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CODING THEORY AND HYPER BCK-ALGEBRAS

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Abstract. In this paper we define the notion of a hyper BCK valued function on a set and investigate some of it's related properties as Y.B. Jun, S.Z. Song and C. Flaut have done for a BCK-algebras. We construct the codes generated by a hyper BCK valued function and provide an algorithm which allow to find a hyper BCK-algebra starting from a given binary block code. Moreover we establish the link between the hyper BCK-algebra constructed from a binary block code and hyper BCK-ideal on a hyper BCK-algebra.

Key Words: Hyper BCK-algebra, Coding theory, Block code, Hyper BCK-ideal.
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1. Introduction

The hyperstructure theory (called also multialgebra) is introduced in 1934 by F. Marty [7]. Since then a great deal of literature has been produced on the applications of the hyperstructures. Later K. Iseki [4] initiated in 1966 the study of BCK-algebras as a generalization of the concept of set-theoretic difference and propositional calculi. Y.B. Jun et al. [6] applied for the first time the hyperstructures to BCK-algebra and introduced in 2000 the notion of a *hyper BCK-algebra* with is a generalization of BCK-algebra.

Y.B. Jun and S.Z. Song, C. Flaut and T.S. Atamewoue et al. [1, 3, 5] study the connection between BCK-algebras, residuated lattices and coding theory.

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The main purpose of this paper is to study coding theory in the context of hyper BCK-algebras. This work is organized as follows: In section 2, we present some basic notions about hyper BCK-algebraic that we will use in the sequel. In section 3, we introduce the notion of hyper BCK-valued functions and investigated several of their properties. In section 4, we give the construction of the block codes by using the notion of hyper BCK-valued functions, and after haves show that in some circumstances every finite hyper BCK-algebras determines a binary block code, we end by a link between the constructed block codes and some hyper BCK-ideal.

2. Preliminaries

We will recall some known concepts related to hyper BCK-algebra which will be helpful in further section. For more about hyper BCK-algebra we refer the reader to [2, 6, 8]. Let H be a non-empty set endowed with a hyperoperation "$*$", i.e. a mapping of $H \times H$ into the family of nonempty subsets of H. For two subsets A and B of H, denote by $A*B$ the set $\bigcup_{a\in A; b\in B} a*b$. We shall use $x*y$ instead of $x*\{y\}$, $\{x\}*y$, or $\{x\}*\{y\}$.

Definition 2.1. By a hyper BCK-algebra we mean a non-empty set H endowed with a hyperoperation $*$ and a constant θ satisfying the following axioms for all $x, y, z \in H$:
(i) $(x*z)*(y*z) \ll x*y$,
(ii) $(x*y)*z = (x*z)*y$,
(ii) $x*y \ll \{x\}$,
(vi) $x \ll y$ and $y \ll x$ imply $x = y$,
Where $x \ll y$ is defined by $\theta \in x*y$ and $A \ll B$ by for all $a \in A$, there exists $b \in B$ such that $a \ll b$, for every $A, B \subseteq H$. Note that " $\ll$ " is called hyper order in H.

In any hyper BCK-algebra $(H, *, \theta)$ the following hold for all $x, y, z \in H$:
(a_1) $x*\theta = \{x\}$, $\theta*x = \{\theta\}$ and $\theta*\theta = \{\theta\}$,
(a_2) $\theta \ll x$,
(a_3) $x*\theta \ll \{y\}$ implies $x \ll y$ and $y*x \ll z*x$
(a_4) $y \ll z$ implies $x*z \ll x*y$,
(a_5) $x*y = \{\theta\}$ implies $(x*z)*(y*z) = \{\theta\}$.

Definition 2.2. Let I be a non-empty subset of a hyper BCK-algebra H. Then I is called a hyper BCK-ideal of H if the following hold:
(i) $\theta \in I$,
(ii) $x * y \ll I$ and $y \in I$ imply $x \in I$ for all $x, y \in H$.

Definition 2.3. Let I be a non-empty subset of a hyper BCK-algebra H. Then I is called a weak hyper BCK-ideal of H if the following hold:
(i) $\theta \in I$,
(ii) $(x * y) \cap I \neq \emptyset$ and $y \in I$ imply $x \in I$ for all $x, y \in H$.

Remark 2.4. Every hyper BCK-ideal of a hyper BCK-algebra H is a weak hyper BCK-ideal of H, but the converse may not be true [6].

3. Hyper BCK-valued functions

In what follows let A and H denote a nonempty set and a hyper BCK-algebra respectively, unless otherwise specified.

Definition 3.1. A mapping $\widetilde{A} : A \to H$ is called a hyper BCK-valued function (briefly, hyper BCK-function) on A.

Definition 3.2. A cut function of $\widetilde{A}$, for $q \in H$ is defined to be a mapping $\widetilde{A}_q : A \to \{0, 1\}$ such that $(\forall x \in A)$ $(\widetilde{A}_q(x) = 1 \Leftrightarrow \theta \in q * \widetilde{A}(x))$.

Obviously, $\widetilde{A}_q$ is the characteristic function of $A_q = \{x \in A | \theta \in q * \widetilde{A}(x)\}$, called a cut subset or a q-cut of $\widetilde{A}$. Note that $A_\theta = A$.

Example 3.3. Let $A = \{x, y\}$ be a set and let $H = \{\theta, a, b\}$ be a hyper BCK-algebra with the following table:

$*$	θ	a	b
θ	$\{\theta\}$	$\{\theta\}$	$\{\theta\}$
a	$\{a\}$	$\{\theta, a\}$	$\{a\}$
b	$\{b\}$	$\{b\}$	$\{\theta, b\}$

The mapping $\widetilde{A} : A \to H$ given by $\widetilde{A} = \begin{pmatrix} x & y \\ a & b \end{pmatrix}$ is a hyper BCK-function. Its cut subsets are $A_\theta = A$, $A_a = \{x\}$, $A_b = \{y\}$.

Proposition 3.4. *Every hyper BCK-function* $\widetilde{A} : A \to H$ *on* A *is represented by the supremum of the set* $\{q \in X | \widetilde{A}_q(x) = 1\}$, *that is*
$(\forall x \in A)$ $(\widetilde{A}(x) = sup\{q \in X | \theta \in q * \widetilde{A}(x)\})$.

Proof. . For any $x \in A$, let $\widetilde{A}(x) = r \in H$. Then $\theta \in r * \widetilde{A}(x)$ and so $\widetilde{A}_r(x) = 1$.

Assume that $\widetilde{A}_p(x) = 1$ for $p \in H$, then $\theta \in p * \widetilde{A}(x) = p * r$. Thus $p \ll r$.
Since $r \in \{p \in H|\widetilde{A}_p(x) = 1\}$, it follows that $\widetilde{A}(x) = r = sup\{p \in H|\widetilde{A}_p(x) = 1\}$. □

For a hyper BCK-function $\widetilde{A} : A \to H$ on A, consider the following sets:
$A_H := \{A_q|q \in H\}$; $\widetilde{A}_H := \{\widetilde{A}_q|q \in H\}$.

Proposition 3.5. *If $\widetilde{A} : A \to H$ is a hyper BCK-function on A, we can easily obtain the following results:*
i)$(\forall x \in A)$ $(\widetilde{A}(x) = sup\{q * \widetilde{A}_q(x)|q \in H\})$,
where $q * \widetilde{A}_q(x) = \begin{cases} q, & \textit{if } \widetilde{A}_q(x) = 1; \\ \theta, & \textit{otherwise.} \end{cases}$
(ii) $(\forall q, p \in H)$ $(\theta \in p * q \Leftrightarrow A_q \subseteq A_p)$,
(iii) $(\forall x, y \in A)$ $(\widetilde{A}(x) \neq \widetilde{A}(y) \Leftrightarrow A_{\widetilde{A}(x)} \neq A_{\widetilde{A}(y)})$,
(iv) $(\forall q \in H)$ $(\forall x \in A)$ $(q * \widetilde{A}(x) = \{\theta\} \Leftrightarrow A_{\widetilde{A}(x)} \subseteq A_q)$,
(v) $(\forall x, y \in A)$ $(\widetilde{A}(x) * \widetilde{A}(y) = \{\theta\} \Leftrightarrow A_{\widetilde{A}(y)} \subseteq A_{\widetilde{A}(x)})$,
(vi) $(\forall Y \subseteq H)$ $(\exists \sup Y in H \Rightarrow A_{sup\{q|q \in Y\}} = \bigcap\{A_q|q \in Y\})$. *(Note here that the* sup *is define via the hyperorder " $\ll$ "),*
(vii) *For a bounded hyper BCK-algebra H, we have*
$(\forall S \subseteq H)$ $(A_{sup\{q|q \in S\}} = \bigcap\{A_q|q \in S\})$,
$(viii)$ *If for any subset Y of H there exists a supremum of Y, then*
$(\forall p, q \in Y)$ $(A_p \bigcap A_q \in A_H)$,
(iX) $\bigcup\{A_q|q \in H\} = A$.

The following example shows that the converse of $(viii)$ may not be true in general.

Example 3.6. Let $A = \{x, y\}$ be a set and let $H = \{\theta, a, b, c\}$ be a hyper BCK-algebra with the following table:

$*$	θ	a	b	c
θ	$\{\theta\}$	$\{\theta\}$	$\{\theta\}$	$\{\theta\}$
a	$\{a\}$	$\{\theta, a\}$	$\{\theta\}$	$\{a\}$
b	$\{b\}$	$\{a\}$	$\{\theta\}$	$\{b\}$
c	$\{c\}$	$\{c\}$	$\{c\}$	$\{\theta\}$

The function $\widetilde{A} : H \to A$ given by $\widetilde{A} = \begin{pmatrix} x & y \\ a & c \end{pmatrix}$ is a hyper BCK-function on A and the cut sets of $\widetilde{A}$ are as follows: $A_\theta = A$, $A_a = \{x\}$,

$A_b = \emptyset$, $A_c = \{y\}$.
$sup\{a, c\}$ does not exist but $A_a \cap A_c \in A_H$.

4. Codes generated by hyper BCK-functions

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Let $\widetilde{A} : A \to H$ be a hyper BCK-function on A and let $\sim$ be a binary relation on H defined by $(\forall p, q \in H)$ $(p \sim q \Leftrightarrow A_p = A_q)$. Then $\sim$ is clearly an equivalence relation on H.
Let $\widetilde{A}(A) := \{q \in H | \widetilde{A}(x) = q$ for some $x \in A\}$.
Let $x/\sim = \{y \in H | x \sim y\}$, for any $x \in H$. $x/\sim$ is called equivalence class containing x. It is also easy to see that $\widetilde{A}(x) = \sup(x/\sim)$ is the greatest element of $\sim$-class to which it belongs and that every $\sim$-class contains exactly one element.

4.1. **From a hyper BCK-algebra to a block code.** Let $A = \{1, 2, ..., n\}$ and let H be a finite hyper BCK-algebra. Every hyper BCK-function $\widetilde{A} : A \to H$ on A determines a binary block code V of length n in the following way:
To every $x/\sim$, where $x \in H$, there corresponds a codeword $v_x = x_1x_2...x_n$ such that $x_i = j \Leftrightarrow \widetilde{A}_x(i) = j$ for $i \in A$ and $j \in \{0, 1\}$.
Let $v_x = x_1x_2...x_n$ and $v_y = y_1y_2...y_n$ be two codewords belonging to a binary block code V. We can define an order relation $\leq_c$ on the set codewords belonging to a binary block code V as follows:
$v_x \leq_c v_y \Leftrightarrow y_i \leq x_i$ for $i = 1, 2, ..., n$.

Example 4.1. (1) Let $H = \{0, 1, 2\}$ be a hyper BCK-algebra defined by the following table:

$*$	0	1	2
0	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0\}$	$\{0\}$
2	$\{2\}$	$\{2\}$	$\{0, 2\}$

Let $\widetilde{A} : H \to H$, $x \mapsto \begin{cases} 0, & \text{if x=0;} \\ 1, & \text{if x=1;} \\ 2, & \text{if x=2.} \end{cases}$ be a hyper BCK-function on H

Then

	0	1	2
$\widetilde{A}_0$	1	1	1
$\widetilde{A}_1$	0	1	0
$\widetilde{A}_2$	0	0	1

Thus $V = \{111, 010, 001\}$ and

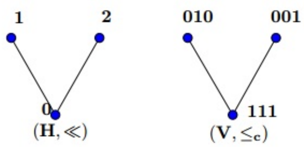

FIGURE 1.

(2) Let $H = \{\theta, a, b, c\}$ be a hyper BCK-algebra defined by the following table:

$*$	θ	a	b	c
θ	$\{\theta\}$	$\{\theta\}$	$\{\theta\}$	$\{\theta\}$
a	$\{a\}$	$\{\theta, a\}$	$\{\theta, a\}$	$\{\theta, a\}$
b	$\{b\}$	$\{b\}$	$\{\theta, a\}$	$\{\theta, a\}$
c	$\{c\}$	$\{c\}$	$\{c\}$	$\{\theta, a\}$

Let $\widetilde{A} : H \to H$ be a hyper BCK-function on H given by $\widetilde{A} = \begin{pmatrix} \theta & a & b & c \\ \theta & a & b & c \end{pmatrix}$

Then

	θ	a	b	c
$\widetilde{A}_\theta$	1	1	1	1
$\widetilde{A}_a$	0	1	1	1
$\widetilde{A}_b$	0	0	1	1
$\widetilde{A}_c$	0	0	0	1

Thus $V = \{1111, 0111, 0011, 0001\}$ and

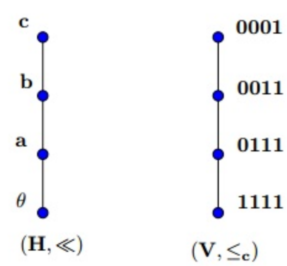

FIGURE 2.

Theorem 4.2. *Every finite hyper BCK-algebra H determines a block codes V such that $(H, \ll)$ is isomorphic to $(V, \leq_c)$.*

Proof. Let $H = \{a_1, a_2, ..., a_n\}$ be a finite hyper BCK-algebra in which $a_1 = \theta$ and let $\widetilde{A} : H \to H$ be the identity hyper BCK-function on H. The decomposition of $\widetilde{A}$ provides a family $\{\widetilde{A}_q | q \in H\}$ which is the desired code under the order $(\leq_c)$. Let $f : H \to \{\widetilde{A}_q | q \in H\}$ be a function defined by $f(q) = \widetilde{A}_q$ for all $q \in H$. Since every $\sim$-class contains exactly one element, hence f is one-to-one.
Let $x \in H$ and $p, q \in H$ be such that $p \ll q$.
If $\widetilde{A}_q(x) = 0$, then $\widetilde{A}_q(x) \leq \widetilde{A}_p(x)$.
If $\widetilde{A}_q(x) = 1$, then $\theta \in q * \widetilde{A}(x)$, i.e. $q \ll \widetilde{A}(x)$. Thus $P \ll q$ and $q \ll \widetilde{A}(x)$ by using the transitivity of the relation " $\ll$ ", we obtain $p \ll \widetilde{A}(x)$, i.e. $\theta \in p * \widetilde{A}(x)$. Therefore $\widetilde{A}_p(x) = 1$ and we conclude that $\widetilde{A}_p \leq_c \widetilde{A}_q$.
Therefore f is an isomorphism. □

4.2. From a binary block code to a hyper BCK-functions.

Example 4.3. Let $(H, \leq)$ be a finite partial ordered set with the minimum element denoted by θ. We define the following hyper operation $*$ on H:

$$\begin{cases} \theta * x = \{\theta\} \text{ and } x * x = \{\theta\}, & x \in H; \\ x * y = \{\theta\}, \text{ if } x \leq y & x, y \in H; \\ x * y = \{x\}, \text{ if } y < x & x, y \in H; \\ x * y = \{y\}, \text{ if } x, y \text{ can't be compared} & x, y \in H. \end{cases}$$

It is easy to see that $(H, *, \theta)$ is a hyper BCK-algebra.

If the above example of hyper BCK-algebra has n elements, we will denote it with C_n. Let V be a binary block code with n codewords of length n. We consider the matrix $M_V = (m_{i,j})_{i,j \in \{1,2,...,n\}} \in \mathcal{M}_n(\{0, 1\})$ with the rows consisting of the codewords of V. This matrix is called the matrix associated to the code V.

Theorem 4.4. *With the above notations, if the codeword $\underbrace{11...1}_{n-time}$ is in V and the matrix M_V is upper triangular with $m_{ii} = 1$, for all $i \in \{1, 2, ..., n\}$, there are a set A with n elements, a hyper BCK-algebra H and a hyper BCK-function $f : A \to H$ such that f determines V.*

Proof. We consider on V the lexicographic order, denoted by $\leq_{lex}$. It is clear that $(V, \leq_{lex})$ is a totally ordered set.
Let $V = \{w_1, w_2, ..., w_n\}$, with $w_1 \geq_{lex} w_2 \geq_{lex} ... \geq_{lex} w_n$. This implies that $w_1 = \underbrace{11\cdots1}_{n\text{-time}}$ and $w_n = \underbrace{00\cdots0}_{(n-1)\text{-time}}1$. On V, we define a partial order $\leq_c$ as in construction of the code by the hyper BCK-function. Now, $(V, \leq_c)$ is a partially ordered set with $w_1 \leq_c w_i \leq_c w_n$, $i \in \{2, ..., n-1\}$. We remark that w_1 correspond to θ and w_n is the maximal element in $(V, \leq_c)$.
We define on $(V, \leq_c, \theta)$ a hyper operation " $*$ " as in Example 4.3.
Then $H = (V, *, \theta)$ is a hyper BCK-algebra and V is isomorphic to H.
We consider $A = V$ and the identity map $f : A \to H$, $w \mapsto w$, as a hyper BCK-function on A. The decomposition of f provides a family $V_H = \{f_r : A \to \{0, 1\} \mid f_r(x) = 1 \Leftrightarrow \theta \in r * f(x), \forall x \in A, r \in H\}$.
This family is the binary block-code V relative to the order relation $\leq_c$.
Indeed, let $w_k \in V$, $1 < k < n$, then $w_k = \underbrace{00...0}_{(k-1)-time} x_{i_k}...x_{i_n}$; with $x_{i_k}, ..., x_{i_n} \in \{0, 1\}$.
$\forall j \in$ If $x_{i_j} = 0$, it result that $w_k \leq_c w_{i_j}$ and $\theta \in w_k * w_{i_j}$.
If $x_{i_j} = 1$, we obtain that $w_{i_j} \leq_c w_k$ or w_{i_j} and w_k can't be compared, therefore $w_k * w_{i_j} = \{w_k\}$ or $w_k * w_{i_j} = \{w_{i_j}\}$. □

The following example show that a binary block code as in Theorem 4.4 can be determined by two or more hyper BCK-algebras.

Example 4.5. Let $V = \{000010, 000110, 011101, 111111, 001011, 000001\}$ be a binary block code. Using the lexicographic order, the code V can be written $V = \{111111, 011101, 001011, 000110,$
$000010, 000001\} = \{w_1, w_2, w_3, w_4, w_5, w_6\}$. Define the partial order $\leq_c$ on V, we remark that $w_1 \leq_c w_i$ for $i \in \{2, 3, 4, 5, 6\}$; $w_2 \leq_c w_6$; $w_3 \leq_c w_5, w_6$; $w_4 \leq_c w_5$; w_2 can't be compared with w_3, w_4, w_5; w_4 cant be compared with w_3, w_6; and w_5 cant be compared with w_6. The operation " $*$ " on V is given in the following table:

$*$	w_1	w_2	w_3	w_4	w_5	w_6
w_1	$\{w_1\}$	$\{w_1\}$	$\{w_1\}$	$\{w_1\}$	$\{w_1\}$	$\{w_1\}$
w_2	$\{w_2\}$	$\{w_1\}$	$\{w_3\}$	$\{w_4\}$	$\{w_5\}$	$\{w_1\}$
w_3	$\{w_3\}$	$\{w_2\}$	$\{w_1\}$	$\{w_4\}$	$\{w_1\}$	$\{w_1\}$
w_4	$\{w_4\}$	$\{w_2\}$	$\{w_3\}$	$\{w_1\}$	$\{w_1\}$	$\{w_6\}$
w_5	$\{w_5\}$	$\{w_2\}$	$\{w_5\}$	$\{w_5\}$	$\{w_1\}$	$\{w_6\}$
w_6	$\{w_6\}$	$\{w_6\}$	$\{w_6\}$	$\{w_4\}$	$\{w_5\}$	$\{w_1\}$

Obviously, V with the operation " $*$ " is a hyper BCK-algebra. We remark that the same binary block code V can be obtained from the hyper BCK-algebra $(H, *, \theta)$

$*$	θ	a	b	c	d	e
θ	$\{\theta\}$	$\{\theta\}$	$\{\theta\}$	$\{\theta\}$	$\{\theta\}$	$\{\theta\}$
a	$\{a\}$	$\{\theta\}$	$\{b\}$	$\{c\}$	$\{d\}$	$\{\theta\}$
b	$\{b\}$	$\{a\}$	$\{\theta\}$	$\{c\}$	$\{\theta\}$	$\{\theta\}$
c	$\{c\}$	$\{a\}$	$\{b\}$	$\{\theta\}$	$\{\theta\}$	$\{e\}$
d	$\{d\}$	$\{a\}$	$\{d\}$	$\{d\}$	$\{\theta\}$	$\{e\}$
e	$\{e\}$	$\{e\}$	$\{e\}$	$\{c\}$	$\{d\}$	$\{\theta\}$

with hyper BCK-function $\widetilde{A} : V \to V$, $\widetilde{A}(x) = x$. From the associated Cayley multiplication tables, it is obvious that the hyper BCK-algebras $(H, *, \theta)$ and $(V, *, w_1)$ are not isomorphic. From here, we obtain that hyper BCK-algebra associated to a binary block-code as in Theorem 4.4 is not unique up to an isomorphism.

Proposition 4.6. *With the above notations, we consider V as a binary block code with n codewords of length m (with $n \neq m$), or a block-code with n codewords of length n such that the codeword $\underbrace{11...1}_{n-time}$ is not in V, or a block-code with n codewords of length n such that the matrix M_V is not upper triangular. There are a natural number $q = max\{m, n\}$, a set A with m elements and a hyper BCK-function $\widetilde{A} : A \to C_q$, where C_q denote the hyper BCK-algebra with q elements, such that the obtained block-code V_{C_q} contains the block-code V as a subset.*

Proof. Let C be a binary block-code, $C = \{w_1, w_2, ..., w_n\}$, with codewords of length m. We consider the codewords $w_1, w_2, ..., w_n$ lexicographic ordered, $w_1 \leq_{lex} w_2 \leq_{lex} ... \leq_{lex} w_n$. Let $M \in \mathcal{M}_{n,m}(\{0, 1\})$ be the associated matrix with the rows $w_1, w_2, ..., w_n$ in this order. Using Proposition 2.8 in [3], we can extend the matrix M to a square matrix $M' \in \mathcal{M}_p(\{0, 1\})$, $p = m + n$; such that $M' = (m'_{ij})_{i,j\in\{1,2,...,p\}}$ is an upper triangular matrix with $m'_{ii} = 1$, for all $i \in \{1, 2, ..., p\}$. If the first line of the matrix M' is not $\underbrace{11...1}_{p-time}$, then we insert the row $\underbrace{11...1}_{p+1-time}$ as a first row and the 1 $\underbrace{0...0}_{p-time}$ as a first column. Let $q = p + 1$, applying Theorem 4.4 for the matrix M', we obtain a residuated lattice $C_q = \{x_1, x_2, ..., x_q\}$, with x_1 correspond to 0 and x_q correspond to 1, and a binary block-code V_{C_q}. Assuming that the initial column of the matrix

M have in the new matrix M' positions $i_{j_1}, i_{j_2}, ..., i_{j_n} \in \{1, 2, ..., q\}$, let $A = \{x_{j_1}, x_{j_2}, ..., x_{j_n}\} \in Cq$. The hyper BCK-function $f : X \to Cq$ is such that $f(x_{j_i}) = x_{j_i}$, $i \in \{1, 2, ..., m\}$, determines the binary-block code C_q such that $C \subseteq V_{C_q}$ as restriction of the hyper BCK-function $f : C_q \to C_q$ on A such that $f(x_i) = x_i$. □

Let C be a binary block code with m codewords of length q, with the above notations, let H be the associated BCK-algebra and $W = \{\theta, w_1, ..., w_{m+q}\}$ the associated binary block code which include the code C. We consider the codewords θ, w_1,...w_{m+q} lexicographic ordered, $\theta \geq_{lex} w_1 \geq_{lex} w_2 \geq_{lex} ... \geq_{lex} w_{m+q}$. Let $M \in \mathcal{M}_{m+q+1}(\{0,1\})$ be the associated matrix with the rows θ, w_1,...,w_{m+q} in this order. We denote with L_{w_i} and C_{w_j} the lines and columns in the matrix M. The submatrix M' of the matrix M with the rows L_{w_1},...,L_{w_m} and the columns $C_{w_{m+1}}$,...,$C_{w_{m+q}}$ is the matrix associated to the code C.

Remark 4.7. 1) If there exists $x \in \{w_1, w_2, ..., w_m\}$ and $y \in \{\theta, w_{m+1}, ..., w_{m+q}\}$ such that $x \ll y$, then the set $I = \{\theta, w_{m+1}, ..., w_{m+q}\}$ can't be hyper BCK-ideal.

2) On W, due to the order $\leq_c$ given in the construction of code from a hyper BCK-function and to the hyper operation $*$ define in Example 4.3, for the product of two elements of W, we can have only two possibilities $w_i * w_j = \{\theta\}$ or $w_i * w_j = \{w_i\}$, ($w_i, w_j \in W$ and $i, j \in \{1, m+q\}$).

Example 4.8. Let $V = \{101, 110\}$ be a binary block code. Using the lexicographic order, the code V can be written $V = \{110, 101\} = \{w_1, w_2\}$. Let $M_V = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ be the associated matrix. By Proposition 2.8 in [3],

we construct the matrix $\begin{pmatrix} 1 & 1 & 1 & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{pmatrix}$.

The binary block code $W = \{w_1, w_2, w_3, w_4, w_5, w_6\}$, determines a hyper BCK-algebra $(H, *, w_1)$. Let $X = \{w_4, w_5, w_6\}$ and $\widetilde{A} : X \to W$, $w_i \mapsto w_i$, ($i \in \{4, 5, 6\}$) be a hyper BCK-function which determines the binary block code $U = \{111, 110, 101, 100, 010, 001\}$. Remark that the code V is a subset of the code U.

Since $w_2 \leq_c w_4$ and $w_3 \leq_c w_4$, then the set $I = \{w_1, w_4, w_5, w_6\}$ is not a hyper BCK-ideal.

Proposition 4.9. *Out of the above notations, if we assume that there is not $x \in \{w_1, w_2, ..., w_m\}$ such that for any $y \in I$; $x \ll y$. Then, I determines a hyper BCK-ideal in the hyper BCK-algebra H.*

Proof. since $\theta \in I$, it will be sufficient to prove the property (HI_2) for these hyper BCK-ideal.
Let $x, y \in H$ such that $x * y \ll I$ and $y \in I$.
If x, y are not compared or $x \gg y$ and if $x \notin I$, then $x * y = \{x\} \ll I$. Since $x \notin I$, then $\theta \in x * z = \{x\}$. Thus $x = \theta \in I$ contradiction. Therefore $x \in I$.
If $x \ll y$, with $y \in I$, then $x \in I$. □

Example 4.10. Let $V = \{000\} = \{w_1\}$ be a binary block code. Let $M_V = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$ be the associated matrix. By Proposition 2.8 in [3], we construct the matrix $\begin{pmatrix} 1 & 1 & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{pmatrix}$.
The binary block code $W = \{w_1, w_2, w_3, w_4, w_5\}$, determines a hyper BCK-algebra $(H, *, w_1)$. Let $X = \{w_3, w_4, w_5,\}$ and $\widetilde{A} : X \to W$, $w_i \mapsto w_i$, $(i \in \{3, 4, 5\})$ be a hyper BCK-function which determines the binary block code $U = \{111, 000, 1000, 010, 001\}$. It is clear that the code V is a subset of the code U.

Since there are not $w_2 \leq_c w_3$, w_4 and w_5, then set $I = \{w_1, w_4, w_5, w_6\}$ is a hyper BCK-ideal.

5. CONCLUSION

In this work, we have studied the connection between hyper BCK-algebras and coding theory. We have also proved that to each hyper BCK-algebras (hyper BCK-function) we can associated a binary block codes. Moreover we establish the link between the hyper BCK-algebra constructed from a binary block code and hyper BCK-ideal on a hyper BCK-algebra.

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References

[1] T.S. Atamewoue, Y.B. Jun, C. Lele, S. Ndjeya, and S.Z. Song, *Codes based on residuated lattices*, Commun. Korean Math. Soc., **31** (2016), 27–40.
[2] R.A. Borzooei and M. Bakhshi, *Some Results in Hyper BCK-algebras*, Quasigroups and Related Systems **11** (2004), 9–24.
[3] C. Flaut, *BCK-algebras Arising from Block Codes*, J. Intell. Fuzzy Syst., **28** (2015), 1829–1833.
[4] K. Iseki, S. Tanaka, *An Introduction to the Theory of BCK-algebras*, Math. Japonica, **23** (1978), 1–26.
[5] Y. B. Jun and S. Z. Song, *Codes based on BCK-algebras*, Inform. Sci., **181** (2011), 5102–5109.
[6] Y.B. Jun, M.M. Zahedi, X.L. Xin and R.A. Borzoei, *On Hyper BCK-algebras*, Italian Journal of Pure and Applied Mathematics, **8** (2000), 127–136.
[7] F. Marty, *Sur une generalization de la notion de groupe*, 8^{iem} congres Math. Scandinaves,Stockholm, (1934), 45–49.
[8] E.H. Roh, Q. Zhang and Y.B. Jun, *Some Results in Hyper BCK-algebras*, Scientiae Mathematicae Japonica, **5** (2001), 265–272.

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