\varphi(\eta,\kappa)\right]+N_\vartheta\left[\varphi(\eta,\kappa)\right]=\omega(\eta,\kappa),\\&\varphi(\eta,0)=\phi(\eta).\end{aligned} \tag{2.12}$$

where $L_\vartheta=\frac{\partial^{2\vartheta}}{\partial\kappa^{2\vartheta}}$ and R_ϑ are linear local fractional operators, N_ϑ is a nonlinear local fractional operator and $\omega(\eta,\kappa)$ is an inhomogenuous term.
By taking the local fractional reduce differential transform on both sides of (2.12), we have

$$\begin{aligned}&\frac{\Gamma(1+(\xi+2)\vartheta)}{\Gamma(1+\xi\vartheta)}\Phi_{\xi+2}(\eta)=\Omega_\xi(\eta)-R_\vartheta\left[\Phi_\xi(\eta)\right]+N_\vartheta\left[\Phi_\xi(\eta)\right],\\&\Phi_0(\eta)=\phi(\eta).\end{aligned} \tag{2.13}$$

where $\Phi_\xi(\eta)$ and $\Omega_\xi(\eta)$ are reduce differential transformed with local fractional operators of the functions $\varphi(\eta,\kappa)$ and $\omega(\eta,\kappa)$ respectively.

3. Illustrative examples

In this section, to give a clear overview of the local fractional reduce differential transform method for linear and nonlinear wave equations within local fractional operator, we present the following examples.

Example 3.1. Let us consider the following linear wave equation within local fractional operator:

$$\frac{\partial^{2\vartheta}\varphi(\eta,\kappa)}{\partial\kappa^{2\vartheta}}-\frac{\partial^{2\vartheta}\varphi(\eta,\kappa)}{\partial\eta^{2\vartheta}}=0, 0<\vartheta\leqslant 1 \tag{3.1}$$

subject to the initial conditions given by

$$\varphi(\eta,0)=0,\ \frac{\partial^{\vartheta}\varphi(\eta,0)}{\partial\kappa^{\vartheta}}=E_{\vartheta}(\eta^{\vartheta}). \tag{3.2}$$

Implementing the RDTM via local fractional derivative to (3.1), we have the following relation

$$\frac{\Gamma(1+(\xi+2)\vartheta)}{\Gamma(1+\xi\vartheta)}\Phi_{\xi+2}(\eta)-\frac{\partial^{2\vartheta}\Phi_{\xi}(\eta)}{\partial\eta^{2\vartheta}}=0, \tag{3.3}$$

which reduce to the following formula

$$\Phi_{\xi+2}(\eta)=\frac{\Gamma(1+\xi\vartheta)}{\Gamma(1+(\xi+2)\vartheta)}\frac{\partial^{2\vartheta}\Phi_{\xi}(\eta)}{\partial\eta^{2\vartheta}}, \tag{3.4}$$

where

$$\Phi_0(\eta)=0,\ \Phi_1(\eta)=\frac{1}{\Gamma(1+\vartheta)}E_{\vartheta}(\eta^{\vartheta}). \tag{3.5}$$

Following (3.4) and (3.5), we obtain the following relations

$$\begin{aligned}
\Phi_2(\eta) &= \frac{1}{\Gamma(1+2\vartheta)}\frac{\partial^{2\vartheta}\Phi_0(\eta)}{\partial\eta^{2\vartheta}}=0,\\
\Phi_3(\eta) &= \frac{\Gamma(1+\vartheta)}{\Gamma(1+3\vartheta)}\frac{\partial^{2\vartheta}\Phi_1(\eta)}{\partial\eta^{2\vartheta}}\\
&= \frac{\Gamma(1+\vartheta)}{\Gamma(1+3\vartheta)}\frac{1}{\Gamma(1+\vartheta)}E_{\vartheta}(\eta^{\vartheta})\\
&= \frac{1}{\Gamma(1+3\vartheta)}E_{\vartheta}(\eta^{\vartheta}),\\
\Phi_4(\eta) &= \frac{\Gamma(1+2\vartheta)}{\Gamma(1+4\vartheta)}\frac{\partial^{2\vartheta}\Phi_2(\eta)}{\partial\eta^{2\vartheta}}=0,\\
\Phi_5(\eta) &= \frac{\Gamma(1+3\vartheta)}{\Gamma(1+5\vartheta)}\frac{\partial^{2\vartheta}\Phi_3(\eta)}{\partial\eta^{2\vartheta}}\\
&= \frac{\Gamma(1+3\vartheta)}{\Gamma(1+5\vartheta)}\frac{1}{\Gamma(1+3\vartheta)}E_{\vartheta}(\eta^{\vartheta})\\
&= \frac{1}{\Gamma(1+5\vartheta)}E_{\vartheta}(\eta^{\vartheta}),\\
&\ \ \vdots
\end{aligned}$$

Therefore, $\varphi(\eta,\kappa)$ is evaluated as follows

$$\begin{aligned}\varphi(\eta,\kappa) &= \sum_{\xi=0}^{\infty}\Phi_\xi(\eta)\kappa^{\xi\vartheta}\\ &= E_\vartheta(\eta^\vartheta)\left[\frac{\kappa^\vartheta}{\Gamma(1+\vartheta)}+\frac{\kappa^{3\vartheta}}{\Gamma(1+3\vartheta)}+\frac{\kappa^{5\vartheta}}{\Gamma(1+5\vartheta)}+\cdots\right]\\ &= E_\vartheta(\eta^\vartheta)sinh_\vartheta(\kappa^\vartheta). \end{aligned} \tag{3.6}$$

Example 3.2. The following linear wave equation within local fractional operator:

$$\frac{\partial^{2\vartheta}\varphi(\eta,\kappa)}{\partial\kappa^{2\vartheta}}-\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\frac{\partial^{2\vartheta}\varphi(\eta,\kappa)}{\partial\eta^{2\vartheta}}=0, 0<\vartheta\leqslant 1 \tag{3.7}$$

is presented and its initial valuses are defined as follows:

$$\varphi(\eta,0)=\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)},\frac{\partial^{\vartheta}\varphi(\eta,0)}{\partial\kappa^{\vartheta}}=0. \tag{3.8}$$

By utilizing the local fractional reduce differential transform on both sides of (3.7), we get

$$\frac{\Gamma(1+(\xi+2)\vartheta)}{\Gamma(1+\xi\vartheta)}\Phi_{\xi+2}(\eta)-\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\frac{\partial^{2\vartheta}\Phi_\xi(\eta)}{\partial\eta^{2\vartheta}}=0, \tag{3.9}$$

or equivalently

$$\Phi_{\xi+2}(\eta)=\frac{\Gamma(1+\xi\vartheta)}{\Gamma(1+(\xi+2)\vartheta)}\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\frac{\partial^{2\vartheta}\Phi_\xi(\eta)}{\partial\eta^{2\vartheta}}, \tag{3.10}$$

where

$$\Phi_0(\eta)=\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)},\Phi_1(\eta)=0. \tag{3.11}$$

Following (3.10) and (3.11), we obtain the following relations

$$\begin{aligned}
\Phi_2(\eta) &= \frac{1}{\Gamma(1+2\vartheta)}\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\frac{\partial^{2\vartheta}\Phi_0(\eta)}{\partial\eta^{2\vartheta}},\\
&= \frac{1}{\Gamma(1+2\vartheta)}\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)},\\
\Phi_3(\eta) &= \frac{\Gamma(1+\vartheta)}{\Gamma(1+3\vartheta)}\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\frac{\partial^{2\vartheta}\Phi_1(\eta)}{\partial\eta^{2\vartheta}}\\
&= 0,\\
\Phi_4(\eta) &= \frac{\Gamma(1+2\vartheta)}{\Gamma(1+4\vartheta)}\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\frac{\partial^{2\vartheta}\Phi_2(\eta)}{\partial\eta^{2\vartheta}},\\
&= \frac{\Gamma(1+2\vartheta)}{\Gamma(1+4\vartheta)}\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\frac{1}{\Gamma(1+2\vartheta)},\\
&= \frac{1}{\Gamma(1+4\vartheta)}\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)},\\
\Phi_5(\eta) &= \frac{\Gamma(1+3\vartheta)}{\Gamma(1+5\vartheta)}\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\frac{\partial^{2\vartheta}\Phi_3(\eta)}{\partial\eta^{2\vartheta}}\\
&= 0,\\
&\vdots
\end{aligned}$$

Therefore, $\varphi(\eta,\kappa)$ is evaluated as follows

$$\begin{aligned}
\varphi(\eta,\kappa) &= \sum_{\xi=0}^{\infty}\Phi_\xi(\eta)\kappa^{\xi\vartheta}\\
&= \frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\left[1+\frac{\kappa^{2\vartheta}}{\Gamma(1+2\vartheta)}+\frac{\kappa^{4\vartheta}}{\Gamma(1+4\vartheta)}+\frac{\kappa^{6\vartheta}}{\Gamma(1+6\vartheta)}+\cdots\right]\\
&= \frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}cosh_\vartheta(\kappa^\vartheta). \quad (3.12)
\end{aligned}$$

Example 3.3. Consider the following nonlinear wave equation within local fractional derivative operator:

(3.13)
$$\frac{\partial^{2\vartheta}\varphi(\eta,\kappa)}{\partial\kappa^{2\vartheta}}-\varphi(\eta,\kappa)\frac{\partial^{2\vartheta}\varphi(\eta,\kappa)}{\partial\eta^{2\vartheta}}+\varphi^2(\eta,\kappa)=1-\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}-\frac{\kappa^{2\vartheta}}{\Gamma(1+2\vartheta)},$$

with the initial conditions

$$\varphi(\eta,0)=\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)},\frac{\partial^{\vartheta}\varphi(\eta,0)}{\partial\kappa^{\vartheta}}=0. \tag{3.14}$$

By applying the local fractional reduce differential transform on both sides of (3.13), we have

$$\begin{aligned}&\tfrac{\Gamma(1+(\xi+2)\vartheta)}{\Gamma(1+\xi\vartheta)}\Phi_{\xi+2}(\eta)-\textstyle\sum_{l=0}^{\xi}\Phi_l(\eta)\tfrac{\partial^{2\vartheta}}{\partial\eta^{2\vartheta}}\Phi_{\xi-l}(\eta)\\ &=\delta_{\vartheta}(\xi)-\tfrac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\tfrac{\delta_{\vartheta}(\xi)}{\Gamma(1+\vartheta)}-\tfrac{\delta_{\vartheta}(\xi-2)}{\Gamma(1+2\vartheta)},\end{aligned} \tag{3.15}$$

which reduces to

$$\begin{aligned}\Phi_{\xi+2}(\eta)=&\tfrac{\Gamma(1+\xi\vartheta)}{\Gamma(1+(\xi+2)\vartheta)}\Big[\textstyle\sum_{l=0}^{\xi}\Phi_l(\eta)\tfrac{\partial^{2\vartheta}}{\partial\eta^{2\vartheta}}\Phi_{\xi-l}(\eta)+\delta_{\vartheta}(\xi)\\ &-\tfrac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\delta_{\vartheta}(\xi)-\tfrac{\delta_{\vartheta}(\xi-2)}{\Gamma(1+2\vartheta)},\end{aligned} \tag{3.16}$$

where

$$\Phi_0(\eta)=\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)},\Phi_1(\eta)=0. \tag{3.17}$$

By iterative calculations on (3.16) and (3.17) we obtain

$$\begin{aligned}
\Phi_2(\eta) &= \frac{1}{\Gamma(1+2\vartheta)}\left[\Phi_0(\eta)\frac{\partial^{2\vartheta}}{\partial\eta^{2\vartheta}}\Phi_0(\eta)+1-\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\right]\\
&= \frac{1}{\Gamma(1+2\vartheta)}\left[\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}+1-\frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}\right]\\
&= \frac{1}{\Gamma(1+2\vartheta)},\\
\Phi_3(\eta) &= \frac{\Gamma(1+\vartheta)}{\Gamma(1+3\vartheta)}\left[\Phi_0(\eta)\frac{\partial^{2\vartheta}}{\partial\eta^{2\vartheta}}\Phi_1(\eta)+\Phi_1(\eta)\frac{\partial^{2\vartheta}}{\partial\eta^{2\vartheta}}\Phi_0(\eta)\right]\\
&= 0,\\
\Phi_4(\eta) &= \frac{\Gamma(1+2\vartheta)}{\Gamma(1+4\vartheta)}\left[\Phi_0(\eta)\frac{\partial^{2\vartheta}}{\partial\eta^{2\vartheta}}\Phi_2(\eta)+\Phi_1(\eta)\frac{\partial^{2\vartheta}}{\partial\eta^{2\vartheta}}\Phi_1(\eta)+\Phi_2(\eta)\frac{\partial^{2\vartheta}}{\partial\eta^{2\vartheta}}\Phi_0(\eta)\right]\\
&= \frac{\Gamma(1+2\vartheta)}{\Gamma(1+4\vartheta)}\left[\frac{1}{\Gamma(1+2\vartheta)}-\frac{1}{\Gamma(1+2\vartheta)}\right]\\
&= 0,\\
&\vdots
\end{aligned}$$

Hence, the approximate solution $\varphi(\eta,\kappa)$ is evaluted as follows

$$\begin{aligned}
\varphi(\eta,\kappa) &= \sum_{\xi=0}^{\infty}\Phi_\xi(\eta)\kappa^{\xi\vartheta}\\
&= \frac{\eta^{2\vartheta}}{\Gamma(1+2\vartheta)}+\frac{\kappa^{2\vartheta}}{\Gamma(1+2\vartheta)}. \qquad (3.18)
\end{aligned}$$

4. Conclusions

The local fractional wave equations have been analyzed using the reduced differential transform method within local fractional differential operators. All the examples show that the local fractional reduced differential transform method is a powerful mathematical tool to solve linear and nonlinear wave equations. It is also a promising method to solve other nonlinear equations. The local fractional RDTM introduces a significant improvement in the fields over existing techniques because it takes less calculations and the number of iteration is less compared by the other methods.

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