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Research Paper

INTUITIONISTIC FUZZY SOFT HYPERALGEBRAS

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Abstract. This study introduces the idea of intuitionistic fuzzy soft hyperalgebra. First, intuitionistic fuzzy soft hyperalgebra is defined and this definition is supported with an example. A level soft set is formed over a hyperalgebra of an intuitionistic fuzzy soft set. The relation between soft hyperalgebras and intuitionistic fuzzy soft hyperalgebras is given. Finally, intuitionistic fuzzy soft hyperalgebra homomorphism is established and demonstrated if the image and inverse image of an intuitionistic fuzzy soft hyperalgebra are both intuitionistic fuzzy soft hyperalgebras under homomorphism.

Key Words: Hyperalgebra, fuzzy soft hyperalgebra, intuitionistic fuzzy soft set.
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1. Introduction

The composition of elements under an operation indicates an element in classical algebraic structures while in algebraic hyperstructures, the composition of two elements indicates a set. As a result, hyperstructures are a generalization of classical algebraic structures. This generalization provides various models for algebraically expressing the problems. Applications in biology, conchology, chemistry, and hadronic mechanics have already been observed. The theory of hyperstructure was first introduced by Marty [10] in 1934. Utilizing the idea of hyperoperation, he defined the hypergroups and analyzed some of their properties. Many

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mathematicians have explored different hyperstructures extensively over the years, both theoretically and in terms of their applicability to various pure and applied mathematics topics. Hyperstructure theory has been covered in many books, see [4, 6, 15]. The application of hyperstructures in rough set theory, cryptography, codes, automata, probability, geometry, lattices, binary relations, graphs, and hypergraphs is highlighted in a recent book on the topic [17]. There are several types of hyperrings introduced and examined. A set with one or more hyperoperations, i.e., multi-valued operations mapping a pair of elements to a set of elements, is known as an algebraic hyperstructure (a hyperalgebra). Hyperalgebras are generalizations of classical algebras, such as hypergroup, hyperlattice, and superlattice, which are generalizations of group and lattice.

Real-world problems frequently exhibit uncertainty and imprecision, and these features can't be dealt with successfully by the mathematical techniques that are typically employed to address these issues. Fuzzy set theory, intuitionistic fuzzy set theory, and soft set theory are a few of the pioneering theories used to address these problems. Zadeh [16] proposed a fuzzy set in 1965 the notion of set μ on a non-empty set X as a function from X to the unite real interval $I = [0, 1]$ as a fuzzy set. Plenty of researchers have applied the fuzzy set structure to abstract algebra concepts. Rosenfeld [18] introduced fuzzy sets in the framework of group theory and developed the idea of a fuzzy subgroup of a group. There was no scope in the fuzzy set theory to consider the membership degree hesitancy that occurs in a variety of real-world circumstances. Atanassov [3] proposed the idea of an intuitionistic fuzzy set in 1986 to cope with these circumstances. He introduced an intuitionistic fuzzy set on a nonempty set X, which is a generalization of the fuzzy set concept established by adding the degree of non-membership to a fuzzy set. The sum of these two degrees must be less than or equal to 1, which is the only restriction on them. It can be useful in decision-making problems, sales analysis, financial services, etc. Then, Molodtsov [12] initiated the notion of soft set theory. The challenge of determining the membership function does not occur in soft set theory, making the theory applicable to a wide range of domains. After that, the ideas of fuzzy set and intuitionistic fuzzy set were combined with soft sets further to create the idea of fuzzy soft set and intuitionistic fuzzy soft set by Maji et al.[9]. Presently soft set theory is used to build algebraic structures in a variety of mathematical fields, including hyperalgebra, fuzzy algebra, and fuzzy hyperalgebra.

Under a hyperoperation, every pair of elements of H goes to a non-empty subset of H, whereas in a fuzzy hyperoperation, every pair of elements of H goes to a non-zero fuzzy set on H. This concept was introduced by Corsini and Tofan [19] and then studied by Kehagias et al. [20, 21]. Recently, Sen, Ameri, and Chowdhury introduced and analyzed fuzzy semihypergroups in [22]. The fuzzy hyperring notion is defined and studied in [24]. Yamak et al. [23] initiated the research of soft hyperstructures as an extension of the theories of soft sets and hyperstructures.

As a generalization of the notation of fuzzy hypergroup, fuzzy hyperring, and fuzzy hyperideal, Davvaz [25] developed a new type of intuitionistic fuzzy ideal, intuitionistic fuzzy hyperideal based on Atanassov's intuitionistic fuzzy sets. Fuzzy hyperalgebras were introduced rather recently. Ameri and Nozari studied the notions of fuzzy hyperalgebras and fuzzy soft hyperalgebras [2, 14].

In this paper, we define the intuitionistic fuzzy soft hyperalgebra. Therefore, we show the image and the inverse image of an intuitionistic fuzzy soft hyperalgebra are also intuitionistic fuzzy soft hyperalgebras.

2. Preliminaries

In this section, we give some basic definitions that we need to define the intuitionistic fuzzy soft hyperalgebras. H is used for a nonvoid set, $P^*(H)$ means the family of all nonvoid subsets of H and H^n is the set of n-tuples over H for a positive integer n .

Let β be an n-ary hyperoperation on H, β be a function $\beta : H^n \rightarrow P^*(H)$, for a positive integer n which is called arity of β. When $\mathbf{S}$ is a subset of H, we say that $\mathbf{S}$ is closed under β if $(x_1, ..., x_n) \in \mathbf{S}^n$ implies that $\beta(x_1, ..., x_n) \subseteq \mathbf{S}$. A hyperalgebra $\mathbf{H} = \langle H, (\beta_i)_{i \in I} \rangle$ is a set which is collection $(\beta_i)_{i \in I}$ of hyperoperations with H. $\mathbf{S}$ is called a subhyperalgebra of $\mathbf{H}$ if $\mathbf{S}$ is closed under β_i, for all $i \in I$. The type of $\mathbf{H}$ is the map from I into the set $\mathbb{N}^*$ of non-negative integers which assigns the arity of β_i to each $i \in I$. If two hyperalgebras are the same type, they are named similar hyperalgebras [2].

Suppose that $\mathbf{H} = \langle H, (\beta_i \mid i \in I) \rangle$ and $\bar{\mathbf{H}} = \langle \bar{H}, (\bar{\beta}_i \mid i \in I) \rangle$ are two similar hyperalgebras and h is a map from $\mathbf{H}$ into $\bar{\mathbf{H}}$. Then, h is called a homomorphism if $h(\beta_i((a_1, a_2, ...a_{n_i})) \subseteq \bar{\beta}_i(h(a_1), ..., h(a_{n_i}))$, for every

$i \in I$ and $(a_1, a_2, ...a_{n_i}) \in H^{n_i}$. Moreover if it is equal, then the map is called a good homomorphism [2].

Suppose that U is the initial universe set and K is a set of parameters. (F, K) is called a soft set over U if and only if $F : K \to P(U)$ is a mapping where $P(U)$ is the power set of U [13]. We can write $(F, K) = \{F(e) \mid e \in K\}$. A soft set can be used to represent a fuzzy set, as we can see. Assume that K is a set of parameters and $A \subset K$. A pair (μ, A) is known as a fuzzy soft set over U, where $\mu : A \to I^U$.

The following definitions are overtaken from [9].

Suppose that U is initial universe set, K is the set of parameters, IF^U denote the collection of all intuitionistic fuzzy subsets of U and $A \subseteq K$. Assume F is a map that $F : A \to IF^U$. Then a pair (F, A) is called an intuitionistic fuzzy soft set over U. For any $a \in A$, $F(a)$ is an intuitionistic fuzzy set of U and it also means an intuitionistic fuzzy value set of parameter a. Further, $F(a)$ can be written as $F(a) = \{(x, \mu_{F(a)}(x), \nu_{F(a)}(x)) : x \in U\}$. If $A \subseteq B$ and $F(a) \subseteq G(a)$, for all $a \in A$, then an intuitionistic fuzzy soft set (F, A) is said to be an intuitionistic fuzzy soft subset of the intuitionistic fuzzy soft set (G, B) over U.

Assume that H is a nonempty set, β_i is a fuzzy n_i-ary hyperoperation on H for every $i \in I$, $\mathbf{H} = \langle H, (\beta_i)_{i \in I} \rangle$ is a fuzzy hyperalgebra and $(n_i : i \in I)$ is the type of this fuzzy hyperalgebra [2]. Suppose that $\langle H, (\beta_i)_{i\in I} \rangle$ is a hyperalgebra and (F, A) is a soft set over H. If for all $a \in Supp(F, A)$, $F(a)$ is a subhyperalgebra, then (F, A) is a soft hyperalgebra of H. If $\langle H, (\beta_i)_{i\in I} \rangle$ is a hyperalgebra and (μ, A) is a fuzzy soft set over H and for all $a \in A$, $\forall i \in I$, $\forall x_1, ..., x_{n_i} \in H$, $\forall z \in \beta_i(x_1, ..., x_{n_i})$,

$$\bigwedge_{z \in \beta_i(x_1,...,x_{n_i})} \mu_a(z) \geq \min\{\mu_a(x_1), ..., \mu_a(x_{n_i})\}$$

then (μ, A) is a fuzzy soft hyperalgebra over H [14].

3. Intuitionistic Fuzzy Soft Hyperalgebras

Throughout this section, we shall use briefly X instead of $X = (\mu, \nu) = \{h, \mu_X(h), \nu_X(h) : h \in H\}$ for the sake of simplicity.

Let us start out by defining intuitionistic fuzzy soft hyperalgebra.

Definition 3.1. Suppose that $\langle H, (\beta_i)_{i\in I} \rangle$ is a hyperalgebra and (X, A) is an intuitionistic fuzzy soft set over H. Let

$$\bigwedge_{z \in \beta_i(x_1,...,x_{n_i})} \mu_a(z) \geq \min\{\mu_a(x_1), ..., \mu_a(x_{n_i})\}$$

and

$$\bigvee_{z\in\beta_i(x_1,...,x_{n_i})} \nu_a(z) \leq \max\{\nu_a(x_1),...,\nu_a(x_{n_i})\}$$

for every $a \in A$, $x_1,...,x_{n_i} \in H$ and $z \in \beta_i(x_1,...,x_{n_i})$ such that $i \in I$. Then (X, A) is an intuitionistic fuzzy soft hyperalgebra over H.

Moreover, if $\langle H,(\beta_i)_{i\in I}\rangle$ has null hyperoperation β, then for all $z \in \beta$, $\mu_a(z)$ and $\nu_a(z)$ are constant. Here $\mu_a(z)$ is greatest and $\nu_a(z)$ is least.

Example 3.2. Let $H = \{e, a, b\}$. Consider the tables below:

$\oplus$	e	a	b
e	e	a	b
a	a	a	H
b	b	H	b

and

$\odot$	e	a	b
e	e	e	e
a	e	a	b
b	e	a	b

It is simple to demonstrate that $\mathbf{H} = (H, \oplus, \odot)$ is a hyperalgebra. Suppose that

$$\begin{aligned}\mu_x(z) &= 0.8/e + 0.7/a + 0.3/b\\ \mu_y(z) &= 0.5/e + 0.5/a + 0.3/b\\ \nu_x(z) &= 0.1/e + 0.2/a + 0.5/b\\ \nu_y(z) &= 0.4/e + 0.5/a + 0.6/b\end{aligned}$$

where $x, y \in A$. Then (X, A) is an intuitionistic fuzzy soft hyperalgebra.

After that, we will use briefly IFSHA instead of intuitionistic fuzzy soft hyperalgebra.

Theorem 3.3. *Suppose that $\langle H,(\beta_i)_{i\in I}\rangle$ is a hyperalgebra, (X, A) and (Y, B) are IFSHAs over H where $X = (\mu, \nu)$ and $Y = (\rho, \omega)$.*
i) *$(X, A) \sqcap (Y, B)$ is an IFSHA over H.*
ii) *If $A \cap B = \emptyset$, then $(X, A) \sqcup (Y, B)$ is an IFSHA over H.*
iii) *$(X, A) \wedge (Y, B)$ is an IFSHA over H.*

Proof. **i)** For all $a \in A$ and $b \in B$, we have

$$\bigwedge_{z\in\beta_i(x_1,...,x_{n_i})} \mu_a(z) \geq \min\{\mu_a(x_1),...,\mu_a(x_{n_i})\}$$

$$\bigvee_{z\in\beta_i(x_1,...,x_{n_i})} \nu_a(z) \leq \max\{\nu_a(x_1),...,\nu_a(x_{n_i})\}$$

$$\bigwedge_{z\in\beta_i(x_1,...,x_{n_i})}\rho_b(z)\geq\min\{\rho_b(x_1),...,\rho_b(x_{n_i})\}$$

$$\bigvee_{z\in\beta_i(x_1,...,x_{n_i})}\omega_b(z)\leq\max\{\omega_b(x_1),...,\omega_b(x_{n_i})\}$$

for every $\forall x_1,...,x_{n_i}\in H$, $\forall z\in\beta_i(x_1,...,x_{n_i})$ where $i\in I$. Let $(X,A)\sqcap(Y,B)=(Z,C)$ such that $Z=(\vartheta,\varsigma)$. We know that $A\cap B=C$ and $Z=\{\langle h,\min\{\mu(h),\rho(h)\},$
$\max\{\nu(h),\omega(h)\}\rangle : h\in H\}$. Then, for all $c\in C$

$$\bigwedge_{z\in\beta_i(x_1,...,x_{n_i})}\vartheta_c(z)=\bigwedge_{z\in\beta_i(x_1,...,x_{n_i})}(\mu_a(z)\wedge\rho_b(z))$$

$$\begin{aligned}&=\left(\bigwedge_{z\in\beta_i(x_1,...,x_{n_i})}\mu_a(z)\right)\wedge\left(\bigwedge_{z\in\beta_i(x_1,...,x_{n_i})}\rho_b(z)\right)\\&\geq\min\{\mu_a(x_1),...,\mu_a(x_{n_i})\}\wedge\min\{\rho_b(x_1),...,\rho_b(x_{n_i})\}\\&=\min\{\mu_a(x_1)\wedge\rho_b(x_1),...,\mu_a(x_{n_i})\wedge\rho_b(x_{n_i})\}\end{aligned}$$

and

$$\bigvee_{z\in\beta_i(x_1,...,x_{n_i})}\varsigma_c(z)=\bigvee_{z\in\beta_i(x_1,...,x_{n_i})}(\nu_a(z)\vee\omega_b(z))$$

$$\begin{aligned}&=\left(\bigvee_{z\in\beta_i(x_1,...,x_{n_i})}\nu_a(z)\right)\vee\left(\bigvee_{z\in\beta_i(x_1,...,x_{n_i})}\omega_b(z)\right)\\&\leq\max\{\nu_a(x_1),...,\nu_a(x_{n_i})\}\vee\max\{\omega_b(x_1),...,\omega_b(x_{n_i})\}\\&=\max\{\nu_a(x_1)\vee\omega_b(x_1),...,\nu_a(x_{n_i})\vee\omega_b(x_1)\}.\end{aligned}$$

If β is null hyperoperation over H, for $z,y\in\beta$, we have

$$(\mu_a\wedge\rho_b)(z)=\mu_a(z)\wedge\rho_b(z)\geq\mu_a(y)\wedge\rho_b(y)=(\mu_a\wedge\rho_b)(y)$$

and

$$(\nu_a\vee\omega_b)(z)=\nu_a(z)\vee\omega_b(z)\leq\nu_a(y)\vee\omega_b(y)=(\nu_a\vee\omega_b)(y).$$

ii) Let $A\cap B=\emptyset$, $(X,A)\sqcup(Y,B)=(Z,C)$ where $Z=(\vartheta,\varsigma)$ and $A\cup B=C$. We know that $Z=\{\langle h,\max\{\mu(h),\rho(h)\},\min\{\nu(h),\omega(h)\}\rangle : h\in H\}$. Since $A\cap B=\emptyset$, for all $c\in C$,

Case 1: If $c\in A-B$,

$$\bigwedge_{z\in\beta_i(x_1,...,x_{n_i})}\vartheta_c(z)=\bigwedge_{z\in\beta_i(x_1,...,x_{n_i})}\mu_a(z)\text{ and }\bigvee_{z\in\beta_i(x_1,...,x_{n_i})}\varsigma_c(z)=\bigvee_{z\in\beta_i(x_1,...,x_{n_i})}\nu_a(z)$$

Case 2: If $c \in B - A$,

$$\bigwedge_{z\in\beta_i(x_1,\ldots,x_{n_i})} \vartheta_c(z) = \bigwedge_{z\in\beta_i(x_1,\ldots,x_{n_i})} \rho_b(z) \text{ and } \bigvee_{z\in\beta_i(x_1,\ldots,x_{n_i})} \varsigma_c(z) = \bigvee_{z\in\beta_i(x_1,\ldots,x_{n_i})} \omega_b(z).$$

So the proof is over since (X, A) and (Y, B) are IFSHAs.
If β is null hyperoperation over H, $z \in \beta$, we have

$$(\mu_a \vee \rho_b)(z) = \mu_a(z) \vee \rho_b(z) \geq \mu_a(y) \vee \rho_b(y) = (\mu_a \vee \rho_b)(y)$$

and

$$(\nu_a \wedge \omega_b)(z) = \nu_a(z) \wedge \omega_b(z) \leq \nu_a(y) \wedge \omega_b(y) = (\nu_a \wedge \omega_b)(y)$$

for every $y \in \beta$.

iii) Let $(X, A) \wedge (Y, B) = (Z, C)$ where $Z = (\vartheta, \varsigma)$ and $A \times B = C$. We know $Z = \{(x, y), \min(\mu(x), \rho(y)), \max(\nu(x), \omega(y)) : x, y \in H\}$ and $z = (x, y) \in A \times B$,

$$\bigwedge_{z\in\beta_i(x_1,\ldots,x_{n_i})} \vartheta(x, y) = \bigwedge_{z\in\beta_i(x_1,\ldots,x_{n_i})} \mu_a(x) \wedge \rho_b(y)$$

$$= \left(\bigwedge_{z\in\beta_i(x_1,\ldots,x_{n_i})} \mu_a(x)\right) \wedge \left(\bigwedge_{z\in\beta_i(x_1,\ldots,x_{n_i})} \rho_b(y)\right)$$
$$\geq \min\{\mu_a(x_1), \ldots, \mu_a(x_{n_i})\} \wedge \min\{\rho_b(x_1), \ldots, \rho_b(x_{n_i})\}$$
$$= \min\{\mu_a(x_1) \wedge \mu_b(x_1), \ldots, \mu_a(x_{n_i}) \wedge \mu_b(x_{n_i})\}$$

and

$$\bigvee_{z\in\beta_i(x_1,\ldots,x_{n_i})} \varsigma(x, y) = \bigvee_{z\in\beta_i(x_1,\ldots,x_{n_i})} \nu_a(x) \vee \omega_b(y)$$

$$= \left(\bigvee_{z\in\beta_i(x_1,\ldots,x_{n_i})} \nu_a(x)\right) \vee \left(\bigvee_{z\in\beta_i(x_1,\ldots,x_{n_i})} \omega_b(y)\right)$$
$$\leq \max\{\nu_a(x_1), \ldots, \nu_a(x_{n_i})\} \vee \max\{\omega_b(x_1), \ldots, \omega_b(x_{n_i})\}$$
$$= \max\{\nu_a(x_1) \vee \omega_b(x_1), \ldots, \nu_a(x_{n_i}) \vee \omega_b(x_1)\}.$$

□

Let us define the level soft set of an intuitionistic fuzzy soft set over hyperalgebra.

Definition 3.4. Let (X, A) be an IF soft set over hyperalgebra $\langle H, (\beta_i)_{i\in I}\rangle$. The soft set $(X, A)_\alpha^\gamma = ((\mu_x)_\alpha, (\nu_x)^\gamma) = \{h \in H : \mu_x(h) \geq \alpha,\ \nu_x(h) \leq \gamma,\ x \in X\}$ for all $\alpha, \gamma \in (0, 1]$, is named as (α, γ)-level soft set of the IF soft set X, where $(\mu_x)_\alpha$ is an α-level subset of the fuzzy set μ_x and $(\nu_x)^\gamma$ is a γ-level subset of the fuzzy set ν_x.

In the next theorem, we see the connection between IFSHA and soft hyperalgebra.

Theorem 3.5. *Let (X, A) be an IF soft set over hyperalgebra $\langle H, (\beta_i)_{i\in I}\rangle$. (X, A) is an IFSHA over H if and only if for arbitrary $\alpha, \gamma \in (0, 1]$ with $(\mu_x)_\alpha \neq \emptyset$ and $(\nu_x)^\gamma \neq \emptyset$, the (α, γ)-level soft set $(X, A)_\alpha^\gamma$ is a soft hyperalgebra over H.*

Proof. Suppose that $\langle H, (\beta_i)_{i\in I}\rangle$ is a hyperalgebra and (X, A) is an IFSHA over H, then μ_x and ν_x are fuzzy subhyperalgebras of H. For $\alpha, \gamma \in (0, 1]$ with $(\mu_x)_\alpha \neq \emptyset, (\nu_x)^\gamma \neq \emptyset$ and $\forall i \in I$, we take $x_1, x_2, ..., x_{n_i} \in (\mu_x)_\alpha, (\nu_x)^\gamma$, then $\mu_x(x_1) \geq \alpha, \mu_x(x_2) \geq \alpha, ..., \mu_x(x_{n_i}) \geq \alpha$ and $\nu_x(x_1) \leq \gamma, \nu_x(x_2) \leq \gamma, ..., \nu_x(x_{n_i}) \leq \gamma$. So $z \in \beta_i(x_1, ..., x_{n_i})$ we have

$$\bigwedge_{z\in\beta_i(x_1,...,x_{n_i})} \mu_x(z) \geq \min\{\mu_x(x_1), ..., \mu_x(x_{n_i})\} \geq \min\{\alpha, ..., \alpha\} = \alpha$$

and

$$\bigvee_{z\in\beta_i(x_1,...,x_{n_i})} \nu_x(z) \leq \max\{\nu_x(x_1), ..., \nu_x(x_{n_i})\} \leq \max\{\gamma, ..., \gamma\} = \gamma.$$

Hence, $\beta_i(x_1, ..., x_{n_i}) \subseteq (\mu_x)_\alpha$ and $\beta_i(x_1, ..., x_{n_i}) \supseteq (\nu_x)^\lambda$.

If β is a null hyperoperation in $\langle H, (\beta_i)_{i\in I}\rangle$ for all $z \in \beta$, since μ_x and ν_x are fuzzy subhyperalgebras over H, we have $\mu_x(z) \geq \mu_x(a)$ and $\nu_x(z) \leq \nu_x(a)$, $\forall a \in H$ and so $\mu_x(z) \geq \alpha$ and $\nu_x(z) \leq \gamma$. Thus $\beta \subseteq (\mu_x)_\alpha$, $\beta \supseteq (\nu_x)^\gamma$. We get that $(\mu_x)_\alpha$ and $(\nu_x)^\gamma$ are subhyperalgebras of H for all $a \in X$.

Conversely, assume that (X, A) is not an IFSHA over H, then μ_x and ν_x are not fuzzy soft hyperalgebras of H. Hence, there is $i \in I$, $x_1, x_2, ..., x_{n_i} \in H$ and $z \in \beta_i(x_1, ..., x_{n_i})$ such that

$$\bigwedge_{z\in\beta_i(x_1,...,x_{n_i})} \mu_x(z) < \min\{\mu_x(x_1), ..., \mu_x(x_{n_i})\}$$

and

$$\bigvee_{z\in\beta_i(x_1,...,x_{n_i})} \nu_x(z) > \max\{\nu_x(x_1), ..., \nu_x(x_{n_i})\}.$$

Let $\bigwedge_{z\in\beta_i(x_1,\ldots,x_{n_i})} \mu_x(z) = \xi$, $\bigvee_{z\in\beta_i(x_1,\ldots,x_{n_i})} \nu_x(z) = \eta$, $\mu_x(x_1)$ $= \delta_1, ..., \mu_x(x_{n_i}) = \delta_{n_i}$, $\nu_x(x_1) = \lambda_1, ..., \nu_x(x_{n_i}) = \lambda_{n_i}$.

We have $\xi < \min\{\delta_1, ..., \delta_{n_i}\}$ and $\eta > \max\{\lambda_1, ..., \lambda_{n_i}\}$. Assume

$$\alpha = \frac{\xi + \min\{\delta_1, ..., \delta_{n_i}\}}{2} \text{ and } \gamma = \frac{\eta + \max\{\lambda_1, ..., \lambda_{n_i}\}}{2},$$

then $\xi < \alpha < \min\{\delta_1, ..., \delta_{n_i}\}$ and $\eta > \gamma > \max\{\lambda_1, ..., \lambda_{n_i}\}$. Since

$$\bigwedge_{z\in\beta_i(x_1,\ldots,x_{n_i})} \mu_x(z) = \xi < \alpha \text{ and } \bigvee_{z\in\beta_i(x_1,\ldots,x_{n_i})} \nu_x(z) = \eta > \gamma,$$

we obtain $\beta_i(x_1, ..., x_{n_i}) \nsubseteq (\mu_x)_\alpha$ and $\beta_i(x_1, ..., x_{n_i}) \nsupseteq (\nu_x)^\gamma$.

On the other hand, $\min\{\delta_1, ..., \delta_{n_i}\} > \alpha$ and $\max\{\lambda_1, ..., \lambda_{n_i}\} < \gamma$ imply that $\mu_x(x_1) \geq \alpha, ..., \mu_x(x_{n_i}) \geq \alpha$ and $\nu_x(x_1) \leq \gamma, ..., \nu_x(x_{n_i}) \leq \gamma$ that is; $x_1, x_2, ..., x_{n_i} \in (\mu_x)_\alpha, (\nu_x)^\gamma$. That is a contradiction with $(X, A)_\alpha^\gamma$ being a soft hyperalgebra over H. Therefore, for all $i \in I$, $x_1, x_2, ..., x_{n_i} \in H$ and $z \in \beta_i(x_1, ..., x_{n_i})$ we obtain

$$\bigwedge_{z\in\beta_i(x_1,\ldots,x_{n_i})} \mu_x(z) \geq \min\{\mu_x(x_1), ..., \mu_x(x_{n_i})\}$$

and

$$\bigvee_{z\in\beta_i(x_1,\ldots,x_{n_i})} \nu_x(z) \leq \max\{\nu_x(x_1), ..., \nu_x(x_{n_i})\}.$$

Furthermore, assume that β is null hyperoperation in $\langle H, (\beta_i)_{i\in I}\rangle$ for all $z \in \beta$ such that $\mu_x(z) < \mu_x(a)$ and $\nu_x(z) > \nu_x(a)$, for some $a \in H$, then let $\alpha = \mu_x(a)$ and $\gamma = \nu_x(a)$. Since $(\mu_x)_\alpha$ and $(\nu_x)^\gamma$ are subhyperalgebras of H, we have $\beta \subseteq (\mu_x)_\alpha$ and $\beta \supseteq (\nu_x)^\gamma$, which is a contradiction. Hence, for all $a \in H$, $\mu_x(a) \leq \mu_x(z)$ and $\nu_x(a) \geq \nu_x(z)$. Moreover, if $z_1, z_2 \in \beta$ and $\mu_x(z_1) \neq \mu_x(z_2)$ and $\nu_x(z_1) \geq \nu_x(z_2)$, we can't conclude that for all $a \in H$, $z \in \beta$, $\mu_x(a) \leq \mu_x(z)$ and $\nu_x(a) \geq \nu_x(z)$. □

Theorem 3.6. *Assume that $\langle H, (\beta_i)_{i\in I}\rangle$ is a hyperalgebra and (X, A) is an IFSHA over H. If β is a null hyperoperation in H, the soft set*

$$(X, A)\mid_\beta = \{(\mu_x\mid_\beta, \nu_x\mid_\beta) = \{a \in H : \mu_x(a) = \mu_x(\beta), \nu_x(a) = \nu_x(\beta), x \in X\}$$

is a soft hyperalgebra over H.

Proof. Suppose that $i \in I$, $x_1, ..., x_{n_i} \in \mu_x\mid_\beta, \nu_x\mid_\beta$ and $z \in \beta_i(x_1, ..., x_{n_i})$. Then,

$$\bigwedge_{z\in\beta_i(x_1,\ldots,x_{n_i})} \mu_x(z) \geq \min\{\mu_x(x_1), ..., \mu_x(x_{n_i})\}$$

$$= \min\{\mu_x(\beta), ..., \mu_x(\beta)\} = \mu_x(\beta)$$

and

$$\bigvee_{z\in\beta_i(x_1,...,x_{n_i})} \nu_x(z) \leq \max\{\nu_x(x_1), ..., \nu_x(x_{n_i})\}$$

$$= \max\{\nu_x(\beta), ..., \nu_x(\beta)\} = \nu_x(\beta).$$

Besides that, we always have

$$\bigwedge_{z\in\beta_i(x_1,...,x_{n_i})} \mu_x(z) \leq \mu_x(\beta) \text{ and } \bigvee_{z\in\beta_i(x_1,...,x_{n_i})} \nu_x(z) \geq \nu_x(\beta).$$

So $\bigwedge_{z\in\beta_i(x_1,...,x_{n_i})} \mu_x(z) = \mu_x(\beta)$ and $\bigvee_{z\in\beta_i(x_1,...,x_{n_i})} \nu_x(z) = \nu_x(\beta)$. Thus, $\beta_i(x_1, ..., x_{n_i}) \subseteq \mu_x \mid_\beta, \nu_x \mid_\beta$. Obviously, if $z \in \mu_x \mid_\beta, \nu_x \mid_\beta$ for every $z \in \beta'$ and a null hyperoperation β then $\mu_x \mid_\beta$ and $\upsilon_x \mid_\beta$ are subhyperalgebra of H and $(X, A) \mid_\beta$ is soft hyperalgebra over H. □

Definition 3.7. Assume that $\langle H, (\beta_i)_{i\in I}\rangle$ is a hyperalgebra, (X, A) is an IFSHA over H and β is a null hyperoperation in H. For $\sigma \in (0, 1]$,

(1) If for every $x \in H$,

$$\mu_x(a) = \begin{cases} \sigma, & if\ a \in \beta \\ 0, & otherwise \end{cases} \text{ and } \nu_x(a) = \begin{cases} 1-\sigma, & if\ a \in \beta \\ 0, & otherwise \end{cases}$$

then (X, A) is called a σ_β-identity IFSHA over H.

(2) If $\mu_x(a) = \sigma$ and $\nu_x(a) = 1 - \sigma$, for all $a \in H$, then (X, A) is called a σ-absolute IFSHA over H.

Note that $f(X)$ means $f(\mu, \nu) = \{(f(h), f(\mu_X)(h), f(\nu_X)(h) : h \in H\}$.

Theorem 3.8. *Assume that $\langle H, (\beta_i)_{i\in I}\rangle$ and $\langle H', (\beta'_i)_{i\in I}\rangle$ are hyperalgebras, $f : H \to H'$ is a homomorphism and β' is a null hyperoperation in H'. Then $(\ker f)_{\beta'} = \{a \in H : f(a) \in \beta'\}$.*

i) *Let (X, A) be an IFSHA over H and for every $x \in H$,*

$$\mu_x(a) = \begin{cases} \sigma, & if\ a \in (\ker f)_{\beta'} \\ 0, & otherwise \end{cases} \text{ and } \nu_x(a) = \begin{cases} 1-\sigma, & if\ a \in (\ker f)_{\beta'} \\ 0, & otherwise \end{cases}$$

Then $(f(X), A)$ is $\sigma_{\beta'}$-identity IFSHA over H'.

ii) *Let (X, A) be a σ-absolute IFSHA over H, then $(f(X), A)$ is a σ-absolute IFSHA over H'.*

Proof. **i)** For every $z \in \beta'$,

$$f(\mu_x)(z) = \bigvee_{a \in f^{-1}(z)} \mu_x(a) = \bigvee_{a \in (\ker f)_{\beta'}} \mu_x(a) = \sigma$$

and

$$f(\nu_x)(z) = \bigwedge_{a \in f^{-1}(z)} \nu_x(a) = \bigwedge_{a \in (\ker f)_{\beta'}} \nu_x(a) = 1 - \sigma.$$

If $y \in H'$ and $y \notin \beta'$, then $f(\mu_x)(y) = 0$ and $f(\nu_X)(y) = 0$. Thus $(f(X), A)$ is $\sigma_{\beta'}$-identity IFSHA over H'.

ii) For all $h \in H'$,

$$f(\mu_x)(h) = \bigvee_{a \in f^{-1}(h)} \mu_x(a) = \sigma \text{ and } f(\nu_x)(h) = \bigwedge_{a \in f^{-1}(h)} \nu_x(a) = 1 - \sigma.$$

So, $(f(X), A)$ is a σ-absolute IFSHA over H'. □

Now, we will define IFSHA homomorphism and we will show if (X, A) is an IFSHA over H, then the image and the inverse image of (X, A) are also IFSHAs over H' under homomorphism where (Ψ, Φ) assigns H to H'.

Definition 3.9. Assume that (X, A) and (Y, B) are two IFSHAs over H and H' respectively. Let (Ψ, Φ) be an IFS function from H to H'. If Ψ is a homomorphism, then (Ψ, Φ) is called IFSHA homomorphism. Moreover, if Ψ is an isomorphism and Φ is a one-to-one mapping from (X, A) onto (Y, B), then (Ψ, Φ) is called IFSHA homomorphism.

Theorem 3.10. *Assume that $\langle H, (\beta_i)_{i \in I} \rangle$ and $\langle H', (\beta'_i)_{i \in I} \rangle$ are hyperalgebras, (X, A) is an IFSHA over H and (Ψ, Φ) is an IFS homomorphism from H to H'. Then $(\Psi, \Phi)(X, A)$ is an IFSHA over H'.*

Proof. Let $k \in \Phi(X, A)$, $i \in I$ and $y_1, y_2, ..., y_{n_i} \in H'$. If $\Psi^{-1}(y_1) = \emptyset$ or $\Psi^{-1}(y_2) = \emptyset$ or ... or $\Psi^{-1}(y_{n_i}) = \emptyset$, the proof is obvious. Suppose that $x_1, x_2, ..., x_{n_i} \in H$ such that $\Psi(x_1) = y_1, \Psi(x_2) = y_2$,...,$\Psi(x_{n_i}) = y_{n_i}$. For all $z \in \beta_i(x_1, ..., x_{n_i})$ and $z' \in \beta'_i(y_1, y_2, ..., y_{n_i})$ such that $\Psi(z) = z'$, we have

$$\bigwedge_{z' \in \beta'_i(y_1, y_2, ..., y_{n_i})} \Psi(\mu)_k(z') = \bigwedge_{z' \in \beta'_i(y_1, y_2, ..., y_{n_i})} \left(\bigvee_{\Psi(z) = z'} \bigvee_{\Phi(x) = k} \mu_x(z) \right)$$

$$\geq \bigvee_{\Psi(z)=z'} \bigvee_{\Phi(x)=k} \left(\bigwedge_{z\in\beta_i(x_1,x_2,...,x_{n_i})} \mu_x(z) \right) \geq \bigvee_{\Phi(x)=k} \left(\bigwedge_{z\in\beta_i(x_1,x_2,...,x_{n_i})} \mu_x(z) \right)$$

$$\geq \bigvee_{\Phi(x)=k} \min\{\mu_x(x_1),...,\mu_x(x_{n_i})\} = \min\left\{ \bigvee_{\Phi(x)=k} \mu_x(x_1),..., \bigvee_{\Phi(x)=k} \mu_x(x_{n_i}) \right\}$$

and

$$\bigvee_{z'\in\beta_i'(y_1,y_2,...,y_{n_i})} \Psi(\nu)_k(z') = \bigvee_{z'\in\beta_i'(y_1,y_2,...,y_{n_i})} \left(\bigwedge_{\Psi(z)=z'} \bigwedge_{\Phi(x)=k} \nu_x(z) \right)$$

$$\leq \bigwedge_{\Psi(z)=z'} \bigwedge_{\Phi(x)=k} \left(\bigvee_{z\in\beta_i(x_1,x_2,...,x_{n_i})} \nu_x(z) \right) \leq \bigwedge_{\Phi(x)=k} \left(\bigvee_{z\in\beta_i(x_1,x_2,...,x_{n_i})} \nu_x(z) \right)$$

$$\leq \bigwedge_{\Phi(x)=k} \max\{\nu_x(x_1),...,\nu_x(x_{n_i})\} = \max\left\{ \bigwedge_{\Phi(x)=k} \nu_x(x_1),..., \bigwedge_{\Phi(x)=k} \nu_x(x_{n_i}) \right\}.$$

Since this inequalities are satisfied for each $x_1,...,x_{n_i} \in H$, such that $\Psi(x_1) = y_1,...,\Psi(x_{n_i}) = y_{n_i}$, we set $\delta_1 = \bigvee_{\Phi(x)=k} \mu_x(x_1),...,\delta_{n_i} = \bigvee_{\Phi(x)=k} \mu_x(x_{n_i})$ and $\lambda_1 = \bigwedge_{\Phi(x)=k} \nu_x(x_1),...,\lambda_{n_i} = \bigwedge_{\Phi(x)=k} \nu_x(x_{n_i})$ since min and max are continues t-norms we have

$$\bigwedge_{z'\in\beta_i'(y_1,y_2,...,y_{n_i})} \Psi(\mu)_k(z') \geq \bigvee_{\Psi(x_{n_i})=y_{n_i}} \left(... \bigvee_{\Psi(x_1)=y_1} \min\{\delta_1,...,\delta_{n_i}\} \right)$$

$$= \min\left\{ \bigvee_{\Psi(x_1)=y_1} \delta_1,..., \bigvee_{\Psi(x_1)=y_1} \delta_{n_i} \right\} = \min\{\Psi(\mu)_k(y_1),...,\Psi(\mu)_k(y_{n_i})\}$$

and

$$\bigvee_{z'\in\beta_i'(y_1,y_2,...,y_{n_i})} \Psi(\nu)_k(z') \leq \bigwedge_{\Psi(x_{n_i})=y_{n_i}} \left(... \bigwedge_{\Psi(x_1)=y_1} \max\{\lambda_1,...,\lambda_{n_i}\} \right)$$

$$= \max\left\{ \bigwedge_{\Psi(x_1)=y_1} \lambda_1,..., \bigwedge_{\Psi(x_1)=y_1} \lambda_{n_i} \right\} = \max\{\Psi(\nu)_k(y_1),...,\Psi(\nu)_k(y_{n_i})\}.$$

Moreover if β' is a null hyperoperation in H' and $z \in \beta'$, we have

$$\Psi(\mu)_k(z) = \bigvee_{\Psi(a)=z} \bigvee_{\Phi(x)=k} \mu_x(a). \quad (*)$$

We know $\Psi^{-1}(\beta')$ in H is a null hyperoperation. Thus, $\mu_x(a)$ and $\nu_x(a)$ such that $\Psi(a) = z$ are constant therefore $\Psi(\mu)_k(z)$ and $\Psi(\nu)_k(z)$ are constant.

Therefore, in $(*)$, since $\mu_x(a) \geq \mu_x(t)$ and $\nu_x(a) \leq \nu_x(t)$ for all $t \in H$, we conclude that $\Psi(\mu)_k(z) \geq \Psi(\mu)_k(s)$ and $\Psi(\nu)_k(z) \leq \Psi(\nu)_k(s)$ for all $s \in H'$. The proof is done. □

Theorem 3.11. *Assume that $\langle H, (\beta_i)_{i\in I}\rangle$ and $\langle H', (\beta'_i)_{i\in I}\rangle$ are hyperalgebras, (Y, B) is an IFSHA over H' and (Ψ, Φ) is an IFS homomorphism from H to H'. Then $(\Psi, \Phi)^{-1}(Y, B)$ is an IFSHA over H.*

Proof. Let $y \in \Phi^{-1}(Y, B)$, $i \in I$ and $x_1, ..., x_{n_i} \in H$. For all $z \in \beta_i(x_1, ..., x_{n_i})$, we have

$$\bigwedge_{z\in\beta_i(x_1,...,x_{n_i})} \Psi^{-1}(\mu)_k(z) = \bigwedge_{z\in\beta_i(x_1,...,x_{n_i})} \mu_{\Phi(k)}(\Psi(z))$$

$$\geq \min\{\mu_{\Phi(k)}(\Psi(x_1)), ..., \mu_{\Phi(k)}(\Psi(x_{n_i}))$$
$$= \min\{\Psi^{-1}(\mu)_a(x_1)), ..., \Psi^{-1}(\mu)_a(x_{n_i}))\}$$

and

$$\bigvee_{z\in\beta_i(x_1,...,x_{n_i})} \Psi^{-1}(\nu)_k(z) = \bigvee_{z\in\beta_i(x_1,...,x_{n_i})} \nu_{\Phi(k)}(\Psi(z))$$

$$\leq \max\{\nu_{\Phi(k)}(\Psi(x_1)), ..., \nu_{\Phi(k)}(\Psi(x_{n_i}))$$
$$= \max\{\Psi^{-1}(\nu)_a(x_1)), ..., \Psi^{-1}(\nu)_a(x_{n_i}))\}.$$

Moreover, if β is a null hyperoperation in H and $z \in \beta$, we have

$$\Psi^{-1}(\mu)_a(z) = \mu_{\Phi(k)}(\Psi(z)) \text{ and } \Psi^{-1}(\nu)_a(z) = \nu_{\Phi(k)}(\Psi(z)).$$

Since $\Psi(\beta)$ is a null hyperoperation in H and $\Psi(z) \in \Psi(\beta)$, $\mu_{\Phi(k)}(\Psi(z))$ and $\nu_{\Phi(k)}(\Psi(z))$ are constant, for all $z \in \beta$. Therefore, since $\mu_{\Phi(k)}(\Psi(z))$ has greatest degree, $\Psi^{-1}(\mu)_a(z)$ is too and since $\nu_{\Phi(k)}(\Psi(z))$ has least degree, $\Psi^{-1}(\nu)_a(z)$ is too. □

4. Conclusion

In this paper, we applied intuitionistic fuzzy soft sets to algebraic hyperalgebras, to generalize classical algebraic structures. We analyzed the concept of intuitionistic fuzzy soft hyperalgebras. Homomorphisms of intuitionistic fuzzy soft hyperalgebras were studied.

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