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# FUZZY DOT HYPER KU-IDEALS

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Abstract. In [12,13] Mostafa et al. applied the hyper structure theory to KU-algebras and introduced the concept of hyper KU-algebras, which is a generalization of KU-algebras. In this paper, some properties of fuzzy dot (s-weak-strong) hyper KU-ideals in hyper KU-algebras are given and discussed the relations among them.

**Key Words:** KU-algebra, hyper KU-algebra, hyper KU-ideal.
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## 1. Introduction

Prabpayak and Leerawat [16,17] introduced a new algebraic structure which is called KU-algebras. They studied ideals and congruences in KU-algebras. Also, they introduced the concept of homomorphism of KU-algebra and investigated some related properties. Moreover, they derived some straightforward consequences of the relations between quotient KU-algebras and isomorphism. Mostafa et al. [11 ,18 ]introduced the notion of fuzzy KU-ideals of KU-algebras and then they investigated several basic properties which are related to fuzzy KU-ideals. Since then numerous mathematical papers [8,9,14,15] have been written investigating the fuzzy algebraic structures of ku- ideals (subalgebras) on KU-algebras.The hyper structure theory (called also multi-algebras) is introduced in 1934 by Marty [10]. Hyper structures have many applications to the pure and applied sciences. Jun and Xin [7] considered

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the fuzzification of the notion of a (weak, strong, reflexive) hyper BCK-ideal, and investigated the relations among them. Jun [4] introduced the concept of fuzzy dot ideals of BCK-algebras. In [12,13], Mostafa et al. applied the hyper structures to KU-algebras and introduced the concept of a hyper KU-algebra which is a generalization of a KU-algebra, and investigated some related properties. They also introduced the notion of a hyper KU-ideal, a weak hyper KU-ideal and gave relations between hyper KU-ideals and weak hyper KU-ideals. In this paper, some properties of fuzzy dot (s-weak-strong) hyper K U-ideals in hyper KU-algebras are given and discussed the relations among them.

## 2. Some properties of hyper KU-algebra

In this section some properties of hyper KU- algebra are discussed. Let $H$ be a nonempty set and $P^*(H) = P(H) \setminus \{\phi\}$ the family of the nonempty subsets of $H$. A multi valued operation (said also hyper operation) " $\circ$ " on $H$ is a function, which associates with every pair $(x, y) \in H \times H = H^2$ denoted by $x \circ y$. An algebraic hyper structure or simply a hyper structure is a non empty set $H$ endowed with one or more hyper operations.

**Definition 2.1.** [12, 13]. Let $H$ be a nonempty set and " $\circ$ " a hyper operation on $H$, such that $\circ : H \times H \to P^*(H)$. Then $H$ is called a hyper KU-algebra if it contains a constant "0" and satisfies the following axioms. For all $x, y, z \in H$
$(HKU_1)$ $[(y \circ z) \circ (x \circ z)] \ll x \circ y$
$HKU_2)$ $x \circ 0 = \{0\}$
$(HKU_3)$ $0 \circ x = \{x\}$
$(HKU_4)$ if $x \ll y$ is defined by $0 \in y \circ x$ and for every $A, B \subseteq H, A \ll B$ is defined by $\forall a \in A, \exists b \in B$ such that $a \ll b$. In such case, we call " $\ll$ " the hyper order in $H$.

We shall use the $x \circ y$ instead of $x \circ \{y\}, \{x\} \circ y$ or $\{x\} \circ \{y\}$.
Note that if $A, B \subseteq H$, then by $A \circ B$ we mean the subset $\bigcup_{a \in A, b \in B} a \circ b$ of $H$.

*Example* 2.2. [12, 13] (A) $H = \{0, 1, 2\}$ be a set. Define hyper operation $\circ$ on $H$ as follows:

| $\circ$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | $\{0\}$ | $\{1\}$ | $\{2\}$ |
| 1 | $\{0\}$ | $\{0,1\}$ | $\{1,2\}$ |
| 2 | $\{0\}$ | $\{0,1\}$ | $\{0,1,2\}$ |

Then $(H, \circ, 0)$ is a hyper KU-algebra.
(B) Let $(H, *, 0)$ be a KU-algebra. Define $xcircy = \{x * y\}$. for all $x, y \in H$. Now,
$(HKU_1)$ $(y \circ z) \circ (x \circ z) = \{(y \circ z) * (x \circ z)\} = \{(y \circ z) * (x * z)\} \leq \{x * y\} \leq x \circ y$
$(HKU_2)$ $x \circ 0 = \{x * 0\} = \{0\}$
$(HKU_3)$ $0 \circ x = \{0 * x\} = \{x\}$
$(HKU_4)$ if $x \ll y, y \ll x$ implies that $y \circ x = \{y * x = 0\}, x \circ y = \{x * y = 0\}$, hence $x = y$
Therefore, $(H, \circ, 0)$ is a hyper KU-algebras .

**Lemma 2.3.** [12, 13] *For all $x, y \in H$ and $A \subseteq H$*
*(i)* $A \circ (y \circ x) = y \circ (A \circ x)$
*(ii)* $(0 \circ x) \circ x = \{0\}$

**Proposition 2.4.** [12, 13] *In any hyper KU-algebra H,* $0 \circ x = \{x\} \forall x \in H$.

**Theorem 2.5.** [12, 13] *For all $x, y, z \in H$ and $A, B.C \subseteq H$*
*(i)* $x \circ y \ll z \Rightarrow z \circ y \ll x$
*(ii)* $x \circ y \ll y$
*(iii)* $x \ll 0 \circ x$
*(iv)* $A \ll B, B \ll C \Rightarrow A \ll C$
*(v)* $x \circ A \ll A$
*(vi)* $A \circ z \ll z \Leftrightarrow z \circ x \ll A$
*(vii)* $A \ll B \Rightarrow C \circ A \ll C \circ B$ *and* $B \circ C \ll A \circ C$
*(viii)* $A \ll 0 \circ A$
*(ix)* $x \in 0 \circ x$
*(x)* $x \in 0 \circ 0 \Leftrightarrow x = 0$
*(xi)* $x \circ x = \{x\} \Leftrightarrow x = 0$

**Lemma 2.6.** [12, 13] *In hyper KU-algebra $(H, \circ, 0)$, we have*

$$z \circ (y \circ x) = y \circ (z \circ x) \text{ for all } x, y, z \in H$$

**Definition 2.7.** [12, 13] Let $S$ be a non-empty subset of a hyper KU-algebra $H$. Then $S$ is said to be a hyper sub-algebra of $H$ if $S_2 : x \circ y \subseteq S, \forall x, y \in S$

**Proposition 2.8.** [12, 13] *Let $S$ be a non-empty subset of a hyper KU-algebra $(H, \circ, 0)$. If $y \circ x \subseteq S$ for all $x, yS$, then $0 \in S$.*

**Theorem 2.9.** [12, 13] *Let $S$ be a non-empty subset of a hyper KU-algebra $(H, \circ, 0)$. Then $S$ is a hyper subalgebra of $H$ if and only if $y \circ x \subseteq S$ for all $x, y \in S$.*

**Definition 2.10.** [12, 13] Let $I$ be a non-empty subset of a hyper KU-algebra $H$ and $0 \in I$. Then
(1) $I$ is said to be a weak hyper KU- ideal of $H$ if $x \circ (y \circ z) \subseteq I$ and $x \in I$ imply $y \circ z \in I$, for all $x, y, z \in H$
(2) $I$ is said to be hyper KU-ideal of $H$ if $x \circ (y \circ z) \ll I$ and $x \in I$ imply $y \circ z \in I$, for all $x, y, z \in H$
(3) $I$ is said a strong hyper KU-ideal of $H$ if $x \circ (y \circ z) \cap I \neq \Phi$ and $x \in I$ imply $y \circ z \in I$, for all $x, y, z \in H$.
(4) $I$ is said to be reflexive if $x \circ x \subseteq I$ for all $x \in H$.

**Definition 2.11.** [12, 13] Let $A$ be a non-empty subset of a hyper KU-algebra $H$. Then $A$ is said to be a hyper ideal of $H$ if
$(HI_1)$ $0 \in A$,
$(HI_2)$ $y \circ x \ll A$ and $y \in A$ imply $x \in A$ for all $x, y \in H$.

**Definition 2.12.** [12, 13] A non-empty set $A$ of a hyper KU-algebra $H$ is called a distributive hyper ideal if it satisfies $(HI_1)$ and
$(HI_3$ $(x \circ y) \circ (z \circ (z \circ x)) \ll A$ and $y \in A$ imply $x \in A$.

**Definition 2.13.** [12, 13] Let $I$ be a non-empty subset of a hyper KU-algebra $H$ and $0 \in I$. Then,
(1) $I$ is called a weak hyper ideal of $H$ if $y \circ x \subseteq I$ and $y \in I$ imply that $x \in I$, for all $x, y \in H$.
(2) $I$ is called a strong hyper ideal of $H$ if $(y \circ x) \cap I \neq \phi$ and $y \in I$ imply that $x \in I$, for all $x, y \in H$.

**Lemma 2.14.** [12, 13] *Let $A$ be a subset of a hyper KU-algebra $H$. If $I$ is a hyper ideal of $H$ such that $A \ll I$ then $A \subseteq I$.*

**Lemma 2.15.** [12, 13] *In hyper KU-algebra $(H, \circ, 0)$, we have*
*(i) Any strong hyper KU- ideal of $H$ is a hyper ideal of $H$.*
*(ii) Any weak hyper KU-ideal of $H$ is a weak ideal of $H$.*

## 3. Fuzzy dot hyper KU - subalgebras (ideals)

In what follows, $H$ denotes a hyper KU-algebra unless otherwise specified.

Now some fuzzy logic concepts are reviewed .A fuzzy set $\mu$ in a set $H$ is a function $\mu : H \to [0, 1]$. A fuzzy set $\mu$ in a set $H$ is said to satisfy the inf (resp. sup) property if for any subset $T$ of $H$ there exists $x_0 \in T$ such that $\mu(x_0) = \inf_{x\in T} \mu(x)$ (resp. $\mu(x_0) = \sup_{x\in T} \mu(x)$).
For a fuzzy set $\nu$ in $X$ and $a \in [0, 1]$ the set $\mu_t := \{x \in H, \mu(x) \geq t\}$, which is called a level set of $\mu$.

**Definition 3.1.** A fuzzy set $\mu$ in H is said to be a fuzzy dot hyper KU-subalgebra of $H$ if it satisfies the inequality:

$$\inf_{z\in y\circ x} \mu(z) \geq \min\{\mu(x), \nu(y)\} \geq \mu(x) \bullet \mu(y) \forall\, x, y \in H.$$

**Proposition 3.2.** *Let $\mu$ be a fuzzy dot hyper KU-sub-algebra of H. Then $\mu(0) \geq (\mu(x))^2$ for all $x \in H$.*

*Proof.* Using Proposition 2.5 (xi), we see that $0 \in x \circ x$ for all $x \in H$. Hence

$$\inf_{0\in x\circ x} \mu(0) \geq \min\{\mu(x), \mu(x)\} = \mu(x) \bullet \mu(x) = (\mu(x))^2 \text{ for all } x \in H.$$

□

*Example* 3.3. Let $H = \{0, a, b\}$ be a set. Define hyper operation $\circ$ on $H$ as follows:

| $\circ$ | $0$ | $a$ | $b$ |
|---|---|---|---|
| $0$ | $\{0\}$ | $\{a\}$ | $\{b\}$ |
| $a$ | $\{0\}$ | $\{0, a\}$ | $\{a, b\}$ |
| $b$ | $\{0\}$ | $\{0, a\}$ | $\{0, a, b\}$ |

Then $(H, \circ, 0)$ is a hyper KU-algebra. Define a fuzzy set $\mu : H \to [0, 1]$ by $\mu(0) = \mu(a) = \alpha_1 > \alpha_2 = \mu(b)$. Then $\mu$ is a fuzzy dot hyper sub-algebra of $H$.
A fuzzy set $\nu : H[0, 1]$ defined by $\nu(0) = 0.7, \nu(a) = 0.5$ and $\nu(b) = 0.2$ is also a fuzzy dot hyper sub-algebra of $H$.

**Theorem 3.4.** *Let $\{\mu_i \mid i \in \Lambda\}$ be a family of fuzzy dot hyper KU-sub algebras of $H$. Then $\bigcap_{i\in\Lambda} \mu_i$ is also a fuzzy dot hyper KU - sub algebras of $H$.*

*Proof.* Let $x, y \in H$, we have:

$$\inf_{z\in y\circ x}(\bigcap_{i\in\Lambda}\mu_i)(z)) = \inf_{z\in y\circ x}\{\inf_{i\in\Lambda}\{\mu_i(z)\}\} \geq \inf_{i\in\Lambda}\{\min\{\mu_i(y), \mu_i(x)\} =$$

$$\min\{\inf_{i\in\Lambda}\{\mu_i(y), \inf_{i\in\Lambda}\{\mu_i(x)\} = \min\{\bigcap_{i\in\Lambda}\mu_i(y), \bigcap_{i\in\Lambda}\mu_i(x)\} \geq \bigcap_{i\in\Lambda}\mu_i(y) \bullet \bigcap_{i\in\Lambda}\mu_i(x)$$

This proves the theorem. □

**Definition 3.5.** For a "hyper KU-algebra" $H$, a "fuzzy set" $\mu$ in H is called a fuzzy dot hyper ideal of $H$, if

$F_1 : x \ll y$ implies $\mu(x) \geq \mu(y)$

$F_2 : \mu(z) \geq \min\{\inf_{u\in((y\circ z)}\mu(u), \mu(y)\} \geq \inf_{u\in((y\circ z)}\mu(u) \bullet \mu(y)$.

$FW$ : fuzzy dot weak hyper ideal of H" if, for any $y; z \in H$

$$\mu(0) \geq \mu(z) \geq \min\{\inf_{u\in(y\circ z)}\mu(u), \mu(y)\} \geq \inf_{u\in(y\circ z)}\mu(u) \bullet \mu(y)$$

$FS$ : fuzzy dot strong hyper ideal of H" if, for any $y; z \in H$

$$\inf_{u\in(y\circ z)}\mu(u) \geq \mu(z) \geq \min\{\inf_{u\in(y\circ z)}\mu(u), \mu(y)\} \geq \inf_{u\in(y\circ z)}\mu(u) \bullet \mu(y).$$

**Definition 3.6.** For a "hyper KU-algebra" $H$, a "fuzzy set" $\mu$ in $H$ is called:

(I) fuzzy dot hyper KU-ideal of $H$, if

$$\mu(x \circ z) \geq \min\{\inf_{u\in(x\circ(y\circ z)}\mu(u), \mu(y)\} \geq \inf_{u\in(x\circ(y\circ z)}\mu(u) \bullet \mu(y)$$

(II) "fuzzy dot weak hyper KU-ideal of H" if, for any $x; y; z \in H$

$$\mu(0) \geq \mu(x \circ z) \geq \min\{\inf_{u\in(x\circ(y\circ z)}\mu(u), \mu(y)\} \geq \inf_{u\in(x\circ(y\circ z)}\mu(u) \bullet \mu(y)$$

(III) "fuzzy dot hyper KU-ideal of $H$" if, $x \ll y$ implies $\mu(x) \geq \mu(y)$ and for any $x; y; z \in H$

$$\mu(x \circ z) \geq \min\{\inf_{u\in(x\circ(y\circ z))}\mu(u), \mu(y)\} \geq \inf_{u\in(x\circ(y\circ z))}\mu(u) \bullet \mu(y)$$

(IV) "fuzzy dot strong hyper KU-ideal of $H$" if, for any $x; y; z \in H$

$$\inf_{u\in x\circ(y\circ z)}\mu(u) \geq \mu(x \circ z)\min\{\inf_{u\in x\circ(y\circ z)}\mu(u), \mu(y)\} \geq \inf_{u\in x\circ(y\circ z)}\mu(u) \bullet \mu(y)$$

**Theorem 3.7.** *Any "fuzzy dot (weak, strong) hyper KU-ideal" is a "fuzzy dot (weak, strong) hyper ideal"*

*Proof.* Straight forward. □

*Example* 3.8. Let H be the hyper KU-algebra as in Example 3.3. Define a fuzzy set in H by $\mu(0) = 0.9; \mu(a) = 0.3$ and $\mu(b) = 0.1$. It is easily verified that $\mu$ is a fuzzy dot hyper KU-ideal of $H$.

*Remark* 3.9. Every fuzzy hyper ideal is a fuzzy dot hyper ideal, but the converse is not true.

**Definition 3.10.** A fuzzy set $\mu$ in $H$ is called a fuzzy dot s-weak hyper KU-ideal of H if
(i) $\mu(0) \geq \mu(x)$, $\forall x \in H$,
(ii) for every $x, y, z \in H$ there exists $a \in x \circ (y \circ z)$ such that
$\mu(x \circ z) \geq \min\{\mu(a), \mu(y)\} \geq \mu(a) \bullet \mu(y)$.

**Proposition 3.11.** *Let $\mu$ be a fuzzy dot weak hyper KU-ideal of $H$. If $\mu$ satisfies the inf property, then $\mu$ is a fuzzy dot s-weak hyper KU -ideal of $H$.*

*Proof.* Since $\mu$ satisfies the inf property, there exists $a_0 \in x \circ (y \circ z)$, such that $\mu(a_0) = \inf\limits_{a_0 \in x\circ(y\circ z)} \mu(a_0)$. It follows that
$\mu(x \circ z) \geq \min\{\inf\limits_{a \in x\circ(y\circ z)} \mu(a), \mu(y)\} \geq \inf\limits_{a \in x\circ(y\circ z)} \mu(a) \bullet \mu(y)$. Ending the proof. Note that, in a finite hyper KU-algebra, every fuzzy dot set satisfies inf (also sup) property. Hence the concept of fuzzy dot weak hyper KU -ideals and fuzzy dot s-weak hyper KU-ideals coincide in a finite hyper KU -algebra. □

**Proposition 3.12.** *Let $\mu$ be a fuzzy dot strong hyper KU-ideal of $H$ and let $x; y; z \in H$. Then*
*(i)* $\mu(0) \geq \mu(x)$
*(ii)* $x \ll y$ *implies* $\mu(x) \geq \mu(y)$.
*(iii)* $\mu(x \circ z) \geq \min\{\mu(a), \mu(y)\} \geq \mu(a) \bullet \mu(y), \forall\, a \in x \circ (y \circ z)$

*Proof.* (i) Since $0 \in x \circ x \forall\, x \in H$, we have
$\mu(0) \geq \inf\limits_{a \in x\circ x} \mu(a) \geq \{\,\}\mu(x)$. Which proves (i).
(ii) Let $x; y \in H$ be such that $x \ll y$. Then $0 \in y \circ x \forall x, y \in H$ and so $\sup\limits_{b \in (y\circ x)} \mu(b) \geq \mu(0)$. It follows from (i) that,
$\mu(x) \geq \min\{\sup\limits_{a \in (y\circ x)} \mu(a), \mu(y)\} \geq \min\{\mu(0), \mu(y)\} = \mu(y)$.
(iii) $\mu(x \circ z) \geq \min\{\sup\limits_{a \in x\circ(y\circ z)} \mu(a), \mu(y)\} \geq \min\{\mu(a), \mu(y)\} \geq$
$\mu(a) \bullet \mu(y), \forall\, a \in x \circ (y \circ z)$
we conclude that (iii) is true. Ending the proof. □

**Corollary 3.13.** *Every fuzzy dot strong hyper KU-ideal is both a fuzzy dot s-weak hyper KU-ideal (and hence a fuzzy dot weak hyper ideal) and a fuzzy dot hyper KU-ideal.*

*Proof.* Straight forward. □

**Proposition 3.14.** *Let $\mu$ be a fuzzy dot hyper KU-ideal of $H$ and let $x, y, z \in H$. Then*
*(i)* $\mu(0) \geq \mu(x)$
*(ii) If $\mu$ satisfies the inf property, then*
$\mu(x \circ z) \geq \min\{\mu(a), \mu(y)\} \geq \mu(a) \bullet \mu(y)$, *for some* $a \in x \circ (y \circ z)$.

*Proof.* (i) Since $0 \ll x$ for each $x \in H$; we have $\mu(0) \geq \mu(x)$ by Definition 3.11(i) and hence (i) holds.
(ii) Since $\mu$ satisfies the inf property, there is $a_0 \in x \circ (y \circ z)$, such that $\mu(a_0) = \inf\limits_{a \in x \circ (y \circ z)} \mu(a)$. Hence
$\mu(x \circ z) \geq \min\{\inf\limits_{a \in x \circ (y \circ z)} \mu(a), \mu(y)\} = \min\{\mu(a_0), \mu(y)\} \geq \mu(a_0) \bullet \mu(y)$,
which implies that (ii) is true. The proof is complete. □

**Corollary 3.15.** *(i) Every fuzzy dot hyper KU-ideal of $H$ is a fuzzy dot weak hyper KU-ideal of $H$.*
*(ii) If $\mu$ is a fuzzy dot hyper KU-ideal of $H$ satisfying inf property, then $\mu$ is a fuzzy dot s-weak hyper KU-ideal of $H$.*

*Proof.* Straightforward. □

The following example shows that the converse of Corollary 3.11 and 3.13 (i). may not be true.

*Example* 3.16. (1) Consider the hyper KU -algebra H

| $\circ$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | $\{0\}$ | $\{1\}$ | $\{2\}$ |
| 1 | $\{0\}$ | $\{0,1\}$ | $\{1,2\}$ |
| 2 | $\{0\}$ | $\{0,1\}$ | $\{0,1,2\}$ |

Define a fuzzy set $\mu$ in H by :

$$\mu(x) = \begin{cases} 1 & if\ x = 0 \\ \frac{1}{2} & if\ x = 1 \\ 0 & if\ x = 2 \end{cases}$$

Then we can see that $\mu$ is a fuzzy dot hyper KU-ideal of $H$. and hence it is also a fuzzy dot weak hyper KU-ideal of $H$. But $\mu$ is not fuzzy dot strong hyper KU-ideal of $H$ since

$\min\{\sup_{a\in 0\circ(1\circ 2)} \mu(a), \mu(y)\} \geq \min\{\mu(1), \mu(1)\} \geq \frac{1}{4} \geq 0 = \mu(2)$,
$\forall\, a \in 0 \circ (1 \circ 2)$.

(2) Consider the hyper KU-algebra $H$ in Example 2.2(3). Define a fuzzy set $\mu$ in $H$ by

$$\mu(x) = \begin{cases} 1 & if\ x = 0 \\ \frac{1}{2} & if\ x = 2 \\ 0 & if\ x = 1 \end{cases}$$

Then $\mu$ is a fuzzy dot weak hyper KU-ideal of $H$ but it is not a fuzzy dot hyper KU-ideal of $H$, since $1 \ll 2$ but $\mu(1) \ngeq \mu(2)$.

Note that a fuzzy subset $\mu$ of H is a fuzzy ideal of H if and only if a non empty level subset $\mu_t = \{x \in H, \mu(x) \geq t\}$ is an ideal of $H$ for every $t \in [0,1]$. But, we know that if $\mu$ is a fuzzy dot ideal of $H$, then there exists $t \in [0,1]$ such that $\mu_t = \{x \in H, \mu(x) \geq t\}$ is not an ideal of H as seen in the following example.

*Example* 3.17. Let $X = \{0, 1, 2, \}$ be a proper KU-algebra with Cayley Table 2 as follows

| $\circ$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | $\{0\}$ | $\{1\}$ | $\{2\}$ |
| 1 | $\{0\}$ | $\{0\}$ | $\{1,2\}$ |
| 2 | $\{0\}$ | $\{0\}$ | $\{0,1,2\}$ |

Define a fuzzy set $\mu$ in $H$ by $\mu(0) = 0.6, \mu(1) = 0.4$ and $\mu(2) = 0.5$. It is easy to verify that $\mu$ is a fuzzy dot ideal of $H$. But it is not a fuzzy ideal of $H$ because $\mu(1) < \min\{\mu(1 * 2), \mu(2)\}$. Note that $\mu_{0.5} = \{0, 2\}$ is not an ideal of $H$. Since $2 \circ 1 \subseteq I, 2 \in I$, but $1 \notin I$.

**Theorem 3.18.** *If $\mu$ is fuzzy dot strong hyper KU-ideal of $H$, then the set $\mu_1 = \{x \in H, \mu(x) = t\}$ is a strong dot hyper KU-ideal of $H$, when $\mu_1 \neq \Phi$, for $t \in [0,1]$.*

*Proof.* Let $\mu$ be a hyper dot fuzzy strong hyper KU-ideal of $H$ and $\mu_t \neq \Phi$, for $t \in [0,1]$. Obviously $0 \in \mu_t$. Let $x, y, z \in H$ such that $x \circ (y \circ z) \cap \mu_t \neq \Phi$ and $y \in \mu_t$. Then there exist $a_0 \in x \circ (y \circ z) \cap \mu_t$ and hence $\mu(a_0) = t$. By definition 3.4 (iv), we have
$\mu(x \circ z) \geq \min\{\sup_{a\in x\circ(y\circ z)} \mu(a), \mu(y)\} \geq \min\{t, t\} = t \geq t^2$, and so $(x \circ z) \in \mu_t$. It follows that $\mu_t$ is a strong hyper KU-ideal of $H$. □

## Conclusion

In the present paper, we have introduced the concept of fuzzy dot (s-weak-strong) hyper KU-ideals in hyper KU-algebras and investigated some of their useful properties. In our opinion, these definitions and main results can be similarly extended to some other fuzzy algebraic systems such as hyper groups, hyper semigroups, hyper rings, hyper .
It is our hope that this work would other foundations for further study of the theory of BC K/BC I-KU-algebras. Our obtained results can be perhaps applied in engineering, soft computing or even in medical diagnosis. In our future study of fuzzy structure of KU -algebras, may be the following topics should be considered:
(1) To establish intuitionistic fuzzy dot (s-weak-strong) hyper KU-ideals in hyper KU-algebras;
(2) To consider the structure of quotient hyper KU -algebras by using these fuzzy dot hyper ideals;
(3) To get more results in fuzzy dot hyper K U -algebras and application.
(4) To consider the structure Neutrosophic dot (s-weak-strong) hyper KU-Ideals of hyper KU-Algebras.

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## Conflicts of Interest

State any potential conflicts of interest here or "The author declare no conflict of interest".

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