Journal of Hyperstructures 6 *(Spec. 13th AHA)* (*2017*), *56-71.*
ISSN: 2322-1666 print/2251-8436 online

INTUITIONISTIC FUZZY 2-ABSORBING IDEALS OF COMMUTATIVE RINGS

S.YAVUZ, S.ONAR, D.SONMEZ AND B.A.ERSOY

Abstract. The aim of this paper is to give a definition of an intuitionistic fuzzy 2- absorbing ideal and an intuitionistic fuzzy weakly completely 2- absorbing ideal of commutative rings and to give their properties. Moreover, we give a diagram of transition between definitions of intuitionistic fuzzy 2- absorbing ideals of commutative rings.

Key Words: Intuitionistic Fuzzy 2-Absorbing Ideal, Intuitionistic Fuzzy Strongly 2-Absorbing Ideal, Intuitionistic Fuzzy K-2-Absorbing Ideal.

2010 Mathematics Subject Classification: Primary: 08A72; Secondary: 03E72, 13A99.

1. Introduction

Zadeh [1] defined the notion of fuzzy subset and Rosenfeld [2] studied to apply fuzzy theory on algebraic structures. After that, various researchers studied about it. Liu [3] explained the concept of fuzzy ideal of a ring.

When it comes to intuitionistic fuzzy set theory, Atanassov [4] introduced the concept of an intuitionistic fuzzy set as a generalization of Zadeh's fuzzy sets. Moreover, Hur, Kang and Song [5] presented the notion of an intuitionistic fuzzy subring. Consequently, many researchers have tried to generalize the concept of an intuitionistic fuzzy subring. Marasdeh and Salleh [6] studied the notion of intuitionistic fuzzy rings based on the concept of fuzzy space. Besides, Sharma [7] explained the translates of intuitionistic fuzzy subrings.

Received: 17 October 2017. Communicated by Ali Taghavi;

∗Address correspondence to S. YAVUZ; E-mail: ssanemy@gmail.com.

As for 2-absorbing ideals, Badawi [8] introduced the concept of 2-absorbing ideals, which is a generalization of prime ideals. Furthermore, Badawi studied [9] and [10] as well. Today, work on 2-absorbing ideal theory is developing rapidly and many other authors studied extensively on this theory.(e.g. [11], [12], [13]). Darani [14] explained and examined of L-fuzzy 2-absorbing ideals. Then, Darani and Hashempoor [15] investigated the concept of L- fuzzy 2-absorbing ideals in semiring.

The main purpose of this paper is to deal with algebraic structure of 2-absorbing ideals by applying intuitionistic fuzzy set theory. The concept of intuitionistic fuzzy 2-absorbing ideals are introduced, their characterization and algebraic properties are investigated by giving some several examples. In addition to this, intuitionistic fuzzy strongly 2-absorbing ideals, intuitionistic fuzzy weakly completely 2-absorbing ideals, intuitionistic fuzzy K-2-absorbing ideals and their properties are introduced. Moreover, image and inverse image of these ideals are studied under ring homomorphism. Finally, a table transition between definitions of intuitionistic fuzzy 2-absorbing ideals of a commutative ring are given.

2. Preliminaries

In this section, preliminary will be required to intuitionistic fuzzy 2-absorbing ideals. First of all we give basic concepts of fuzzy set and intuitionistic fuzzy set theory.

Throughout this paper R is a commutative ring with $1 \neq 0$ and $L = [0, 1]$ stands for a complete lattice.

Definition 2.1. [14] A fuzzy subset μ in a set X is a function

$$\mu : X \to [0, 1].$$

Definition 2.2. [19] The intuitionistic fuzzy sets are defined on a non-empty set X as objects having the form

$$A = \{\langle x, \mu(x), v(x)\rangle \,|x \in X\}$$

where the functions $\mu : X \to [0, 1]$ and $v : X \to [0, 1]$ denote the degrees of membership and of non- membership of each element $x \in X$ to set A, respectively, $0 \leq \mu(x) + v(x) \leq 1$ for all $x \in X$.

Definition 2.3. [19] Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point, written as $x_{(\alpha,\beta)}$ is defined to be an intuitionistic fuzzy subset of R, given by

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta), & \text{if } x = y \\ (0, 1), & \text{if } x \neq y \end{cases}$$

an intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to belong in intuitionistic fuzzy set $\langle\mu, v\rangle$ denoted by $x_{(\alpha,\beta)} \in \langle\mu, v\rangle$ if $\mu(x) \geq \alpha$ and $v(x) \leq \beta$ and we have for $x, y \in R$

i) $x_{(t,s)} + y_{(\alpha,\beta)} = (x+y)_{(t\wedge a, s\vee\beta)}$
ii) $x_{(t,s)}y_{(\alpha,\beta)} = (xy)_{(t\wedge a, s\vee\beta)}$
iii) $\left\langle x_{(t,s)}\right\rangle \left\langle y_{(\alpha,\beta)}\right\rangle = \left\langle x_{(t,s)}y_{(\alpha,\beta)}\right\rangle$.

Definition 2.4. [14] Let I be an intuitionistic fuzzy set in a set X and $t, s \in L$ such that $t + s \leq 1$. Then, the set

$$I^{(t,s)} = \{x \in X \mid \mu_I(x) \geq t \text{ and } v_I(x) \leq s\}$$

is called a (t, s) level subset of I.

Definition 2.5. [19] Let R be a ring. An intuitionistic fuzzy set

$$A = \{\langle x, \mu(x), v(x)\rangle \,|x \in R\}$$

is said to be an intuitionistic fuzzy ideal of R if $\forall x, y \in R$

i) $\mu(x-y) \geq \mu(x) \wedge \mu(y)$,
ii) $v(x-y) \leq v(x) \vee v(y)$,
iii) $\mu(xy) \geq \mu(x) \vee \mu(y)$,
iv) $v(xy) \leq v(x) \wedge v(y)$.

Moreover, we will introduce intuitionistic fuzzy prime ideal and 2-absorbing ideal. Then, we will study intuitionistic fuzzy weakly prime and completely prime ideal. Finally, we will give intuitionistic fuzzy K-prime ideal.

Definition 2.6. [14] An intuitionistic fuzzy ideal $I = \langle\mu_I, v_I\rangle$ of R is called an intuitionistic fuzzy prime ideal, if for any intuitionistic fuzzy ideals $A = \langle\mu_A, v_A\rangle$ and $B = \langle\mu_B, v_B\rangle$ of R the condition $AB \subset I$ implies that either $A \subset I$ or $B \subset I$.

Definition 2.7. [14] Let R be a commutative ring with identity. A proper ideal I of R is said to be a 2- absorbing provided that whenever $a, b, c \in R$ with $abc \in I$ then either $ab \in I$ or $ac \in I$ or $bc \in I$. Morever R is called a 2- absorbing ring if and only if its zero ideal is 2- absorbing.

3. Intuitionistic Fuzzy 2-Absorbing Ideals

In this section, we will define intuitionistic fuzzy 2- absorbing ideals of commutative ring, then we will give some properties of these ideals. Throughout the section, R is a commutative ring with identity.

Definition 3.1. An intuitionistic fuzzy ideal $I = \langle \mu_I, v_I \rangle$of R is called an intuitionistic fuzzy completely prime ideal, if for any intuitionistic fuzzy points $x_{(t,s)}$ and $y_{(\alpha,\beta)}$ such that $x_{(t,s)}y_{(\alpha,\beta)} \in I$ implies that $x_{(t,s)} \in I$ or $y_{(\alpha,\beta)} \in I$.

Definition 3.2. Let $I = \langle \mu_I, v_I \rangle$ be an intuitionistic fuzzy ideal of R. I is called an intuitionistic fuzzy 2- absorbing ideal of R if for any intuitionistic fuzzy points $x_{(a,b)}, y_{(c,d)}, z_{(e,f)}$ such that for all $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I$ implies that

$$\text{either } x_{(a,b)}y_{(c,d)} \in I \text{ or } y_{(c,d)}z_{(e,f)} \in I \text{ or } x_{(a,b)}z_{(e,f)} \in I$$

$x, y, z \in R$ and $a, b, c, d, e, f \in L$.

Theorem 3.3. *Every intuitionistic fuzzy prime ideal of R is an intuitionistic fuzzy 2- absorbing ideal of R.*

Example 3.4. Let $R = \mathbb{Z}_6$ and an intuitionistic fuzzy subset I in $\mathbb{Z}_6$ is defined as follows

$$\mu(x) = \begin{cases} 1, & x \in \{\overline{0}, \overline{3}\} \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad v(x) = \begin{cases} 0, & x \in \{\overline{0}, \overline{3}\} \\ 1, & \text{otherwise.} \end{cases}$$

Then I is an intitionistic fuzzy prime ideal and intuitionistic fuzzy 2-absorbing ideal.

The following example shows that the converse of Theorem 3.3 is not necessarily true.

Example 3.5. Let $R = \mathbb{Z}_6$ and an intuitionistic fuzzy subset I in $\mathbb{Z}_6$ is defined as follows

$$\mu(x) = \begin{cases} 1/3, & x \in \{\overline{0}, \overline{3}\} \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad v(x) = \begin{cases} 0, & x \in \{\overline{0}, \overline{3}\} \\ 1/3, & \text{otherwise} \end{cases}$$

then for fuzzy points $\overline{3}_{2/5}, \overline{5}_{1/3} \in \mathbb{Z}_6$ such that $\overline{3}_{2/5}\overline{5}_{1/3} \in \mu$ but $\overline{3}_{2/5} \notin \mu$ and $\overline{5}_{1/3} \notin \mu$. Therefore (μ, v) is not an intuitionistic fuzzy prime ideal of $\mathbb{Z}_6$. On the other hand, (μ, v) is an intuitionistic fuzzy 2- absorbing ideal of $\mathbb{Z}_6$.

Theorem 3.6. *The intersection of distinct intuitionistic fuzzy prime ideals I and J of R is an intuitionistic fuzzy 2- absorbing ideal.*

Proof. Let I and J are two distinct intuitionistic fuzzy prime ideals of R. Suppose that for any intuitionistic points $x_{(a,b),}y_{(c,d)}, z_{(e,f)}$, ($x, y, z \in R$ and $a, b, c, d, e, f \in I$), such that $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I \cap J$ but $x_{(a,b)}z_{(e,f)} \notin I \cap J$ and $x_{(a,b)}y_{(c,d)} \notin I \cap J$.

$Case1$: $x_{(a,b)}y_{(c,d)} \notin I$ and $x_{(a,b)}z_{(e,f)} \notin I$. As I is an intuitionistic fuzzy prime ideal of R, we have $z_{(e,f)} \in I$. Therefore, $x_{(a,b)}z_{(e,f)} \in I$. Hence, this is a contradiction.

$Case2$: $x_{(a,b)}y_{(c,d)} \notin I$ and $x_{(a,b)}z_{(e,f)} \notin J$. In this case from,

$$x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I \cap J.$$

We get $z_{(e,f)} \in I$ and $y_{(c,d)} \in J$. Hence, $y_{(c,d)}z_{(e,f)} \in I \cap J$.

$Case3$: $x_{(a,b)}y_{(c,d)} \notin J$ and $x_{(a,b)}z_{(e,f)} \notin I$. By a similar argument as in $Case2$ we may show that $y_{(c,d)}z_{(e,f)} \in I \cap J$.

$Case4$: $x_{(a,b)}y_{(c,d)} \notin J$ and $x_{(a,b)}z_{(e,f)} \notin J$. As similar argument as in $Case1$ leads us to a contradiction.

Therefore, $I \cap J$ is an intuitionistic fuzzy 2-absorbing ideal of R. □

Corollary 3.7. *The intersection of every pair of distinct intuitionistic fuzzy prime ideals of R is an intuitionistic fuzzy 2- absorbing ideals of R.*

Theorem 3.8. *Let $I = \langle \mu_I, v_I \rangle$ is an intuitionistic fuzzy 2- absorbing ideal of R. Then, $I^{(t,s)}$ is a 2- absorbing ideal of R for every $t, s \in L$ with $I^{(t,s)} \neq R$.*

Proof. Suppose that $a, b, c \in R$ such that $abc \in I^{(t,s)}$. Then, $\mu_I(abc) \geq t$ and $v_I(abc) \leq s$. Moreover $a_{(t,s)}b_{(t,s)}c_{(t,s)} = (abc)_{(t,s)} \in I$. Since I is an intuitionistic fuzzy 2-absorbing ideal of R,

$$a_{(t,s)}b_{(t,s)} \in I \text{ or } a_{(t,s)}c_{(t,s)} \in I \text{ or } b_{(t,s)}c_{(t,s)} \in I.$$

If $x_{(t,s)} \in I$ for some $x \in R$, then $\mu_I(x) \geq t$ and $v_I(x) \leq s$. We have $x \in \mu_t$ and $x \in v_s$ such that $x \in I^{(t,s)}$. Hence, $ab \in I^{(t,s)}$ or $ac \in I^{(t,s)}$ or $bc \in I^{(t,s)}$. Thus, $I^{(t,s)}$ is a 2-absorbing ideal. □

Corollary 3.9. *If $I = \langle \mu_I, v_I \rangle$ is an intuitionistic fuzzy 2- absorbing ideal of R, then*

$$\mu_* = \{x \in R \mid \mu(x) = \mu(0)\} \text{ and } v_* = \{x \in R \mid v(x) = v(1)\}$$

is a 2- absorbing ideal of R.

Now, we will study image and inverse image of an intuitionistic fuzzy 2-absorbing ideal with ring homomorphism.

Theorem 3.10. *Let $f : R \to S$ be a surjective ring homomorphism. If I is an intuitionistic fuzzy 2- absorbing ideal of R which is constant on* $\ker f$ *, then $f(I)$ is an intuitionistic fuzzy 2- absorbing ideal of S.*

Proof. Assume that $x_{(r,k)}y_{(s,l)}z_{(t,m)} \in f(I)$, where $x_{(r,k)}, y_{(s,l),}z_{(t,m)}$ are intuitionistic fuzzy points of S. Since f is a surjective ring homomorphism, then there exist $a, b, c \in R$ such that $f(a) = x$, $f(b) = y$, $f(c) = z$. Thus,

$$\begin{aligned} x_{(r,k)}y_{(s,l)}z_{(t,m)}(xyz) &= ((r \wedge s \wedge t), (k \vee l \vee m)) \\ &\leq (f(\mu_I)(xyz), f(v_I)(xyz)) \\ &= (f(\mu_I)(f(a)f(b)f(c)), f(v_I)(f(a)f(b)f(c)) \\ &= (f(\mu_I)(f(abc)), f(v_I)(f(abc)) \\ &= (\mu_I(abc), v_I(abc)). \end{aligned}$$

and

$$\left\{ \begin{matrix} r \wedge s \wedge t \leq \mu_I(abc) \\ a_r b_s c_t \in \mu_I \end{matrix} \right\} \text{ and } \left\{ \begin{matrix} k \vee l \vee m \leq v_I(abc) \\ a_k b_l c_m \in v_I \end{matrix} \right\}.$$

Since (μ_I, v_I) are constant on $\ker f$, then we get $a_r b_s c_t \in \mu_I$ and $a_k b_l c_m \in v_I$. Because $(\mu_I\ ,v_I)$ is an intuitionistic fuzzy 2-absorbing ideal of R,then

$$\left\{ \begin{matrix} a_r b_s \in \mu_I \\ \text{and} \\ a_k b_l \in v_I \end{matrix} \right\} \text{ or } \left\{ \begin{matrix} b_s c_t \in \mu_I \\ \text{and} \\ b_l c_m \in v_I \end{matrix} \right\} \text{ or } \left\{ \begin{matrix} a_r c_t \in \mu_I \\ \text{and} \\ a_k c_m \in v_I \end{matrix} \right\}.$$

i) If $a_r b_s \in \mu_I$ and $a_k b_l \in v_I$, then

$$r \wedge s \leq \mu_I(ab) = f(\mu_I)(f(ab)) = f(\mu_I)(f(a)f(b)) = f(\mu_I)(xy)$$

and

$$k \vee l \leq v_I(ab) = f(v_I)(f(ab)) = f(v_I)(f(a)f(b)) = f(v_I)(xy)$$

and so $x_r y_s \in f(\mu_I)$ and $x_k y_l \in f(v_I)$. Hence, $x_{(r,k)}y_{(s,l)} \in f(I)$.

ii) If $b_s c_t \in \mu_I$ and $b_l c_m \in v_I$, then proof is similarly that of (i).

iii) If $a_r c_t \in \mu_I$ and $a_k c_m \in v_I$, then proof is similarly that of (i). Hence, $f(I)$ is an intuitionistic fuzzy 2-absorbing ideal of S. □

Theorem 3.11. *Let $f : R \to S$ be a ring homomorphism. If I is an intuitionistic fuzzy 2- absorbing ideal of S , then $f^{-1}(I)$ is an intuitionistic fuzzy 2- absorbing ideal of R.*

Proof. Suppose that $x_{(a,b)}y_{(c,d)}z(e,f) \in f^{-1}(I)$ where $x_{(a,b)}, y_{(c,d)}, z(e,f)$ are intuitionistic fuzzy points of R. Then,

$$\begin{aligned} ((a \wedge c \wedge e), (b \vee d \vee f)) &\leq \left(f^{-1}(\mu_I)\right)(xyz), f^{-1}(v_I)(xyz)) \\ &= (\mu_I(f(xyz)), v_I(f(xyz))) \\ &= (\mu_I((f(x)f(y)f(z)), v_I(f(x)f(y)f(z))) \end{aligned}$$

Take $f(x) = r$, $f(y) = s$, $f(z) = t$. We have that $(a \wedge c \wedge e) \leq \mu_I(rst)$ and $(b \vee d \vee f) \leq v_I(rst)$. Besides, $r_a s_c t_e \in \mu_I$ and $r_b s_d t_f \in v_I$. Since I is an intuitionistic 2-absorbing ideal, we get

$$\begin{Bmatrix} r_a s_c \in \mu_I \\ \text{and} \\ r_b s_d \in v_I \end{Bmatrix} \text{ or } \begin{Bmatrix} r_a t_e \in \mu_I \\ \text{and} \\ r_b t_f \in v_I \end{Bmatrix} \text{ or } \begin{Bmatrix} s_c t_e \in \mu_I \\ \text{and} \\ s_d t_f \in v_I \end{Bmatrix}.$$

i) If $r_a s_c \in \mu_I$ and $r_b s_d \in v_I$, then

$$a \wedge c \leq \mu_I(rs) = \mu_I(f(x)f(y)) = \mu_I(f(xy)) = f^{-1}(\mu_I(xy))$$

and

$$b \vee d \leq v_I(rs) = v_I(f(x)f(y)) = v_I(f(xy)) = f^{-1}(v_I(xy))$$

and so $x_a y_c \in f^{-1}(\mu_I)$ and $x_b y_d \in f^{-1}(v_I)$. Hence $x_{(a,b)}y_{(c,d)} \in f^{-1}(I)$.

ii) Similarly, $x_{(a,b)}z_{(e,f)} \in f^{-1}(I)$.

iii) Similarly, $y_{(c,d)}z_{(e,f)} \in f^{-1}(I)$. Hence, $f^{-1}(I)$ is an intuitionistic fuzzy 2- absorbing ideal of R. □

Finally, we will give definition of an intuitionistic fuzzy strongly 2-aborbing ideal and two important theorem associated with this ideal.

Definition 3.12. Let I be an intuitionistic fuzzy ideal of R. I is called an intuitionistic fuzzy strongly 2- absorbing ideal of R if it is constant or

$$JKL \subseteq I \text{ implies that } JK \subseteq I \text{ or } JL \subseteq I \text{ or } KL \subseteq I$$

for any intuitionistic fuzzy ideals J, K, L of R.

Theorem 3.13. *Every intuitionistic fuzzy prime ideal of R is an intuitionistic fuzzy strongly 2- absorbing ideal of R.*

Proof. The proof is straightforward. □

Theorem 3.14. *Every intuitionistic fuzzy strongly 2- absorbing ideal is an intuitionistic fuzzy 2- absorbing ideal of R.*

Proof. Suppose that I is an intuitionistic fuzzy strongly 2-absorbing ideal of R. Assume that $x_{(a,b)}y_{(c,d)}z_{(e,f)} \in I$ for some intuitionistic fuzzy points. Then, we have

$$\left\langle x_{(a,b)}\right\rangle \left\langle y_{(c,d)}\right\rangle \left\langle z_{(e,f)}\right\rangle = \left\langle x_{(a,b)}y_{(c,d)}z_{(e,f)}\right\rangle \subseteq I.$$

Since, I is an intuitionistic fuzzy 2-absorbing ideal, we have

$$\begin{aligned}\left\langle x_{(a,b)}y_{(c,d)}\right\rangle &= \left\langle x_{(a,b)}\right\rangle \left\langle y_{(c,d)}\right\rangle \subseteq I \text{ or}\\ \left\langle x_{(a,b)}z_{(e,f)}\right\rangle &= \left\langle x_{(a,b)}\right\rangle \left\langle z_{(e,f)}\right\rangle \subseteq I \text{ or}\\ \left\langle y_{(c,d)}z_{(e,f)}\right\rangle &= \left\langle y_{(c,d)}\right\rangle \left\langle z_{(e,f)}\right\rangle \subseteq I.\end{aligned}$$

Therefore, $x_{(a,b)}y_{(c,d)} \in I$ or $y_{(c,d)}z_{(e,f)} \in I$ or $x_{(a,b)}z_{(e,f)} \in I$, that is I is an intuitionistic fuzzy 2-absorbing ideal of R. □

4. Intuitionistic Fuzzy Weakly Completely 2-Absorbing Ideals

In this section,we will define an intuitionistic fuzzy weakly completely 2- absorbing ideal and an intuitionistic fuzzy K-2- absorbing ideals of R. Then we will prove some fundamental properties between these classes of ideals.

Definition 4.1. An intuitionistic fuzzy ideal $I = \langle \mu_I, v_I\rangle$ of R is called an intuitionistic fuzzy weakly completely prime ideal, if for any $x, y \in R$ such that

$$I(xy) = \max\{I(x), I(y)\}, \text{ ie.}$$
$$\mu_I(xy) = max\{\mu_I(x), \mu_I(y)\} \text{ and } v_I(xy) = min\{v_I(x), v_I(y)\}.$$

Definition 4.2. Let $I = \langle \mu_I, v_I\rangle$ be a non-constant intuitionistic fuzzy ideal of a ring R. I is called an intuitionistic fuzzy K- prime ideal, if for any $x, y \in R$ such that

$$I(xy) = I(0) \text{ implies that } I(x) = I(0) \text{ or } I(y) = I(0).$$

Definition 4.3. Let I be an intuitionistic fuzzy ideal of R. I is called an intuitionistic fuzzy weakly completely 2- absorbing ideal of R provided that for all $a, b, c \in R$,

$$I(abc) \leq I(ab) \text{ or } I(abc) \leq I(ac) \text{ or } I(abc) \leq I(bc)$$

for all $a, b, c \in R$, ie.

$$\left(\begin{Bmatrix}\mu(abc) \leq \mu(ab)\\ \text{and}\\ v(abc) \geq v(ab)\end{Bmatrix}\right) \text{ or } \left(\begin{Bmatrix}\mu(abc) \leq \mu(ac)\\ \text{and}\\ v(abc) \geq v(ac)\end{Bmatrix}\right) \text{ or } \left(\begin{Bmatrix}\mu(abc) \leq \mu(bc)\\ \text{and}\\ v(abc) \geq v(bc)\end{Bmatrix}\right).$$

Corollary 4.4. *Let I be a non-constant intuitionistic fuzzy ideal of R. It is easy to see that I is an intuitionistic fuzzy weakly completely 2-absorbing ideal of R if and only if*

$$I(abc) = max\{I(ab), I(ac), I(bc)\}$$

for every $a, b, c \in R$, ie.

$$\mu(abc) = \max\{\mu(ab), \mu(ac), \mu(bc)\} \text{ and } v(abc) = \min\{v(ab), v(ac), v(bc)\}.$$

Definition 4.5. Let I be an intuitionistic fuzzy ideal of R. I is called an intuitionistic fuzzy K-2- absorbing ideal of R provided that for all $a, b, c \in R$,

$I(abc) = (0, 1)$ implies that $I(ab) = (0, 1)$ or $I(ac) = (0, 1)$ or $I(bc) = (0, 1)$ ie.

$$I(abc) = (0,1) = \left(\begin{Bmatrix} \mu(abc) = \mu(0) \\ \text{and} \\ v(abc) = v(1) \end{Bmatrix}\right) \text{ implies that } \left(\begin{Bmatrix} \mu(ab) = \mu(0) \\ \text{and} \\ v(ab) = v(1) \end{Bmatrix}\right) \text{ or } \left(\begin{Bmatrix} \mu(ac) = \mu(0) \\ \text{and} \\ v(ac) = v(1) \end{Bmatrix}\right) \text{ or } \left(\begin{Bmatrix} \mu(bc) = \mu(0) \\ \text{and} \\ v(bc) = v(1) \end{Bmatrix}\right).$$

Corollary 4.6. *Every intuitionistic fuzzy weakly completely 2- absorbing ideal is an intuitionistic fuzzy K-2- absorbing ideal.*

But the converse of corollary is not true.

Example 4.7. Let $R = \mathbb{Z}$. Define the intuitionistic fuzzy ideal of $\mathbb{Z}$ by

$$I(x) = \begin{Bmatrix} (1,0) & x = 0 \\ (1/3, 2/3) & x \in 8\mathbb{Z} - \{0\} \\ (1/4, 2/4) & x \in \mathbb{Z} - 8\mathbb{Z} \end{Bmatrix}$$

Then, I is an intuitionistic fuzzy K-2- absorbing ideal of $\mathbb{Z}$. But since

$$\mu(40) = 1/3 > 1/4 = \max\{\mu(20), \mu(4), \mu(20)\},$$

I is not an intuitionistic fuzzy weakly completely 2- absorbing ideal.

Now, we will present some important theorems linked with intuitionistic these two ideals.

Theorem 4.8. *Every intuitionistic fuzzy weakly completely prime ideal of R is an intuitionistic fuzzy weakly completely 2- absorbing ideal.*

Proof. Let I be an intuitionistic fuzzy weakly completely prime ideal of R. For every $x, y, z \in R$,

$$\left\{\begin{matrix} \mu_I(xyz) = \mu_I(x) \\ and \\ v_I(xyz) = v_I(x) \end{matrix}\right\} or \left\{\begin{matrix} \mu_I(xyz) = \mu_I(y) \\ and \\ v_I(xyz) = v_I(y) \end{matrix}\right\} or \left\{\begin{matrix} \mu_I(xyz) = \mu_I(z) \\ and \\ v_I(xyz) = v_I(z) \end{matrix}\right\}.$$

Assume that $\mu_I(xyz) = \mu_I(x)$ and $v_I(xyz) = v_I(x)$.Then from,

$$\begin{aligned} \mu_I(xyz) &\geq \mu_I(xy) \geq \mu_I(x) weget \mu_I(xyz) = \mu_I(xy) \text{ and} \\ v_I(xyz) &\leq v_I(xy) \geq v_I(x) weget v_I(xyz) = v(xy). \end{aligned}$$

In a similar way, we can show that

$$\begin{aligned} \mu_I(xyz) &= \mu_I(yz) and v_I(xyz) = v_I(yz) \text{ or} \\ \mu_I(xyz) &= \mu_I(xz) and v_I(xyz) = v_I(xz). \end{aligned}$$

Hence, every intuitionistic fuzzy weakly completely prime ideal of R is an intuitionistic fuzzy weakly completely 2- absorbing ideal. □

Theorem 4.9. *Every intuitionistic fuzzy K- prime ideal of R is an intuitionistic fuzzy K-2- absorbing ideal.*

Proof. The proof is clear. □

Theorem 4.10. *Let I be an intuitionistic fuzzy ideal of R. Then, following statements are equivalent:*

i) I is an intuitionistic fuzzy weakly completely 2- absorbing ideal of R.

ii) For every $s, t \in L$, the level subset $I^{(t,s)}$ of I is a 2- absorbing ideal of R.

Proof. $(i \to ii)$: Assume that I is an intuitionistic fuzzy weakly completely 2-absorbing ideal of R. For every $x, y, z \in R$ such that $xyz \in I^{(t,s)}$ for some $t, s \in L$.

$$\begin{aligned} \mu(xyz) &= \max\{\mu(xy), \mu(xz), \mu(yz)\} \geq t \text{ and} \\ v(xyz) &= \min\{v(xy), v(xz), v(yz)\} \leq s. \end{aligned}$$

Hence,

$$\left\{\begin{matrix} \mu(xy) \geq t \\ \text{and} \\ v(xy) \leq s \end{matrix}\right\} or \left\{\begin{matrix} \mu(xz) \geq t \\ \text{and} \\ v(xz) \leq s \end{matrix}\right\} or \left\{\begin{matrix} \mu(yz) \geq t \\ \text{and} \\ v(yz) \leq s \end{matrix}\right\}.$$

Therefore $xy \in I^{(t,s)}$ or $xz \in I^{(t,s)}$ or $yz \in I^{(t,s)}$. Hence, $I^{(t,s)}$ is a 2-absorbing ideal of R.

$(ii \to i)$: Assume that $I^{(t,s)}$ is an 2-absorbing ideal of R for every $t, s \in L$. For $x, y, z \in R$, set $\mu(xyz) = t$ and $v(xyz) = s$. Then, $xyz \in \mu_t$ and $xyz \in v_s$ and (μ_t, v_s) are 2- absorbing gives that $xy \in \mu_t$ and $xy \in v_s$ or $xz \in \mu_t$ and $xz \in v_s$ or $yz \in \mu_t$ and $yz \in v_s$. Thus,

$$\left\{\begin{matrix} \mu(xy) \geq t \\ \text{and} \\ v(xy) \leq s \end{matrix}\right\} or \left\{\begin{matrix} \mu(xz) \geq t \\ \text{and} \\ v(xz) \leq s \end{matrix}\right\} or \left\{\begin{matrix} \mu(yz) \geq t \\ \text{and} \\ v(yz) \leq s \end{matrix}\right\}.$$

That is,

$$\begin{aligned} \mu(xyz) &= t \leq \max\{\mu(xy), \mu(xz), \mu(yz)\} \text{ and} \\ v(xyz) &= s \geq \min\{\mu(xy), \mu(xz), \mu(yz)\}. \end{aligned}$$

Also I is an intuitionistic fuzzy ideal of R, we have

$$\begin{aligned} \mu(xyz) &\geq \max\{\mu(xy), \mu(xz), \mu(yz)\} \text{ and} \\ v(xyz) &\leq \min\{\mu(xy), \mu(xz), \mu(yz)\}. \end{aligned}$$

Hence,

$$\begin{aligned} \mu(xyz) &= \max\{\mu(xy), \mu(xz), \mu(yz)\} \text{ and} \\ v(xyz) &= \min\{\mu(xy), \mu(xz), \mu(yz)\}. \end{aligned}$$

Therefore, I is an intuitionistic fuzzy weakly completely 2-absorbing ideal of R. □

Finally, we will study image and inverse image of an intuitionistic fuzzy weakly completely 2-absorbing ideal and an intuitionistic fuzzy K-2-absorbing ideal.

Theorem 4.11. *Let $f : R \to S$ be a surjective ring homomorphism. If $I = \langle \mu, v \rangle$ is an intuitionistic fuzzy weakly completely 2- absorbing ideal of R which is constant on $\ker f$, then $f(I)$ is an intuitionistic fuzzy weakly completely 2- absorbing ideal of S.*

Proof. Suppose that $f(\mu)(xyz) \neq f(\mu)(xy)$ and $f(v)(xyz) \neq f(v)(xy)$ for any $x, y, z \in S$. Since f is a surjective ring homomorphism, then
$f(a) = x, f(b) = y, f(c) = z$ for some $a, b, c \in R$.
Hence,

$$\begin{aligned} f(\mu)(xyz) &= f(\mu)(f(a)f(b)f(c)) = f(\mu)(f(abc)) \\ &\neq f(\mu)(xy) = f(\mu)(f(a)f(b)) = f(\mu)(f(ab)) \end{aligned}$$

and

$$\begin{aligned} f(v)(xyz) &= f(v)(f(a)f(b)f(c)) = f(v)(f(abc)) \\ &\neq f(v)(xy) = f(v)(f(a)f(b)) = f(v)(f(ab)). \end{aligned}$$

Since I is constant on $\ker f$,

$$f(\mu)(f(abc)) = \mu(abc) \text{ and } f(\mu)(f(ab)) = \mu(ab).$$

Similarly,

$$f(v)(f(abc)) = v(abc) \text{ and } f(v)(f(ab)) = v(ab).$$

It means that

$$\begin{aligned} f(\mu)(f(abc)) &= \mu(abc) \neq \mu(ab) = f(\mu)(f(ab)) \text{ and} \\ f(v)(f(abc)) &= v(abc) \neq v(ab) = f(v)(f(ab)). \end{aligned}$$

Since I is an intuitionistic fuzzy weakly completely 2-absorbing ideal of R, then

$$\begin{aligned} \mu(abc) &= f(\mu)(f(a)f(b)f(c)) = f(\mu)(xyz) \\ &= \mu(abc) = f(\mu)(f(ac)) = f(\mu)(f(a)f(c)) = f(\mu)(xz). \end{aligned}$$

So, we get $f(\mu)(xyz) = f(\mu)(xz)$. And similarly,

$$\begin{aligned} v(abc) &= f(v)(f(a)f(b)f(c)) = f(v)(xyz) \\ &= v(abc) = f(v)(f(ac)) = f(v)(f(a)f(c)) = f(v)(xz). \end{aligned}$$

Then we have $f(v)(xyz) = f(v)(xz)$ or

$$\begin{aligned} \mu(abc) &= f(\mu)(f(a)f(b)f(c)) = f(\mu)(xyz) \\ &= \mu(abc) = f(\mu)(f(bc)) = f(\mu)(f(b)f(c)) = f(\mu)(yz). \end{aligned}$$

Thus $f(\mu)(xyz) = f(\mu)(yz)$. And similarly,

$$\begin{aligned} v(abc) &= f(v)(f(a)f(b)f(c)) = f(v)(xyz) \\ &= v(abc) = f(v)(f(bc)) = f(v)(f(b)f(c)) = f(v)(yz). \end{aligned}$$

We conclude that $f(v)(xyz) = f(v)(yz)$. Therefore, $f(I)$ is an intuitionistic fuzzy weakly completely 2- absorbing ideal of S. □

Theorem 4.12. *Let $f : R \to S$ be a ring homomorphism. If $I = \langle \mu, v \rangle$ is an intuitionistic fuzzy weakly completely 2- absorbing ideal of S, then $f^{-1}(I)$ is an intuitionistic fuzzy weakly completely 2- absorbing ideal of R.*

Proof. Suppose that $f^{-1}(\mu)(xyz) \neq f^{-1}(\mu)(xy)$ and $f^{-1}(v)(xyz) \neq f^{-1}(v)(xy)$ for any $x, y, z \in R$. Then,

$$\begin{aligned} f^{-1}(\mu)(xyz) &= \mu(f(xyz)) = \mu(f(x)f(y)f(z)) \\ &\neq f^{-1}(\mu)(xy) = \mu(f(xy)) = \mu(f(x)f(y)) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(v)(xyz) &= v(f(xyz)) = v(f(x)f(y)f(z)) \\ &\neq f^{-1}(v)(xy) = v(f(xy)) = v(f(x)f(y)). \end{aligned}$$

Since I is an intuitionistic fuzzy weakly completely 2-absorbing ideal of S we have that

$$\begin{aligned} \mu(f(x)f(y)f(z)) &= f^{-1}(\mu)(xyz) \\ &= \mu(f(x)f(z)) = \mu(f(xz)) \\ &= f^{-1}(\mu)(xz) \end{aligned}$$

and

$$\begin{aligned} v(f(x)f(y)f(z)) &= f^{-1}(v)(xyz) \\ &= v(f(x)f(z)) = v(f(xz)) \\ &= f^{-1}(v)(xz) \end{aligned}$$

or

$$\begin{aligned} \mu(f(x)f(y)f(z)) &= f^{-1}(\mu)(xyz) \\ &= \mu(f(y)f(z)) = \mu(f(yz)) \\ &= f^{-1}(\mu)(yz) \end{aligned}$$

and

$$\begin{aligned} v(f(x)f(y)f(z)) &= f^{-1}(v)(xyz) \\ &= v(f(y)f(z)) = v(f(yz)) \\ &= f^{-1}(v)(yz). \end{aligned}$$

Hence, $f^{-1}(I)$ is an intuitionistic fuzzy weakly completely 2- absorbing ideal of R. □

Theorem 4.13. *Let $f : R \to S$ be a surjective ring homomorphism. If $I = \langle \mu, v \rangle$ is an intuitionistic fuzzy K-2- absorbing ideal of R which is constant on* $\ker f$*, then $f(I)$ is an intuitionistic fuzzy K- 2- absorbing ideal of S.*

Proof. Proof is straightforward. □

Theorem 4.14. *Let $f : R \to S$ be a ring homomorphism. If $I = \langle \mu, v\rangle$ is an intuitionistic fuzzy K-2- absorbing ideal of S, then $f^{-1}(I)$ is an intuitionistic fuzzy K- 2- absorbing ideal of R.*

Proof. Proof is straightforward. □

Remark 4.15. The following table summarizes findings of intuitionistic fuzzy 2- absorbing ideals.

$\nearrow$		i.f.strongly 2-abs. ideal $\downarrow$
i.f. prime ideal $\downarrow$	$\longrightarrow$	i.f. 2-abs. ideal $\downarrow$
i.f.w.c. prime ideal	$\longrightarrow$	i.f.w.c. 2-abs. ideal $\downarrow$
		i.f. K-2-abs. ideal.

5. Conclusion

In this work, the theoretical point of view of intuitionistic fuzzy 2-absorbing ideals is discussed. The work also is focused on an intuitionistic fuzzy strongly 2-absorbing ideal,an intuitionistic fuzzy weakly completely prime ideal, an intuitionistic fuzzy K-2-absorbing ideal of commutative ring and characterized their algebraic properties. Furthermore, under a ring homomorphism, these ideals are investigated. In order to extend this study, one could study other algebraic structures and do some further study on properties them.

6. References

[1] L.A.Zadeh, Fuzzy sets, Inform. and Control, 8 (1965), 338-353.

[2] A.Rossenfeld, Fuzzy groups, J. Math. Anal. Appl., 35 (1971), 512-517.

[3] W.J.Liu, Operation on fuzzy ideals, Fuzzy Sets and Systems, 11 (1983), 31-41.

[4] K.Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1986), 87-96.

[5] K.Hur, H.W.Kang and H.K.Song, Intuitionistic fuzzy subgroups and subrings, Honam Math J., 25 (2003), 19-41.

[6] M.F.Marashdeh, A.R.Salleh, Intuitionistic fuzzy rings, International Journal of Algebra, 5 (2011), 37-47.

[7] P.K.Sharma, Translates of intuitionistic fuzzy subring, Internaational Review of Fuzzy Mathematics, 6 (2), (2011), 77-84.

[8] A.Badavi, On 2- absorbing ideals of commutative rings, Bull. Austral. Math. Soc., 75 (2007), 417-429.

[9] D.F.Anderson and A.Badavi, On n- absorbing ideals of commutative rings, Comm. Algebra, 39 (2011), 1646- 1672.

[10] A.Badavi, U.Tekir, E.Yetkin, On 2- absorbing primary ideals in commutative rings, Bull. Austral. Math. Soc., 51 (2014), 1163-1173.

[11] A.Badawi and A.Y.Darani, On weakly 2-absorbing ideals of commutative rings, Houston Math, 39 (2013), 441-452.

[12] A.Y.Darani and E.R.Puczylowski, On 2-absorbing commutative semigroups and their applications to rings, Semigroup Forum, 86 (2013), 83-91.

[13] P.Kumar, M.K.Dubey and P.Sarohe, Some results on 2-absorbing ideals in commutative semirings, J.Math and Application, 38 (2015), 77-84.

[14] A.Y.Darani, On L- fuzzy 2- absorbing ideals, Italian J. of Pure and Appl. Math., 36 (2016), 147-154.

[15] A.Y.Darani, A. Hashempoor, L-fuzzy 0-(1-or 2-or 3-) 2-absorbing ideals in semiring, Annals of Fuzzy Math. and Inform., 7 (2014), 303-311.

[16] D.S.Malik and J.N.Mordeson, Fuzzy Commutative Algebra, World Scientific Publishing, (1998).

[17] D.S.Malik and J.N.Mordeson, Fuzzy Prime Ideals of a ring, Fuzzy Sets and Systems, 37 (1990), 93-98.

[18] D.S.Malik and J.N.Mordeson, Fuzzy relations on rings and groups, Fuzzy Sets and Systems, 43 (1991), 117-123.

[19] I.Bakhadach, S.Melliani, M.Oukessou and L.S.Chadli, Intuitionistic fuzzy ideal and intuitionistic fuzzy prime ideal in aring, Intuitionistic Fuzzy Sets (ICIFSTA), 22 (2) (2016), 59-63.

Sanem YAVUZ
Department of Mathematics, Yıldız Technical University, Istanbul, Turkey
Email: `ssanemy@gmail.com`

Serkan ONAR
Department of Mathematics, Yıldız Technical University, Istanbul, Turkey
Email: `serkan10ar@gmail.com`

Deniz SÖNMEZ
Department of Mathematics, Yıldız Technical University, Istanbul, Turkey

Email: dnzguel@hotmail.com

Bayram Ali ERSOY
Department of Mathematics, Yıldız Technical University, Istanbul, Turkey
Email: ersoya@gmail.com