

SOME FIXED POINT THEOREM USES INTIMATE MAPPINGS TO PRODUCE INTUITIONISTIC FUZZY METRIC SPACE

RAM MILAN SINGH

ABSTRACT. The aim of this research paper to use the concept of Intimate mappings to demonstrate the existence and uniqueness of common fixed point theorems for self-mappings in intuitionistic fuzzy metric space.

Key Words: Fuzzy metric space, Intuitionistic Fuzzy Metric Space, Intimate mappings, E.A property, Common E.A property.

2010 Mathematics Subject Classification: Primary: 54H25; Secondary: 47H10.

1. INTRODUCTION

The fuzzy set is a new notion that was introduced by L.A. Zadeh [1] as an extension of the classical set. Kramosil and Mechalek later presented the idea of fuzzy metric space in [2]. George and Veeramani [4] further modified this to produce Hausdorff topology for the category of fuzzy metric spaces. Following that, other fixed point theorems in fuzzy metric space were discovered under a variety of circumstances, including ([5],[6],[9],[10],[11], and [13]). Sahu and colleagues [12] introduced the idea of Intimate mappings, which are generalised compatible mappings of type (α) , under different circumstances. Chugh and Madhu Aggarwal [13] expanded these further, leading to the development of some findings in Hausdorff uniform spaces. Additional results are also visible,

Research Paper

Received: 15 March 2024, Accepted: 03 June 2024. Communicated by Nasrin Eghbali;

*Address correspondence to R. M. Singh; E-mail: rammilansinghlig@gmail.com.

© 2024 University of Mohaghegh Ardabili.

such as [14], which makes use of intimate mappings in complex valued metric space. In addition, Praveenkumar and associates [15] established a number of theorems in multiplicative metric space (MMS) by employing the concept of intimate mappings. As a result, numerous findings were made possible on this platform, including. Aamri and Matouwakil [16] established the idea of non-compatible mappings as the E. A property in metric space. Thus, Yicheng Liu et al. [17] presented the idea of enhanced E.A property, which led to the creation of common property E.A.

2. PRELIMINARIES

Definition 2.1. A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t -norm if it satisfies the following conditions:

- (a) $*$ is commutative and associative;
- (b) $*$ is continuous;
- (c) $a*1 = a$ for all $a \in [0, 1]$;
- (d) $a* b \leq c* d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0, 1]$.

Definition 2.2. A binary operation \diamond : $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t -conorm if it satisfies the following conditions:

- (a) \diamond is commutative and associative;
- (b) \diamond is continuous;
- (c) $a\diamond 0 = a$ for all $a \in [0, 1]$;
- (d) $a\diamond b \leq c\diamond d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, b \in [0, 1]$:

Definition 2.3. A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t -norm, \diamond a continuous t -conorm and M, N are fuzzy sets on $X \times X \times (0, \infty)$, satisfying the following conditions for all $x, y, z \in X$ and $s, t > 0$:

- a. $M(x, y, t) + N(x, y, t) \leq 1$;
- b. $M(x, y, t) > 0$;
- c. $M(x, y, t) = 1$ if and only if $x = y$;
- d. $M(x, y, t) = M(y, x, t)$;
- e. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
- f. $M(x, y, \cdot) : (0, \infty) \rightarrow (0, 1]$ is continuous;
- g. $N(x, y, t) \geq 0$;
- h. $N(x, y, t) = 0$ if and only if $x = y$;
- i. $N(x, y, t) = N(y, x, t)$;
- j. $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$;
- k. $N(x, y, \cdot) : (0, \infty) \rightarrow (0, 1]$ is continuous.

Then (M, N) is called an intuitionistic fuzzy metric on X . The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and degree of non-nearness between x and y with respect to t respectively [11].

Definition 2.4. A triplet $(X, M_{KM}, *)$ is a fuzzy metric space (i.e., FMS) if X is a arbitrary set, $*$ is continuous t -norm and M_{KM} is fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions for all $a, b, d \in X$ and $t, s \in (0, \infty)$:

- (KMFM-i) $M_{KM}(a, b, 0) = 0$
- (KMFM-ii) $M_{KM}(a, b, t) = 1, \forall t > 0 \Leftrightarrow a = b$
- (KMFM-iii) $M_{KM}(b, a, t) = M_{KM}(a, b, t)$
- (KMFM-iv) $M_{KM}(a, d, t + s) \geq M_{KM}(a, b, t) * M_{KM}(b, d, s)$
- (KMFM-v) $M_{KM}(a, b,) : [0, 1] \rightarrow [0, 1]$ left continuous.

Definition 2.5. A triplet $(X, M_{KM}, N_{KM}, *, \diamond)$ is an intuitionistic fuzzy metric space (i.e., IFMS) if X is a arbitrary set, $*$ is continuous t -norm, \diamond is continuous t -co norm and M_{KM} and N_{KM} are fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions for all $a, b, d \in X$ such that $t, s \in (0, \infty)$:

- (KMFM-i) $M_{KM}(a, b, 0) = 0$
- (KMFM-ii) $M_{KM}(a, b, t) = 1 \forall t > 0 \Leftrightarrow a = b$
- (KMFM-iii) $M_{KM}(b, a, t) = M_{KM}(a, b, t)$
- (KMFM-iv) $M_{KM}(a, d, t + s) \geq M_{KM}(a, b, t) * M_{KM}(b, d, s)$
- (KMFM-v) $N_{KM}(a, b,) : [0, 1] \rightarrow [0, 1]$ left continuous.
- (KMFM-vi) $N(a, b, 0) = 0$
- (KMFM-vii) $N_{KM}(a, b, t) = 0 \forall t > 0 \Leftrightarrow a = b$
- (KMFM-viii) $N_{KM}(b, a, t) = N_{KM}(a, b, t)$
- (KMFM-ix) $N_{KM}(a, d, t + s) \leq N_{KM}(a, b, t) \diamond N_{KM}(b, d, s)$
- (KMFM-x) $N_{KM}(a, b,) : [0, 1] \rightarrow [0, 1]$ right continuous.

Definition 2.6. Let $\langle a_n \rangle$ be sequence in IFMS $(X, M_{KM}, N_{KM}, *, \diamond)$. We say that $\langle a_n \rangle$ converges to a point $l \in X$ if:

$$\lim_{n \rightarrow \infty} M_{KM}(a_n, l, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N_{KM}(a_n, l, t) = 0 \quad \forall t > 0.$$

Definition 2.7. Let $\langle a_n \rangle$ be a sequence in IFMS $(X, M_{KM}, N_{KM}, *, \diamond)$, this sequence $\langle a_n \rangle$ in X is said to be Cauchy sequence in FMS if

$$\lim_{n \rightarrow \infty} M_{KM}(a_{n+p}, a_n, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N_{KM}(a_{n+p}, a_n, t) = 0$$

for every $t > 0$ and $p > 0$.

Definition 2.8. If every Cauchy sequence is convergent in $(X, M_{KM}, N_{KM}, *, \diamond)$, then we say that X is complete.

Definition 2.9. Let $(X, M_{KM}, N_{KM}, *, \diamond)$ be an IFMS and \mathcal{G} and \mathfrak{J} be two self mappings on X . Then \mathcal{S} and \mathcal{T} are

- (1) compatible if $\lim_{n \rightarrow \infty} M_{KM}(\mathcal{G}\mathfrak{T}_n, \mathfrak{T}\mathcal{G}a_n, t) = 1$ and $\lim_{n \rightarrow \infty} N_{KM}(\mathcal{G}\mathfrak{T}_n, \mathfrak{T}\mathcal{G}a_n, t) = 0$ whenever a sequence $\langle a_n \rangle$ in X provided that $\lim_{n \rightarrow \infty} \mathcal{G}a_n = \lim_{n \rightarrow \infty} \mathfrak{J}a_n = t$ for some $t \in X$.
- (2) compatible of type (α) if $\lim_{n \rightarrow \infty} M_{KM}(\mathcal{G}\mathcal{T}a_n, \mathfrak{T}a_n, t) = 1$ and $\lim_{n \rightarrow \infty} N_{KM}(\mathfrak{T}\mathcal{G}a_n, \mathcal{G}a_n, t) = 0$ whenever $\langle a_n \rangle$ in X such that $\lim_{n \rightarrow \infty} \mathcal{G}a_n = \lim_{n \rightarrow \infty} \mathfrak{T}a_n = t$ for some $t \in X$.

Now we discuss some definitions related to intimate mappings in IFMS.

Definition 2.10. Let A and B be two mappings of the IFMS $(X, M_{KM}, N_{KM}, *, \diamond)$ into itself. Then A and B are said to be \mathcal{A} -intimate mappings if

$$\alpha M_{KM}(Aa_n, Ba_n, t) \geq \alpha M_{KM}(Aa_n, Ba_n, t)$$

and

$$\alpha N_{KM}(Aa_n, Ba_n, t) \leq \alpha N_{KM}(Aa_n, Ba_n, t)$$

where $\alpha = \lim_{n \rightarrow \infty} \text{Sup}$ or $\lim_{n \rightarrow \infty} \text{Inf}$ and $\langle a_n \rangle$ is a sequence in X such that $\lim_{n \rightarrow \infty} Aa_n = \lim_{n \rightarrow \infty} Ba_n = t$ for some $t \in X$.

Definition 2.11. Let A and B be two self maps on the IFMS $(X, M_{KM}, N_{KM}, *, \diamond)$. We say that A and B satisfy the property E.A if there exists a sequence $\langle a_n \rangle \in X$ such that $\lim_{n \rightarrow \infty} Aa_n = \lim_{n \rightarrow \infty} Ba_n = t$ for some $t \in X$.

Definition 2.12. Suppose A, B, C and D are four self maps on the IFMS $(X, M_{KM}, N_{KM}, *, \diamond)$. We say that (A, B) and (C, D) satisfy common property E.A whenever two sequences $\langle a_n \rangle$ and $\langle b_n \rangle$ in X satisfy $\lim_{n \rightarrow \infty} Aa_n = \lim_{n \rightarrow \infty} Ba_n = \lim_{n \rightarrow \infty} Cb_n = \lim_{n \rightarrow \infty} Db_n = t$ for some $t \in X$.

3. Main Result

Theorem 3.1. Let $(X, M_{KM}, N_{KM}, *, \diamond)$ be a complete intuitionistic fuzzy metric space. Suppose P, Q, R and S are self maps on X satisfying the following conditions:

- (i) $P(X) \subseteq R(X)$ and $Q(X) \subseteq S(X)$;

- (ii) For every where $k \in (0, 1)$ and $a, b \in \mathbb{X}$: $M_{KM}(Pa, Qb, kt)$
 $\geq M_{KM}(Sa, Rb, t) * M_{KM}(Pa, Sa, t) * M_{KM}(Qb, Rb, t) * M_{KM}(Pa, Rb, t)$
 and
 $N_{KM}(Pa, Qb, kt)$
 $\leq N_{KM}(Sa, Rb, t) \diamond N_{KM}(Pa, Sa, t) \diamond N_{KM}(Qb, Rb, t) \diamond N_{KM}(Pa, Rb, t)$
- (iii) $S(X)$ is complete;
- (iv) the pair of mappings S and P is \mathcal{A} - intimate and the other pair of mappings also R and Q is \mathcal{S} - intimate.

Then P, Q, R and S have a unique common fixed point in X .

Proof. Let a_0 be an arbitrary point of X . From the condition $P(X) \subseteq R(X)$ of (i), there exists a point $a_1 \in X$ such that

$$Pa_0 = Ra_1 = b_0.$$

Now for a_1 , applying (i), there exists $a_2 \in \mathbb{X}$ such that

$$Qa_1 = Sa_2 = b_1.$$

Continuing this way, we establish two real sequences $\langle a_n \rangle$ and $\langle b_n \rangle$ in X :

$$\exists b_{2n} = Pa_{2n} = Ra_{2n+1} \text{ and } b_{2n+1} = Qa_{2n+1} = Sa_{2n+2} \text{ for } n \geq 0.$$

Taking $a = a_{2n}, b = a_{2n+1}$ in the inequality (ii), we have

$$M_{KM}(Pa_{2n}, Qa_{2n+1}, kt)$$

$$\geq M_{KM}(Sa_{2n}, Ra_{2n+1}, t) * M_{KM}(Pa_{2n}, Sa_{2n}, t) * M_{KM}(Qa_{2n+1}, Ra_{2n+1}, t) * M_{KM}(Pa_{2n}, Ra_{2n+1}, t)$$

and

$$N_{KM}(Pa_{2n}, Qa_{2n+1}, kt)$$

$$\leq N_{KM}(Sa_{2n}, Ra_{2n+1}, t) \diamond N_{KM}(Pa_{2n}, Sa_{2n}, t) \diamond N_{KM}(Qa_{2n+1}, Ra_{2n+1}, t) \diamond N_{KM}(Pa_{2n}, Ra_{2n+1}, t)$$

which implies that as $n \rightarrow \infty$:

$$M_{KM}(b_{2n}, b_{2n+1}, kt) \geq M_{KM}(b_{2n-1}, b_{2n}, t) * M_{KM}(b_{2n}, b_{2n-1}, t) *$$

$$M_{KM}(b_{2n+1}, b_{2n}, t) * M_{KM}(b_{2n}, b_{2n}, t)$$

and

$$N(b_{2n}, b_{2n+1}, kt) \leq N_{KM}(b_{2n-1}, b_{2n}, t) \diamond N_{KM}(b_{2n}, b_{2n-1}, t)$$

$$\diamond N_{KM}(b_{2n+1}, b_{2n}, t) \diamond N_{KM}(b_{2n}, b_{2n}, t)$$

This yields

$$M_{KM}(b_{2n}, b_{2n+1}, kt)$$

$$\geq M_{KM}(b_{2n-1}, b_{2n}, t) * M_{KM}(b_{2n+1}, b_{2n}, t) * M_{KM}(b_{2n}, b_{2n-1}, t) * 1$$

and

$$\begin{aligned} & N_{KM}(b_{2n}, b_{2n+1}, kt) \\ \leq & N_{KM}(b_{2n-1}, b_{2n}, t) \diamond N_{KM}(b_{2n+1}, b_{2n}, t) \text{iamond} N_{KM}(b_{2n}, b_{2n-1}, t) \diamond 0 \end{aligned}$$

Again, by the condition KMIFM-3, we get

$$\begin{aligned} & M_{KM}(b_{2n}, b_{2n+1}, kt) \\ & \geq M_{KM}(b_{2n-1}, b_{2n}, t) * M_{KM}(b_{2n}, b_{2n+1}, t) \end{aligned}$$

and

$$\begin{aligned} & N_{KM}(b_{2n}, b_{2n+1}, kt) \\ & \leq N_{KM}(b_{2n-1}, b_{2n}, t) N_{KM}(b_{2n}, b_{2n+1}, t) \end{aligned}$$

which implies (since $\mathbf{a} * \mathbf{b} = \min\{\mathbf{a}, \mathbf{b}\}$ and $\mathbf{a} \mathbf{b} \mathbf{b} = \max\{\mathbf{a}, \mathbf{b}\}$.)

$$M_{KM}(b_{2n}, b_{2n+1}, kt) \geq M_{KM}(b_{2n-1}, b_{2n}, t).$$

and

$$N_{KM}(b_{2n}, b_{2n+1}, kt) \leq N_{KM}(b_{2n-1}, b_{2n}, t).$$

In general

$$M_{KM}(b_{n+1}, b_{n+2}, kt) \geq M_{KM}(b_n, b_{n+1}, t) \dots (i)$$

and

$$N_{KM}(b_{n+1}, b_{n+2}, kt) \leq N_{KM}(b_n, b_{n+1}, t) \dots (i)$$

for all $n = 1, 2, 3, \dots$, and $t > 0$.

From (i) we have:

$$\begin{aligned} [M_{KM}(b_n, b_{n+1}, t)] & \geq M_{KM}(b_{n-1}, b_n, \frac{t}{k}) \geq M_{KM}(b_{n-2}, b_{n-1}, \frac{t}{k^2}) \\ & \geq \dots \geq M_{KM}(b_0, b_1, \frac{t}{k^n}) \rightarrow 1 \text{ as } n \rightarrow \infty \end{aligned}$$

and

$$\begin{aligned} [N_{KM}(b_n, b_{n+1}, t)] & \leq N_{KM}(b_{n-1}, b_n, \frac{t}{k}) \leq N_{KM}(b_{n-2}, b_{n-1}, \frac{t}{k^2}) \\ & \leq \dots \leq N_{KM}(b_0, b_1, \frac{t}{k^n}) \rightarrow 0 \text{ as } n \rightarrow \infty \dots (ii) \end{aligned}$$

For any $t > 0$ and $\lambda_{MK} \in (0, 1)$ we consider $\forall n > n_0 \in \mathbb{N}$ such that

$$M_{KM}(b_n, b_{n+1}, t) > (1 - \lambda_{MK})$$

and

$$N_{KM}(b_n, b_{n+1}, t) < (-\lambda_{MK}) \dots (iii)$$

For $m, n \in \mathbb{N}$, suppose $m \geq n$. Then we have:

$$\begin{aligned}
& [M_{MK}(b_n, b_m, t)] \\
& \geq \min \left\{ M_{MK} \left(b_n, b_{n+1}, \frac{t}{m-n} \right) * M_{MK} \left(b_{n+1}, b_{n+2}, \frac{t}{m-n} \right) * \dots \right. \\
& M_{MK} \left(b_{m-1}, b_m, \frac{t}{m-n} \right) \geq (1 - \lambda_{MK}) * (1 - \lambda_{MK}) * \dots * (1 - \lambda_{MK}) \dots (m-n) \text{ times} \\
& \text{and} \\
& \left. [N_{MK}(b_n, b_m, t)] \right\}
\end{aligned}$$

$$\leq \max \left\{ N_{MK} \left(b_n, b_{n+1}, \frac{t}{m-n} \right) \diamond N_{MK} \left(b_{n+1}, b_{n+2}, \frac{t}{m-n} \right) \diamond \dots \right.$$

$$\left. N_{MK} \left(b_{m-1}, b_m, \frac{t}{m-n} \right) \leq (-\lambda_{MK}) \diamond (-\lambda_{MK}) \downarrow (-\lambda_{MK}) \dots (m-n) \text{ times.} \right\}$$

This implies

$$M_{MK}(b_{m-1}, b_m, t) \geq (1 - \lambda_{MK}) \text{ and } N_{MK}(b_{m-1}, b_m, t) \leq (-\lambda_{MK})$$

Therefore $\{b_n\}$ is a Cauchy sequence in IFMS.

Since $(X, M_{KM}, N_{KM}, *, \diamond)$ is a complete IFMS, so the sequence $\{b_n\}$ converges $L \in I$.

Further, the fuzzy Cauchy sequence $\{b_n\}$ has a convergent subsequence $\{b_{2n+1}\}$ and $\{b_{2n}\}$.

From the above argument,

$$b_{2n+1} = Qa_{2n+1} = Sa_{2n+2} \rightarrow L \text{ and}$$

$$b_{2n} = Pa_{2n} = \tilde{S}a_{2n+1} \rightarrow L \text{ as } n \rightarrow \infty \dots (iv)$$

Now suppose that the range set $S(X)$ is complete then \exists a point $u \in X \ni Su = L \dots (v)$.

Now we claim that $Pu = L$ from the inequality, put $a = u$ and $b = a_{2n+1}$ we have

$$\begin{aligned}
M_{KM}(Pu, Qn_{2n+1}, kt) & \geq M_{KM}(Su, Ra_{2n+1}, t) * M_{KM}(Pu, Su, t) \\
& * M_{KM}(Qa_{2n+1}, Ra_{2n+1}, t) * M_{KM}(Pu, Sa_{2n+1}, t)
\end{aligned}$$

and

$$\begin{aligned}
N_{KM}(Pu, Qn_{2n+1}, kt) & \leq N_{KM}(Su, Ra_{2n+1}, t) N_{KM}(Pu, Su, t) \\
& \diamond N_{KM}(Qa_{2n+1}, Ra_{2n+1}, t) \diamond N_{KM}(Pu, Sa_{2n+1}, t).
\end{aligned}$$

Taking limit as $n \rightarrow \infty$ we have:

$$\begin{aligned} & M_{KM}(Pu, L, kt) \\ & \geq M_{KM}(L, L, t) * M_{KM}(Pu, L, t) * M_{KM}(L, L, t) * M_{KM}(Pu, L, t) \end{aligned}$$

This gives $Pu = L$. That is $Pu = Su = L \dots \dots$ (vi)

Let us prove that $Qv = p^*$.

Using the equation ((vi) with contained inequality $P(X) \subseteq R(X)$, $L = Pu \in P(X) \subseteq R(X)$ then there exists a point $v \in X$ such that $Rv = Pu = L \dots$ (vii).

Put $a = u$ and $b = v$ in (ii) then we obtain

$$\begin{aligned} & M_{KM}(Pu, Qv, kt) \\ & \geq M_{KM}(Bu, Rv, t) * M_{KM}(Pu, Su, t) * M_{KM}(Qv, Rv, t) * M_{KM}(Pu, Rv, t). \\ & \text{and } N_{KM}(Pu, Qv, kt) \\ & \leq N_{KM}(Bu, Rv, t) \diamond N_{KM}(Pu, Su, t) \triangleright N_{KM}(Qv, Rv, t) \diamond N_{KM}(Pu, Rv, t). \end{aligned}$$

By using (vii) we get

$$\begin{aligned} & M_{KM}(L, Qv, kt) \\ & \geq M_{KM}(L, Qv, t) * M_{KM}(L, L, t) * M_{KM}(Qv, L, t) * M_{KM}(L, L, t) \\ & \text{and} \\ & N_{KM}(L, Qv, kt) \\ & \leq N_{KM}(L, Qv, t) \diamond N_{KM}(L, L, t) \diamond N_{KM}(Qv, L, t) \diamond N_{KM}(L, L, t) \end{aligned}$$

this gives

$$M_{KM}(L, Qv, kt) \geq M_{KM}(Qv, L, ktt)$$

and

$$N_{KM}(L, Qv, kt) \leq N_{KM}(Qv, L, kt).$$

Consequently $M_{KM}(L, Qv, kt) \geq M_{KM}(Qv, L, kt)$ and $N_{KM}(L, Qv, kt) \leq N_{KM}(Qv, L, kt)$. This implies $Qv = L$.

This shows that $Qv = Rv = L$. Since $Pu = Su = L$ and (S, P) is \mathcal{A} -intimate we have $M_{KM}(S L, L, t) \geq M_{KM}(P L, L, t)$ and $N_{KM}(S L, L, t) \leq N_{KM}(P L, L, t) \dots$ (ix).

Suppose that $Pp^* \neq p^*$. Put $a = L, b = v$ in (ii). Then we get,

$$\begin{aligned} & M_{KM}(PL, Qv, kt) \\ & \geq M_{KM}(S L, Rv, t) * M_{KM}(PL, S L, t) * M_{KM}(Qv, Rv, t) * M_{KM}(PL, Rv, t) \\ & \text{and} \\ & N_{KM}(PL, Qv, kt) \\ & \leq N_{KM}(S L, Rv, t) N_{KM}(PL, S L, t) N_{KM}(Qv, Rv, t) \diamond N_{KM}(PL, Rv, t). \end{aligned}$$

Using (viii) we get,
 $M_{KM}(PL, L, kt)$
 $\geq M_{KM}(SL, L, t) * M_{KM}(PL, SL, t) * M_{KM}(L, L, t) * M_{KM}(PL, L, t)$

and

$N_{KM}(PL, L, kt)$
 $\leq N_{KM}(SL, L, t) \diamond N_{KM}(PL, SL, t) \diamond N_{KM}(L, L, t) \diamond N_{KM}(PL, L, t).$

By applying (KMIFM-iv) we get
 $M_{KM}(PL, L, kt) \geq M_{KM}(PL, L, t) * M_{KM}(PL, L, \frac{t}{2})$
 $* M_{KM}\left(L, SL, \frac{t}{2}\right) * M_{KM}(L, L, t) * M_{KM}(PL, L, t)$

and

$N_{KM}(PL, L, kt) \leq N_{KM}(PL, L, t) \diamond N_{KM}\left(PL, L, \frac{t}{2}\right)$
 $\diamond N_{KM}\left(L, SL, \frac{t}{2}\right) \diamond N_{KM}(L, L, t) \diamond N_{KM}(PL, L, t)$

By using (ix) we get $M_{KM}(PL, L, kt) \geq M_{KM}(PL, L, t/2)$
and $N_{KM}(PL, L, kt) \leq N_{KM}(PL, L, t/2)$. This gives $PL = L$.

From (ix) and (x) we write

$$M_{KM}(SL, L, t) \geq 1 \text{ and } N_{KM}(SL, L, t) \leq 1$$

this gives $SL = L \dots \dots (xi)$.

Using (x) and (xi) we get $SL = PL = L \dots \dots (xii)$. Also, $Qv = Rv = L$ and using the pair (R, Q) as \mathcal{A} -Intimate, then we have

$$M_{KM}(RL, L, t) \geq M_{KM}(QL, L, kt)$$

and

$$N_{KM}(RL, L, t) \leq N_{KM}(QL, L, kt) \dots (\dots (xiii)).$$

Suppose that $QL \neq L$. Put $a = u$ and $b = L$ in the inequality. We have:

$M_{KM}(Pu, QL, kt)$
 $\geq M_{KM}(Su, RL, t) * M_{KM}(Pu, Su, t) * M_{KM}(QL, RL, t) * M_{KM}(Pu, RL, t)$

and

$N_{KM}(Pu, QL, kt)$
 $\leq N_{KM}(Su, RL, t) \diamond N_{KM}(Pu, Su, t) \diamond N_{KM}(QL, RL, t) \diamond N_{KM}(Pu, RL, t)$

Using (vi) and (KMIFM-iv) we get,

$$M_{KM}(L, QL, kt) \geq M_{KM}(L, RL, t) * M_{KM}(L, L, t) \\ * M_{KM}\left(PL, L, \frac{t}{2}\right) * M_{KM}\left(L, SL, \frac{t}{2}\right) * M_{KM}(L, RL, t)$$

and $N_{KM}(L, QL, kt) \leq N_{KM}(L, RL, t) \diamond N_{KM}(L, L, t)$

$$\diamond N_{KM}\left(PL, L, \frac{t}{2}\right) \diamond N_{KM}\left(L, SL, \frac{t}{2}\right) \diamond N_{KM}(L, RL, t)$$

Now using (xiii) we get

$$M_{KM}(L, QL, kt) \geq M_{KM}(L, QL, t) * M_{KM}(QL, L, \frac{t}{2}) \\ * M_{KM}(QL, L, t/2) * M_{KM}(L, QL, t)$$

and

$$N_{KM}(L, QL, kt) \leq N_{KM}(L, QL, t) N_{KM}(QL, L, \frac{t}{2}) \\ \downarrow N_{KM}(QL, L, t/2) \diamond N_{KM}(L, QL, t)$$

This implies $M_{KM}(L, L, kt) \geq M_{KM}(L, L, t/2)$ and $N_{KM}(L, L, kt) \leq N_{KM}(L, L, t/2)$. This gives $QL = L \dots$ (xiv).

From (xii) and (xiv) we get

$$M_{KM}(RL, L, t) \geq 1 \text{ and } N_{KM}(RL, L, t) \leq 1$$

$$RL = L \dots \dots (xv).$$

Using (xiv) and (xv) we get

$$QL = SL = L \dots (xvi).$$

Using (xii) and xvi we conclude that $PL = QL = RL = SL = L$.

□

Conclusion

This study demonstrates that fixed point theorems can be effectively applied to intimate mappings within intuitionistic fuzzy metric spaces. By extending classical fixed point principles to these spaces, we have shown that such mappings maintain the necessary conditions for establishing fixed points. This work not only broadens the theoretical framework of fuzzy metric spaces but also suggests potential applications in fields where uncertainty and imprecision are common. Future research may further explore these extensions and their practical implications.

REFERENCES

- [1] L. A. Zadeh , Fuzzy sets, Inform. Contro, **8** (1965), 338-353.
- [2] I. Kramosil and J. Michalek, Fuzzy metrics and statistical metric spaces, Kybernet, **11** (1975) 336-344.
- [3] M. Grabiec, Fixed points in fuzzy metric space, Fuzzy Sets Syst., **27**(3)(1988), 385-389.
- [4] G.Veeramani, P. On some results in fuzzy metric spaces, Fuzzy Sets Syst.,**64**(3) (1994),395-399.
- [5] S. N. Mishra, N. Sharma and S. L. Singh, Common fixed points of maps on fuzzy metric spaces, International Journal of Mathematics and Mathematical Sciences, **17**(2)(1994), 253-258.
- [6] S. Hoon cho, On Common fixed points in fuzzy metric spaces, International Mathematical Forum, **1**(2006), 471-479.
- [7] B. Schweizer and A. Sklar, Statistical metric spaces, Pacific Journal of Mathematics, **10**(1960), 314-334.
- [8] E.P. Klement, R. Mesier and E. Pap, Triangular norms, Kluwer Academic Publisher Dordrecht, Trends in Logic, **8**(2000).
- [9] G. Jungck, P. P. Murthy and Y. J. Cho, Compatible mappings of type (A) and common fixed points, Mathematica Japonica ,**36** (1993), 381-390.
- [10] Y.J. Cho, Fixed points in fuzzy metric spaces, Journal of Fuzzy Mathematics, **5**(4)(1997), 949- 962..
- [11] M. R. Singh, Y. Mahindra singh and L. Shambhu singh, Fixed points of biased maps of Some fixed point results in fuzzy metric space using intimate mappings type (R_M) on fuzzy metric space, International Journal of Contemporary Mathematical Sciences, **4**(16)(2009)), 757 - 768.
- [12] D.R. Sahu, V.B. Dhagat and M. Srivastava, fixed points with intimate mappings (I), Bulletin of Calcutta Mathematical Society, **93**(2001), 107-114.
- [13] R. Chugh and M. Aggarwal, Fixed points of intimate mappings in uniform spaces, International Journal of Mathematical Analysis, **6**(9)(2012), 429-436.
- [14] G. Meena, Common fixed points for intimate mappings in complei valued metric Spaces, General Mathematics Notes,**26**(2)(2015),97-103.
- [15] P. kumar, S. kumar and S. Min Kang, Common fixed for intimate mappings in multiplicative metric spaces, International Journal of Pure and Applied Mathematics, **104**(4)(2015), 709-718.
- [16] M. Aamri and D. El. Moutawakil, Some new fixed point theorems under strict contractive conditions, Journal of Mathematical Analysis and Applications, **270**(1)(2002), 181-188.
- [17] Y. Liu, Jun wu and Z. Li, Common fixed points of single-valued and multi-valued maps, International Journal of Mathematics and Mathematical sciences, **19**(2005), 3045-3055.

Ram Milan Singh

Department of Mathematics,

Institute for Excellence in higher Education, Bhopal (India)

Email: rammilansinghlig@gmail.com