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Research Paper

$(\frac{1}{2}, \frac{1}{2})$-FUZZY QS-IDEALS AND CORRELATION COEFFICIENTS

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ABSTRACT

As an extension of intuitionistic fuzzy sets, we introduce the notion of $(\frac{1}{2}, \frac{1}{2})$-fuzzy set (denoted by SSR) of QS-ideals on a OS-algebra and investigate its properties. Furthermore we study the homomorphic image and inverse image of SSR -fuzzy QS-ideals of a QS-algebra under homomorphism of QS-algebras. Moreover, the Cartesian product of SSR-fuzzy QS-ideals in Cartesian product QS-algebras is given. Finally, novel correlation coefficient between two SSRfuzzy sets are also studied.

1. Introduction

In 1966, Y. Imai and K. Iski introduced two classes of abstract algebras: BCK-algebras and BCI-algebras see[12,13,14]. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. Neggers et al [17] introduced a notions, called Q-algebras, which is a generalization of BCH / BCI / BCK-algebras and generalized some theorems discussed in BCI- algebras. Moreover, Ahn and Kim [1] introduced the notions of QS-algebras which is a proper subclass of Q-algebras. Kondo [16] proved that, each theorem of QS-algebras is provable in the theory of Abelian groups and conversely each theorem of Abelian groups is provable in the theory of QS-algebras. QS algebra in the fuzzy setting have also been considered by many authors see [10,18]. The concept of fuzzy sets was introduced by Zadeh

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[25]. In 1991, Xi [24] applied the concept of fuzzy sets to BCI, BCK, MV-algebras. Since its inception, the theory of fuzzy sets , ideal theory and its fuzzification has been developed in many directions and applied to a wide variety of fields. Yager [22,23] launched a nonstandard fuzzy set referred to as Pythagorean fuzzy set which is the generalization of intuitionistic fuzzy sets. The construct of Pythagorean fuzzy sets can be used to characterize uncertain information more sufficiently and accurately than intuitionistic fuzzy set. [2,20] defined a new generalized Pythagorean fuzzy set is called (3, 2)-Fuzzy sets. In 2020. Fermatean fuzzy sets proposed by Senapati and Yager [21], can handle uncertain information more easily in the process of decision making. They also defined basic operations over the Fermatean fuzzy sets. The main advantage of Fermatean fuzzy sets is that it can describe more uncertainties than Pythagorean fuzzy sets, which can be applied in many decision-making problems. The relevant research can be referred to [20], SR-Fuzzy sets). Pythagorean fuzzy set is one of the successful extensions of the fuzzy set for handling uncertainties in information. Under this environment, Salih et al, [19], we introduce a new type of generalized fuzzy sets is called CR-fuzzy sets and compare CR-fuzzy sets with Pythagorean fuzzy sets and Fermatean fuzzy sets. The set operations, score function and accuracy function of CRfuzzy sets will study along with their several properties. Recently Jun et al [15] introduced the concept of the (m; n)-fuzzy set which is the subclass of intuitionistic fuzzy set, Pythagorean fuzzy set, (3; 2)-fuzzy set, Fermatean fuzzy set and n-Pythagorean fuzzy set and compared with them. They introduced some operations for the (m; n)-fuzzy set, investigate their properties and applied the (m; n)-fuzzy set to BCK-algebras and BCI-algebras. They introduced the (m; n)-fuzzy subalgebra in BCK-algebras and BCI-algebras and investigate their properties. Ahn etal [1] apply the concept of (2, 3)-fuzzy sets to BCK-algebras and BCI-algebas. Ibrahim et al. [11] introduced (3; 2)-fuzzy sets and applied it to topological spaces. Bashar et al.[5]. present the definitions of square and square root of a continuous fuzzy number with examples followed by respective graphs. In first section, present briefly the necessary preliminaries on fuzzy sets, fuzzy numbers and fuzzy arithmetic. In second section, present definition and example, they establishes a common nature of the square of a continuous fuzzy number. In third section, they introduce the definition of the square root of a continuous fuzzy number with example. Proceeding further they find that some continuous fuzzy numbers do not have square roots. Then they develop the necessary and sufficient conditions for a continuous fuzzy number, which possesses its membership grade 1.0 at only one point, to have a square root. Decision-making problems are very important in establishing foreign policy, national defense policy, economic policy, various election strategies, and prevention policy of the recent worldwide coronavirus, etc. So, many mathematicians have dealt with decision-making problems using the algorithms for decision making in three direct ions: aggregation operators, similarity measures and correlation coefficients based on various fuzzy sets. kinds of fuzzy concepts or fused fuzzy concepts. Mainly, they presented algorithms for decision making in three directions: aggregation operators, similarity measures and correlation coefficients based on various fuzzy sets see [4,8,9] .

In this paper, we introduce the notion of $(\frac{1}{2}, \frac{1}{2})$-fuzzy set (denoted by SSR) of QS-ideals on a QS -algebra and investigate its properties. Furthermore we study the homomorphic image and inverse image of SSR-fuzzy QS-ideals of a QS-algebra under homomorphism of QS-algebras. Moreover, the Cartesian product of SSR-fuzzy QS-ideals in Cartesian product

QS-algebras is given. Finally, novel correlation coefficient between two SSR-fuzzy sets are also studied.

2. Preliminaries

Now, we will recall some known concepts related to QS-algebra from the literature, which will be helpful in further study of this article.

Definition 2.1. [1] A QS-algebra $(X, , 0)$ is a non-empty set X with a constant 0 and a binary operation $*$ such that for all $x, y, z \in X$ satisfying the following axioms:
(QS-1) $(x * y) * z = (x * z) * y$
(QS-2) $x * 0 = x$.
(QS-3) $x * x = 0$.
(QS-4) $(x * Y) * (x * z) = z * y$.

Definition 2.2. [1] Let $(X, *, 0)$ be a QS-algebra, we can define a binary relation $\leq$ on X as $x \leq y$ if and only if $x * y = 0$, this makes X as a partially ordered set.

Proposition 2.3. [1] Let $(X, *, 0)$ be a QS-algebra. Then the following hold $\forall\, x, y, z \in X$:

1. $x \leq y$ implies $z * x \leq z * y$.
2. $x \leq y$ and $y \leq z$ imply $x \leq z$.
3. $x * y \leq z$ implies $x * z \leq yz$.
4. $(x * z) * (y * z) \leq x * y$.
5. $x \leq y$ implies $x * z \leq y * z$.
6. $0 * (0 * (0 * x)) = 0 * x$.

Lemma 2.4. [1] Let $(X, *, 0)$ be a QS-algebra. If $x * y = z$, then $x * z = y\, \forall\, x, y, z \in X$.
Lemma 2.5. [1] Let $(X, *, 0)$ be a QS-algebra. $0 * (x * y) = y * x\, \forall\, x, y, z \in X$.
Lemma 2.6. [1] Let $(X, *, 0)$ be a QS-algebra. $0 * (0 * x) = x\, \forall\, x \in X$.
Lemma 2.7. [1] Let $(X, *, 0)$ be a QS-algebra. $x * (0 * y) = y * (0 * x)\, \forall\, x, y \in X$.
Proposition 2.8. [1] Let $(X, *, 0)$ be a QS-algebra. Then the following hold $\forall\, x, y, z \in X$:

1. $x * (x * y) = y$.
2. $x * (x * (x * y)) = x * y$.
3. $(x * (x * y)) * y = 0$.
4. $(x * z) * (y * z) = x * y$.
5. $(x * y) * x = 0 * y$.
6. $x * 0 = 0 \Rightarrow x = 0$.
7. $0 * (x * y) = (0 * x) * (0 * y)$.
8. $x * y = 0, y * x = 0 \Rightarrow x = y$.

Example 2.9. [16] (a) Let $X = \{0, 1, 2\}$ be a set in which the operation $*$ is defined as follows:

Table(1)

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Then $(X, *, 0)$ is a QS-algebra.

(b) Let X be the set of all integers. Define a binary operation $*$ on X by $x * y := x - y$. Then $(X; *, 0)$ is a QS-algebra.

Definition 2.10 [1,16]. Let $(X, *, 0)$ be a QS-algebra and S be a non-empty subset of X , then S is called sub algebra of X if $x * y \in S \, \forall \, x, y \in S$.

Definition 2.11 [1,16]. Let $(X, *, 0)$ be a QS-algebra, $x, y \in X$ we denote $x \wedge y = y * (y * x)$.

Remark 2.12 [1,16]. Every QS-algebra X is a Q-algebra. Every Q-algebra X satisfying the condition

(5) $(x * y) * (x * z) = z * y$ for any $x, y \in X$, is QS-algebra.

Definition 2.13 [14]. A non empty subset I of a BCK-algebra X is called an ideal of X if it satisfies

$(I_1)\, 0 \in I$,

$(I_2)\, x \in I$ and $y * x \in I$ implies $y \in I \, for \, all \, x, y \in X$. or

Definition 2.14 [16]. A non-empty subset I of a QS-algebra $(X, *, 0)$ is called a sub-algebra of X if $x * y \in I$ whenever $x, y \in I$.

Definition 2.15 [10]. A non empty subset I of a OS-algebra X is called an OS-ideal of X if it satisfies

$(I_1)\, 0 \in I$.

$(I_2)\, (x * z) \in I, z * y \in I \Rightarrow xy \in I$.

Definition 2.16 [1] Let $(X, *, 0)$ and $(X^{\backprime}, *^{\backprime}, 0^{\backprime})$ be QS-algebras. A map $f : X \to X^{\backprime}$ is called a homomorphism if $f(x * y) = f(x) *^{\backprime} f(y)$ for all $x, y \in X$.

Proposition 2.17 [1] Let $(X, *, 0)$ and $(X^{\backprime}, *^{\backprime}, 0^{\backprime})$ be QS-algebras and $f : X \to X^{\backprime}$ be a homomorphism if f is a QS-ideal of X.

Definition 2.18.[25] A fuzzy set A of X is of the form $A = \{(x, \mu_A(x)), x \in X\}$, where $\mu_A(x)"X \to [0, 1]$ is called the degree of existence of the element x in the set A and $0 \leq \mu_A(x) \leq I$.

Definition 2.19 [12]. Let X be a BCI-algebra. a fuzzy set μ in X is called a fuzzy BCI-ideal of X if it satisfies:

$(FI_1)\, \mu(0) \geq \mu(x)$,

$(FI_2)\, \mu(x) \geq \min\{\mu(x * y), \mu(y)\}$. for all x, y and $z \in X$.

Definition 2.20 [10]. Let X be a QS-algebra. A fuzzy set μ in X is called a fuzzy QS-ideal of X if it satisfies:

$(FQS_1)\, \mu(0) \geq \mu(x)$,

$(FQS_2)\, \mu(x * y) \geq \min\{\mu((x * z))), \mu(z * y)\}$. for all x, y and $z \in X$.

Lemma 2.21 Any fuzzy QS-ideal of a QS-algebra is a fuzzy BCI-ideal of X.

Proof. Clear

In what follows, let X denotes a QS-algebra unless otherwise specified.

Definition 2.22.[3] (An Intuitionistic fuzzy set (briefly I F S) A in a nonempty set X is an object having the form $A = \{(x, \mu_A(x), \lambda(x)) | x \in X\}$, where the function
$\mu_A : X \to [0, 1]$ and $\lambda_A : X \to [0, 1]$ denote the degree of membership and degree of nonmembership, respectively and $0 \geq \mu_A(x) + \lambda_A(x) \geq 1$ for all $x \in X$. An intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda(x)) | x \in X\}$ in X can be identified as an order pair (μ_A, λ_B) in $I^X \times I^Y g$. We shall use the symbol $A = (\mu_A, \lambda_A)$.

Remark 2.23. Let $\mu_A : X \to [0,1]$ and $\lambda_A : X \to [0,1]$ be fuzzy sets in a set X. The structure $A = \{(x, \mu_A(x), \lambda(x)) | x \in X\}$, is called:
(1) an intuitionistic fuzzy set in X (See [3]), if it satisfies: $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$
(2) an a Pythgorean fuzzy set in X (See [22]), if it satisfies: $0 \leq \mu_A^2(x) + \lambda_A^2(x) \leq 1$
(3) an a n- Pythgorean fuzzy set in X (See [6]), if it satisfies : $0 \leq \mu_A^n(x) + \lambda_A^n(x) \leq 1$
(4) an a Fermatean fuzzy set in X (See [21]), if it satisfies : $0 \leq \mu_A^3(x) + \lambda_A^3(x) \leq 1$
(5) (3; 2)- fuzzy set in X (See [2,19]), if it satisfies : $0 \leq \mu_A^3(x) + \lambda_A^2(x) \leq 1$
(6) (n; m)- fuzzy set in X (See [15]), if it satisfies : $0 \leq \mu_A^n(x) + \lambda_A^m(x) \leq 1$
(7) SR-Fuzzy set fuzzy set in X (See [20]), if it satisfies : $0 \leq \mu_A^2(x) + \sqrt{\lambda_A(x)} \leq 1$

3. $(\frac{1}{2}, \frac{1}{2})$-fuzzy QS-(sub-algebra)ideal on QS-algebras

Definition 3.1 Let X be a non empty set, an $(\frac{1}{2}, \frac{1}{2})$-fuzzy set A of X is an object of the form $A = \{(x, \mu_A(x), \lambda(x)) | x \in X\}$,
where $\mu_A(x), \lambda_A(x) : X \to [0,1]$ are fuzzy sets in X, such that $-1 \leq \sqrt{\mu}(x) + \sqrt{\lambda}(x) \leq 1$. i.e

$$0 \leq \sqrt{\mu_A(x)}^{(+)} + \sqrt{\lambda_A(x)}^{(+)} \leq 1, -1 \leq \sqrt{\mu_A(x)}^{(-)} + \sqrt{\lambda_A(x)}^{(-)} \leq 0.$$

Example 3.2. Let $X = \{0, a, b, c\}$ be a set. Define $A(x) = \{(x, \mu(x), \lambda(x)) : x \in X\}$ as follows: $A(0) = \{0.06, 0.01\}$, $A(a) = \{0.05, 0.02\}$, $A(b) = \{(0.04, 0.03)\}$, $A(c) = \{0.03, 0.04\}$, then A is $(\frac{1}{2}, \frac{1}{2})$-fuzzy se for all $x \in X$.
Since the square root of fuzzy real numbers have two values , then for $A(0) = \{0.06, 0.01\}$, we have

$$\sqrt{0.06} = \pm 0.24494897 \qquad \textit{and} \qquad \sqrt{0.01} = \pm 0.1$$

We denote $\sqrt{0.06}^{(+)} = +0.24494897, \sqrt{0.06}^{(-)} = -0.24494897, \sqrt{0.01}^{(+)} = +0.1$, $\sqrt{0.01}^{(-)} = -0.1$
From the Table (2) ,we show that $-1 \leq \sqrt{\mu(x)} + \sqrt{\lambda(x)} \leq 1$ fore $A(0) = \{0.06, 0.01\}$.

Table (2)

$A(0) = \{0.06, 0.01\}$	$\sqrt{0.06} + \sqrt{0.01}$
Case 1: $\sqrt{0.06}^{(+)} + \sqrt{0.01}^{(+)}$	$0.2449489742783178 + 0.1 \leq 1$
Case 2: $\sqrt{0.06}^{(+)} + \sqrt{0.01}^{(-)}$	$0.2449489742783178 - 0.1 \leq 1$
Case 3: $\sqrt{0.06}^{(-)} + \sqrt{0.01}^{(+)}$	$-0.2449489742783178 + 0.1 > -1$
Case 4: $\sqrt{0.06}^{(-)} + \sqrt{0.01}^{(-)}$	$-0.2449489742783178 - 0.1 > -1$

In the same way we can easy show that :
If $A(a) = \{0, 05, 0.02\}$, then $-1 \leq \sqrt{0.05} + \sqrt{0.02} \leq 1$
If $A(b) = \{(0, 04, 0.03)\}$, then $-1 \leq \sqrt{0.04} + \sqrt{0.03} \leq 1$
If $A(c) = \{0, 04, 0.03\}$, then $-1 \leq \sqrt{0.03} - \sqrt{0.04} \leq 1$,
then $A = \{(x, \mu(x), \lambda(x)) : x \in X\}$ is $(\frac{1}{2}, \frac{1}{2})$-fuzzy set.

Definition 3.3. Let $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x), \lambda_B(x)) : x \in X\}$ be two $(frac12, \frac{1}{2})$-fuzzy sets of X, then we say:
1. $A \subseteq B$ if and only if :
(1) $\sqrt{\mu_A(x)}^{(+)} \leq \sqrt{\mu_B(x)}^{(+)}, \sqrt{\mu_A(x)}^{(-)} \geq \sqrt{\mu_B(x)}^{(-)}$ and

$\sqrt{\lambda_A(x)}^{(+)} \geq \sqrt{\lambda_B(x)}^{(+)}, \sqrt{\lambda_A(x)}^{(-)} \leq \sqrt{\lambda_B(x)}^{(-)}$

Example 3.4. Let $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\} = \{0.2, 0.3\}$ and $B = \{(x, \mu_B(x), \lambda_B(x)) : x \in X\} = \{0.3, 0.2\}$ be two $(frac12, \frac{1}{2})$-fuzzy sets of X, then $A \subseteq B$. Since
$\sqrt{\mu_A(2)}^{(+)} = 0.4472135954999579 < \sqrt{\mu_B(3)}^{(+)} = 0.5477225575051661$
$\sqrt{\mu_A(2)}^{(-)} = -0.4472135954999579 > \sqrt{\mu_B(3)}^{(-)} = -0.5477225575051661$
and
$\sqrt{\lambda_A(3)}^{(+)} = 0.5477225575051661 \geq \sqrt{\lambda_B(2)}^{(+)} = 0.4472135954999579,$
$\sqrt{\lambda_A(3)}^{(-)} = 0.5477225575051661 < \sqrt{\lambda_B(2)}^{(-)} = -0.4472135954999579$

2. $A = B$ if and only if
$\sqrt{\mu_A(x)}^{(+)} = \sqrt{\mu_B(x)}^{(+)}, \sqrt{\mu_A(x)}^{(-)} = \sqrt{\mu_B(x)}^{(-)}$ and

$\sqrt{\lambda_A(x)}^{(+)} = \sqrt{\lambda_B(x)}^{(+)}, \sqrt{\lambda_A(x)}^{(-)} = \sqrt{\lambda_B(x)}^{(-)}$

3. $A^c(x) = \{(x, \mu^c(x), \lambda^c(x)) : x \in X\}$ if and only if
$(\sqrt{\mu_A(x)}^{(+)})^c = 1 - \sqrt{\mu_A(x)}^{(+)}, (\sqrt{\mu_A(x)}^{(-)})^c = -1 - \sqrt{\mu_A(x)}^{(-)}$

$(\sqrt{\lambda_A(x)}^{(+)})^c = 1 - \sqrt{\lambda_A(x)}^{(+)}, (\sqrt{\lambda_A(x)}^{(-)})^c = -1 - \sqrt{\lambda_A(x)}^{(-)}$

Definition 3.5. Let $X \neq \Phi$ be QS-algebras, then the fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ over a set X is called $(\frac{1}{2}, \frac{1}{2})$-fuzzy QS-sub algebras if the following are holed:

$((\frac{1}{2}, \frac{1}{2}) - S)$ (a) $\sqrt{\mu_A(x*z)}^{(+)} \geq \min\{\sqrt{\mu_A(x)}^{(+)}, \sqrt{\mu_A(z)}^{(+)}\}$,
$\sqrt{\mu_A(x*z)}^{(-)} \geq \max\{\sqrt{\mu_A(x)}^{(-)}, \sqrt{\mu_A(z)}^{(-)}\}$ and
$((\frac{1}{2}, \frac{1}{2}) - S)$ (b) $\sqrt{\lambda_A(x*z)}^{(+)} \leq \max\{\sqrt{\lambda_A(x)}^{(+)}, \sqrt{\lambda_A(z)}^{(+)}\}$,
$\sqrt{\lambda_A(x*z)}^{(-)} \geq \min\{\sqrt{\lambda_A(x)}^{(-)}, \sqrt{\lambda_A(z)}^{(-)}\}$ and
where $\mu_A(x), \lambda_A(x) : x \to [-1, 1]$ such that $-1 \leq \sqrt{\mu_A(x)} + \sqrt{\lambda_A(x)} \leq 1$,

Definition 3.6. Let $X \neq \Phi$ be QS-algebras, then the $(\frac{1}{2}, \frac{1}{2})$-fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ over a set X is called $(\frac{1}{2}, \frac{1}{2})$-fuzzy BCK-ideal if the following are holed:

$((\frac{1}{2},\frac{1}{2})-bck)$ (a) $\sqrt{\mu_A(0)} \geq \sqrt{\mu_A(x)}^{(+)}, \sqrt{\mu_A(0)} \leq \sqrt{\mu_A(x)}^{(-)}$,

$$\sqrt{\lambda_A(0)} \leq \sqrt{\lambda_A(x)}^{(+)}, \sqrt{\lambda_A(0)} \geq \sqrt{\lambda_A(x)}^{(-)},$$

$((\frac{1}{2},\frac{1}{2})-bck)$ (b) $\sqrt{\mu_A(x)}^{(+)} \geq \min\{\sqrt{\mu_A(x*z)}^{(+)}, \sqrt{\mu_A(y)}^{(+)}\}$,

$$\sqrt{\mu_A(x)}^{(-)} \leq \max\{\sqrt{\mu_A(x*y)}^{(-)}, \sqrt{\mu_A(y)}^{(-)}\}$$

$$\sqrt{\lambda_A(x)}^{(+)} \leq \max\{\sqrt{\lambda_A(x*y)}^{(+)}, \sqrt{\lambda_A(y)}^{(+)}\},$$

$$\sqrt{\lambda_A(x)}^{(-)} \geq \min\{\sqrt{\lambda_A(x*y)}^{(-)}, \sqrt{\lambda_A(y)}^{(-)}\}$$

such that $-1 \leq \sqrt{\mu_A(x)} + \sqrt{\lambda_A(x)} \leq 1$

Definition 3.7. Let $X \neq \Phi$ be QS-algebras, then the $(\frac{1}{2},\frac{1}{2})$-fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ over a set X is called $(\frac{1}{2},\frac{1}{2})$-fuzzy QS-ideal if the following are holed:

$((\frac{1}{2},\frac{1}{2})-QSI_0)$: $\sqrt{\mu_A(0)} \geq \sqrt{\mu_A(x)}^{(+)}, \sqrt{\mu_A(0)} \leq \sqrt{\mu_A(x)}^{(-)}$,

$$\sqrt{\lambda_A(0)} \leq \sqrt{\lambda_A(x)}^{(+)}, \sqrt{\lambda_A(0)} \geq \sqrt{\lambda_A(x)}^{(-)},$$

$((\frac{1}{2},\frac{1}{2})-QSI_1)$: $\sqrt{\mu_A(x*y)}^{(+)} \geq \min\{\sqrt{\mu_A(x*z)}^{(+)}, \sqrt{\mu_A(z*y)}^{(+)}\}$,

$$\sqrt{\mu_A(x*y)}^{(-)} \leq \max\{\sqrt{\mu_A(x*z)}^{(-)}, \sqrt{\mu_A(z*y)}^{(-)}\}$$

$((\frac{1}{2},\frac{1}{2})-QSI_2)$: $\sqrt{\lambda_A(x*y)}^{(+)} \leq \max\{\sqrt{\lambda_A(x*z)}^{(+)}, \sqrt{\lambda_A(z*y)}^{(+)}\}$,

$$\sqrt{\lambda_A(x*y)}^{(-)} \geq \min\{\sqrt{\lambda_A(x*z)}^{(-)}, \sqrt{\lambda_A(z*y)}^{(-)}\}$$

such that $-1 \leq \sqrt{\mu_A(x)} + \sqrt{\lambda_A(x)} \leq 1$

Example 3.8. Let $X = \{0, 1, 2, 3\}$ be a set with a binary operation $*$ defined by the following Table 3 :

Table 3

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

We can prove that $(X, *, 0)$ is a QS-algebra.

Define $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ as follows:

$A(0) = \{0.04, 0.01\}, A(1) = \{0.03, 0.02\}$

$A(2) = \{0.02, 0.02\}, A(3) = \{0.01, 0.01\}$

It is easy to check that $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ is $(\frac{1}{2},\frac{1}{2})$-fuzzy QS-ideal on QS-algenra X.

Lemma 3.9. If $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ is $(\frac{1}{2},\frac{1}{2})$-fuzzy QS-sub algebra on QS-algenra X, then

$\sqrt{\mu_A(0)} \geq \sqrt{\mu_A(x)}^{(+)}, \sqrt{\mu_A(0)} \leq \sqrt{\mu_A(x)}^{(-)}, sqrt\lambda_A(0) \leq \sqrt{\lambda_A(x)}^{(+)}$,

$\sqrt{\lambda_A(0)} \geq \sqrt{\lambda_A(x)}^{(-)}$

Proof. In (Definition 3.5) put $x = z$,

$((\frac{1}{2},\frac{1}{2})$-QS) (a) :

$$\sqrt{\mu_A\overbrace{(x*x)}^{0}}^{(+)} = \sqrt{\mu_A(0)} \geq \min\{\sqrt{\mu_A(x)}^{(+)}, \sqrt{\mu_A(x)}^{(+)}\} = \sqrt{\mu_A(x)}^{(+)},$$
$$\sqrt{\mu_A\overbrace{(x*x)}^{0}}^{(-)} = \sqrt{\mu_A(0)} \leq \max\{\sqrt{\mu_A(x)}^{(-)}, \sqrt{\mu_A(x)}^{(-)}\} = \sqrt{\mu_A(x)}^{(-)}$$

Similar, we can prove that from $((\frac{1}{2}, \frac{1}{2})$-QS) (b) :

$$\sqrt{\lambda_A\overbrace{(x*x)}^{0}}^{(+)} = \sqrt{\lambda_A(0)} \leq \max\{\sqrt{\lambda_A(x)}^{(+)}, \sqrt{\lambda_A(x)}^{(+)}\} = \sqrt{\lambda_A(x)}^{(+)},$$
$$\sqrt{\lambda_A\overbrace{(x*x)}^{0}}^{(-)} = \sqrt{\lambda_A(0)} \geq \min\{\sqrt{\lambda_A(x)}^{(-)}, \sqrt{\lambda_A(x)}^{(-)}\} = \sqrt{\lambda_A(x)}^{(-)}$$

This completes the proof. This completes the proof.

Lemma 3.10. If $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ is $(\frac{1}{2}, \frac{1}{2})$-fuzzy QS-ideal of X. If $x \leq z$ in X, then

$sqrt\lambda_A(x) \leq \sqrt{\lambda_A(z)}, \sqrt{\mu_A(x)} \geq \sqrt{\mu_A(z)},$

$\sqrt{\mu_A(x)}^{(+)} \geq \sqrt{\mu_A(z)}^{(+)}, \sqrt{\mu_A(x)}^{(-)} \leq \sqrt{\mu_A(z)}^{(-)}$ and

$\sqrt{\lambda_A(x)}^{(+)} \leq \sqrt{\lambda_A(z)}^{(+)}, \sqrt{\lambda_A(z)}^{(-)} \geq \sqrt{\lambda_A(z)}^{(-)}$

for all $x, y \in X$

Proof. Let $x, y \in X$ be such that $x \leq z$, then $x * z = 0$. From (Definition 3.7) put $y = 0$. We have from $((\frac{1}{2}, \frac{1}{2})$-QSI$_0$)

$\sqrt{\mu_A(0)} \geq \sqrt{\mu_A(x)}^{(+)}, \sqrt{\mu_A(0)} \leq \sqrt{\mu_A(x)}^{(-)}, \sqrt{\lambda_A(0)} \leq \sqrt{\lambda_A(x)}^{(+)},$

$\sqrt{\lambda_A(0)} \geq \sqrt{\lambda_A(x)}^{(-)}$

from $((\frac{1}{2}, \frac{1}{2})$-QSI$_1$): $\sqrt{\mu_A\overbrace{(x*0)}^{x}}^{(+)} \geq \min\{\sqrt{\mu_A\overbrace{(x*z)}^{0}}^{(+)}, \sqrt{\mu_A\overbrace{(z*0)}^{z}}^{(+)}\} i.e$

$$\sqrt{\mu_A(x)}^{(+)} \geq \min\{\sqrt{\mu_A(0)}^{(+)}, \sqrt{\mu_A(z)}^{(+)}\} = \sqrt{\mu_A(z)}^{(+)}$$

Similar, we can prove that $\sqrt{\mu_A(x)}^{(-)} \leq \max\{\sqrt{\mu_A(0)}^{(-)}, \sqrt{\mu_A(z)}^{(-)}\} = \sqrt{\mu_A(z)}^{(-)},$

from $((\frac{1}{2}, \frac{1}{2})$-QSI$_2$) : $\sqrt{\lambda_A\overbrace{(x*0)}^{x}}^{(+)} \leq \max\{\sqrt{\lambda_A\overbrace{(x*z)}^{0}}^{(+)}, \sqrt{\lambda_A\overbrace{(z*0)}^{z}}^{(+)}\} i.e$

$\sqrt{\lambda_A(x)}^{(+)} \leq \max\{\sqrt{\lambda_A(0)}^{(+)}, \sqrt{\lambda_A(z)}^{(+)}\} = \sqrt{\lambda_A(z)}^{(+)}$, and Similar, we can prove that $\sqrt{\lambda_A(x)}^{(-)} \geq \min\{\sqrt{\lambda_A(0)}^{(-)}, \sqrt{\lambda_A(z)}^{(-)}\} = \sqrt{\lambda_A(z)}^{(-)}$.

This completes the proof.

Lemma 3.11. let $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ be$(\frac{1}{2}, \frac{1}{2})$-fuzzy QS-ideal of X, if the inequality $x * y \leq z$ hold in X, then

$\sqrt{\mu_A(x)}^{(+)} \geq \min\{\sqrt{\mu_A(y)}^{(+)}, \sqrt{\mu_A(z)}^{(+)}\}, \sqrt{\mu_A(x)}^{(-)} \leq \max\{\sqrt{\mu_A(y)}^{(-)}, \sqrt{\mu_A(z)}^{(-)}\}$ and

$\sqrt{\lambda_A(x)}^{(+)} \leq \max\{\sqrt{\lambda_A(y)}^{(+)}, \sqrt{\lambda_A(z)}^{(+)}\} \geq \min\{\sqrt{\lambda_A(y)}^{(-)}, \sqrt{\lambda_A(z)}^{(-)}\}$

Proof. Assume that the inequality $x * yaleqz$ hence $x * z \leq y$ then holds in X .Then by (Lemma 3. 10),we have. $\sqrt{\mu_A((x*z)}^{(+)} \geq \sqrt{\mu_A(y)}^{(+)}, \sqrt{\mu_A(x*z)}^{(-)} \leq \sqrt{\mu_A(y)}^{(-)}$ and

$\sqrt{\lambda_A(x*z)}^{(+)} \leq \sqrt{\lambda_A(y)}^{(+)}, \sqrt{\lambda_A(x*z)} \geq \sqrt{\lambda_A(y)}^{(-)}$

Thus, put $y = 0$ in (definition 3.7.) and using lemma 3.10,then we get from $((\frac{1}{2}, \frac{1}{2})$-QSI$_0$):

$\sqrt{\mu_A(0)} \geq \sqrt{\mu_A(x)}^{(+)}, \sqrt{\mu_A(0)} \leq \sqrt{\mu_A(x)}^{(-)}, \sqrt{\lambda_A(0)} \geq \sqrt{\lambda_A(x)}^{(+)}, \sqrt{\lambda_A(0)} \geq \sqrt{\lambda_A(x)}^{(-)}$

and

$((\frac{1}{2},\frac{1}{2})\text{-QSI}_1)$:
$\sqrt{\mu_A(x*0)}^{(+)} = \sqrt{\mu_A(x)}^{(+)} \geq \min\{\sqrt{\mu_A(x*z)}^{(+)}, \sqrt{\mu_A(z*0)}^{(+)}\}$
$= \min\{\sqrt{\mu_A(x*z)}^{(+)} = \sqrt{\mu_A(z)}^{(+)}\} \geq \min\{\sqrt{\mu_A(y)}^{(+)}, \sqrt{\mu_A(z)}^{(+)}\}$
Similar, we can prove that
$\sqrt{\mu_A(x)}^{(-)} \leq \max\{\sqrt{\mu_A(y)}^{(-)}, \sqrt{\mu_A(z)}^{(-)}\}$ and
$\sqrt{\lambda_A(x)}^{(+)} \leq \max\{\sqrt{\lambda_A(y)}^{(+)}, \sqrt{\lambda_A(z)}^{(+)}\}, \sqrt{\lambda_A(x)}^{(-)} \geq \min\{\sqrt{\lambda_A(y)}^{(-)}, \sqrt{\lambda_A(z)}^{(-)}\}$
This completes the proof.

Theorem 3.12. In BCK-algebra, every $(\frac{1}{2},\frac{1}{2})$-fuzzy BCK-ideal of X is $(\frac{1}{2},\frac{1}{2})$-fuzzy QS-ideal of X.

proof. Let $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ be $(\frac{1}{2},\frac{1}{2})$-fuzzy BCK-ideal of X and $x, y, z \in X$, then
$\sqrt{\mu_A(0)} \geq \sqrt{\mu_A(x)}^{(+)}, \sqrt{\mu_A(0)} \leq \sqrt{\mu_A(x)}^{(-)}, \sqrt{\lambda_A(0)} \leq \sqrt{\lambda_A(x)}^{(+)}, \sqrt{\lambda_A(0)} \geq \sqrt{\lambda_A(x)}^{(-)}$,
$((\frac{1}{2},\frac{1}{2})$-bck): (a) $\sqrt{\mu_A(x)}^{(+)} \geq \min\{\sqrt{\mu_A(x*y)}^{(+)} \sqrt{\mu_A(y)}^{(+)}\}$,
$\sqrt{\mu_A(x)}^{(-)} \leq \max\{\sqrt{\mu_A(x*y)}^{(-)} \sqrt{\mu_A(y)}^{(-)}\}$
$((\frac{1}{2},\frac{1}{2})$-bck): (b) $\sqrt{\lambda_A(x)}^{(+)} \leq \max\{\sqrt{\lambda_A(x*y)}^{(+)} \sqrt{\lambda_A(y)}^{(+)}\}$,
$\sqrt{\lambda_A(x)}^{(-)} \geq \min\{\sqrt{\lambda_A(x*y)}^{(-)} \sqrt{\lambda_A(y)}^{(-)}\}$
such that $-1 \leq \sqrt{\mu_A(x)} + \sqrt{\lambda_A(x)} \leq 1$
put $x*y$ instate of x, and $z*y$ instate of y in $((\frac{1}{2},\frac{1}{2})$-bck): (a) and $((\frac{1}{2},\frac{1}{2})$-bck): (b) we have

$$\sqrt{\mu_A(x*y)}^{(+)} \geq \min\{\sqrt{\mu_A(\overbrace{(x*y)*(z*y)}^{(x*y)*(z*y)\leq x*z \text{by proposition 2.3(4)}})}^{(+)}, \sqrt{\mu_A(z*y)}^{(+)}\} \geq,$$

$$\min\{\sqrt{\mu_A(\overbrace{(x*z)}^{\text{Lemma 3.10}})}^{(+)}, \sqrt{\mu_A(z*y)}^{(+)}\}$$

i.e. $\sqrt{\mu_A(x*y)}^{(+)} \geq \min\{\sqrt{\mu_A(x*z)}^{(+)}, \sqrt{\mu_A(z*y)}^{(+)}\}$
Similar, we can prove that
$\sqrt{\mu_A(x*y)}^{(-)} \leq \min\{\sqrt{\mu_A(x*z)}^{(-)}, \sqrt{\mu_A(z*y)}^{(-)}\}$
$\sqrt{\lambda_A(x*y)}^{(+)} \leq \max\{\sqrt{\lambda_A(x*z)}^{(+)}, \sqrt{\lambda_A(z*y)}^{(+)}\}$ and
$\sqrt{\lambda_A(x*y)}^{(-)} \geq \min\{\sqrt{\lambda_A(x*z)}^{(-)}, \sqrt{\lambda_A(z*y)}^{(-)}\}$;
then $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ is $(\frac{1}{2},\frac{1}{2})$-fuzzy QS-ideal of X.
This completes the proof.

Theorem 3.13. If $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ is $(\frac{1}{2},\frac{1}{2})$-fuzzy QS-ideal of X, then the set
$X = \{(x, \sqrt{\mu_A(x)}^{(+)} = \sqrt{\mu_A(0)}^{(+)}, \sqrt{\mu_A(x)}^{(-)} = \sqrt{\mu_A(0)}^{(-)}$ and
$\sqrt{\lambda_A(x)}^{(+)} = \sqrt{\lambda_A(0)}^{(+)}, \sqrt{\lambda_A(x)}^{(-)} = \sqrt{\lambda_A(0)}^{(-)} | x \in X\}$
is a QS-ideal of X.

Proof. Let $x*z, z*y \in X$. Then
$\sqrt{\mu_A(x*z)}^{(+)} = \sqrt{\mu_A(0)}^{(+)} = \sqrt{\mu_A(x*y)}^{(+)}, \sqrt{\mu_A(x*z)}^{(-)} = \sqrt{\mu_A(0)}^{(-)} = \sqrt{\mu_A(z*y)}^{(-)}$,
$\sqrt{\lambda_A(x*z)}^{(+)} = \sqrt{\lambda_A(0)}^{(+)} = \sqrt{\lambda_A(z*y)}^{(+)}, \sqrt{\lambda_A(x*z)}^{(-)} = \sqrt{\lambda_A(0)}^{(-)} = \sqrt{\lambda_A(z*y)}^{(-)}$,
and
$((\frac{1}{2},\frac{1}{2})$ QSI1) $\sqrt{\mu_A(x*y)}^{(+)} \geq \min\{\sqrt{\mu_A(x*z)}^{(+)}, sqrt\mu_A(z*y)^{(+)}\} =$
$\min\{\sqrt{\mu_A(0)}^{(-)}, \sqrt{\mu_A(0)}^{(-)}\} = \sqrt{\mu_A(0)}^{(-)}$ and
$\sqrt{\lambda_A(x*y)}^{(+)} \leq \max\{\sqrt{\lambda_A(x*z)}^{(+)}, sqrt\lambda_A(z*y)^{(+)}\} =$
$\max\{\sqrt{\lambda_A(0)}^{(+)}, \sqrt{\lambda_A(0)}^{(+)}\} = \sqrt{\lambda_A(0)}^{(+)}$,

$\sqrt{\lambda_A(x*y)}^{(-)} \geq \min\{\sqrt{\lambda_A(x*z)}^{(-)}, \sqrt{\lambda_A(z*y)}^{(-)}\} =$
$\max\{\sqrt{\lambda_A(0)}^{(-)}, \sqrt{\lambda_A(0)}^{(-)}\} = \sqrt{\lambda_A(0)}^{(-)}$, that is $x*y \in X$
Combining this with conditions of definition 3.7.
$\sqrt{\mu_A(0)}^{(+)} \geq \sqrt{\mu_A(x)}^{(+)}, \sqrt{\mu_A(0)}^{(-)} \geq \sqrt{\mu_A(x)}^{(-)}$
$\sqrt{\lambda_A(0)}^{(+)} \leq \sqrt{\lambda_A(x)}^{(+)}, \sqrt{\lambda_A(0)}^{(-)} \geq \sqrt{\lambda_A(x)}^{(-)}$
we get
$\sqrt{\mu_A(x*z)}^{(+)} = \sqrt{\mu_A(0)}^{(+)}, \sqrt{\mu_A(x*z)}^{(-)} = \sqrt{\mu_A(0)}^{(-)}$ $\sqrt{\lambda_A(x*z)}^{(+)} = \sqrt{\lambda_A(0)}^{(+)}$, $\sqrt{\lambda_A(x*z)}^{(-)} = \sqrt{\lambda_A(0)}^{(-)}$ that is $x*z \in X$. Hence X is a QS-ideal of X.

Proposition 3.14. $(M_i)_{i\in\Lambda} = \{(x(\sqrt{\mu})_{M_i}(x), (\sqrt{\lambda})_{M_i}(x)|x \in X\}$ be a family of fuzzy $(\frac{1}{2},\frac{1}{2})$-QS-ideal of X, then $\bigcap\limits_{i\in\Lambda} M_i$ is $(\frac{1}{2},\frac{1}{2})$-fuzzy QS-ideal of a QS-algebra X. **Proof.** Let $(M_i)_{i\in\Lambda}$ be a family of $(\frac{1}{2},\frac{1}{2})$-fuzzy QS-ideal of a QS-algebra, then for any $x,y,z \in X$, we get
$(\bigcap(\sqrt{\mu_A(0)}^{(+)})_{M_i}) = \inf((\sqrt{\mu_A(0)}^{(+)})_{M_i}) \geq \inf((\sqrt{\mu_A(x)}^{(+)})_{M_i}) =$
$(\bigcap(\sqrt{\mu_A(x)}^{(+)})_{M_i})$
$(\bigcup(\sqrt{\mu_A(0)}^{(-)})_{M_i}) = \sup((\sqrt{\mu_A(0)}^{(-)})_{M_i}) \leq \sup((\sqrt{\mu_A(x)}^{(-)})_{M_i}) = (\bigcup(\sqrt{\mu_A(x)}^{(-)})_{M_i})$
$(\bigcup(\sqrt{\lambda_A(0)}^{(+)})_{M_i}) = \sup((\sqrt{\lambda_A(0)}^{(+)})_{M_i}) \leq \sup((\sqrt{\lambda_A(x)}^{(+)})_{M_i}) = (\bigcup(\sqrt{\lambda_A(x)}^{(+)})_{M_i})$,
$(\bigcap(\sqrt{\lambda_A(0)}^{(-)})_{M_i}) = \inf((\sqrt{\lambda_A(0)}^{(-)})_{M_i}) \geq$
$\inf((\sqrt{\lambda_A(x)}^{(-)})_{M_i}) = (\bigcap(\sqrt{\lambda_A(x)}^{(-)})_{M_i})$,
Now
$(\bigcap(\sqrt{\mu_A(x*z)}^{(+)})_{M_i}) = \inf((\sqrt{\mu_A(x*y)}^{(+)})_{M_i}) \geq$
$\inf\{\min\{(\sqrt{\mu_A(x*z)}^{(+)})_{M_i}, (\sqrt{\mu_A(z*y)}^{(+)})_{M_i}\}\} =$
$\min\{\inf\{(\sqrt{\mu_A(x*z)}^{(+)})_{M_i}, \inf\{(\sqrt{\mu_A(z*y)}^{(+)})_{M_i}\}\}\} =$
$\min\{(\bigcap(\sqrt{\mu_A(x*z)}^{(+)})_{M_i}), (\bigcap(\sqrt{\mu_A(z*y)}^{(+)})_{M_i})\}\}$,
$(\bigcup(\sqrt{\mu_A(x*y)}^{(-)})_{M_i}) = \sup((\sqrt{\mu_A(x*y)}^{(-)})_{M_i}) \leq$
$\sup\{\max\{(\sqrt{\mu_A(x*z)}^{(-)})_{M_i}, (\sqrt{\mu_A(z*y)}^{(-)})_{M_i}\}\} =$
$\max\{\sup\{(\sqrt{\mu_A(x*z)}^{(-)})_{M_i}, \sup\{(\sqrt{\mu_A(z*y)}^{(-)})_{M_i}\}\}\} =$
$\max\{(\bigcup(\sqrt{\mu_A(x*z)}^{(-)})_{M_i}), (\bigcup((\sqrt{\mu_A(z*y)}^{(-)})_{M_i})\}\}$
and
$(\bigcup(\sqrt{\lambda_A(x*y)}^{(+)})_{M_i}) = \sup((\sqrt{\lambda_A(x*y)}^{(+)})_{M_i})) \leq$
$\sup\{\max\{\{(\sqrt{\lambda_A(x*z)}^{(+)})_{M_i}), (\sqrt{\lambda_A(z*y)}^{(+)})_{M_i})\}\} =$
$\max\{\sup\{(\sqrt{\lambda_A(x*z)}^{(+)})_{M_i}), \sup\{(\sqrt{\lambda_A(z*y)}^{(+)})_{M_i})\}\}\} =$
$\max\{(\bigcup(\sqrt{\lambda_A(x*z)}^{(+)})_{M_i})), (\bigcup(\sqrt{\lambda_A(z*y)}^{(+)})_{M_i}))\}\}\}$,
$(\bigcap(\sqrt{\lambda_A(x*y)}^{(-)})_{M_i}) = \inf((\sqrt{\lambda_A(x*y)}^{(-)})_{M_i})) \geq$
$\inf\{\min\{\{(\sqrt{\lambda_A(x*z)}^{(-)})_{M_i}), (\sqrt{\lambda_A(z*y)}^{(-)})_{M_i})\}\} =$
$\min\{\inf\{(\sqrt{\lambda_A(x*z)}^{(-)})_{M_i}), \inf\{(\sqrt{\lambda_A(z*y)}^{(-)})_{M_i})\}\}\} =$
$\min\{(\bigcap(\sqrt{\lambda_A(x*z)}^{(-)})_{M_i})), (\bigcap(\sqrt{\lambda_A(z*y)}^{(-)})_{M_i}))\}\}\}$,
Hence $\bigcap\limits_{i\in\Lambda} M_i$ is $(\frac{1}{2},\frac{1}{2})$- fuzzy QS-ideal of a QS-algebra X. This completes the proof.

4. Image (Pre-image) $(\frac{1}{2},\frac{1}{2})$- fuzzy QS-ideal under homomorphism of QS-algebras

Definition 4.1 Let $(X,*,0)$ and $(Y,*',0')$ be QS-algebras. A mapping $f: X \to Y$ is said to be a homomorphism if $f(x*y) = f(x)*' f(y)$ for all $x,y \in X$.

Definition 4.2 Let X and Y be two QS-algebras, μ a fuzzy subset of X, β a fuzzy subset of Y, and $f : X \to Y$ a QS-homomorphism.
The image of μ under f, denoted by $f(\mu)$, is a fuzzy set of Y defined by

$$f(\mu)(y) = \begin{cases} \sup\limits_{x\in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) \neq \phi \\ 0, & \text{otherwise} \end{cases}$$

The pre-image of β under f denoted $f^{-1}(\beta)$ is a fuzzy set of X defined by: For all $x \in X, f^{-1}(\beta)(x) = \beta(f(x))$.

Let $f : X \to Y$ be a homomorphism of QS-algebras. For any $(\frac{1}{2}, \frac{1}{2})$-fuzzy QS-ideals $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$ in Y, we define new $(\frac{1}{2}, \frac{1}{2})$-fuzzy QS-ideals $A^f = \{(x, \mu_A(x))^f, (\lambda_A(x))^f \mid x \in X\}$ in X defined by:
$(\sqrt{\mu_A(x)}^{(+)})^f = \sqrt{\mu_A(f(x)}^{(+)})$, and
$(\sqrt{\lambda_A(x)}^{(+)})^f = \sqrt{\lambda_A(f(x)}^{(+)})$, for all $x \in X$.

Theorem 4.3 Let $f : X \to Y$ be a homomorphism of QS-algebras. If $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$ is a $(\frac{1}{2}, \frac{1}{2})$-fuzzy QS-ideal of Y, then $A^f = \{(x, \mu_A(x))^f, (\lambda_A(x)))^f \mid x \in X\}$ is a $(\frac{1}{2}, \frac{1}{2})$-fuzzy QS-ideal of X.

Proof. Not that: (1) $(\sqrt{\mu_A(x)}^{(+)})^f := \sqrt{\mu_A(f(x)}^{(+)}) \leq \sqrt{\mu_A(f(0)}^{(+)}) =$
$(\sqrt{\mu_A(0)})^f(\sqrt{\mu_A(x * y)}^{(+)})^f := (\sqrt{\mu_A(f(x * y))}^{(+)} \geq$
$\min\{\sqrt{\mu_A((f(x) * f(z))}^{(+)}, \sqrt{\mu_A(f(z) * f(y))}^{(+)}\} =$
$\min\{\sqrt{\mu_A(f(x) * f(z))}^{(+)}, \sqrt{\mu_A(f(z) * f(y))}^{(+)}\} =$
$\min\{(\sqrt{\mu_A((x * z)}^{(+)})^f, (\sqrt{\mu_A(z * y)}^{(+)})^f\} =$
$\min\{\sqrt{\mu_A((f(x * z))}^{(+)}, \sqrt{\mu_A(f(z * y))}^{(+)}\} =$
$(\sqrt{\mu_A(x)}^{(-)})^f := \sqrt{\mu_A(f(x))}^{(-)} \geq \sqrt{\mu_A(f(0))}^{(-)} = \sqrt{\mu_A(0)})^f$
$(\sqrt{\mu_A(x * y)}^{(-)})^f := (\sqrt{\mu_A(f(x * y))}^{(-)} \geq$
$\max\{\sqrt{\mu_A((f(x) * f(z))}^{(-)}, \sqrt{\mu_A(f(z) * f(y))}^{(-)}\} =$
$\max\{\sqrt{\mu_A(f(x) * f(z))}^{(-)}, \sqrt{\mu_A(f(z) * f(y))}^{(-)}\} =$
$\max\{(\sqrt{\mu_A(x * z)}^{(-)})^f, (\sqrt{\mu_A(z * y)}^{(-)})^f\} =$
$\max\{\sqrt{\mu_A(f(x * z))}^{(-)}, \sqrt{\mu_A(f(z * y))}^{(-)}\} =$
(2) $(\sqrt{\lambda_A(x)}^{(+)})^f := \sqrt{\lambda_A(f(x))}^{(+)} \geq \sqrt{\lambda_A(f(0))}^{(+)} = (\sqrt{\lambda_A(0)}^{(+)})^f$
$(\sqrt{\lambda_A(x * y)}^{(+)})^f := \sqrt{\lambda_A(f(x * y))}^{(+)} \geq$
$\max\{\sqrt{\lambda_A((f(x) * f(z))}^{(+)}, \sqrt{\lambda_A(f(z) * f(y))}^{(+)}\} =$
$\max\{\sqrt{\lambda_A(f(x * z))}^{(+)}, \sqrt{\lambda_A(f(z * y))}^{(+)}\} =$
$\max\{(\sqrt{\lambda_A(x * z)}^{(+)})^f, (\sqrt{\lambda_A(z * y)}^{(+)})^f\}$.
$(\sqrt{\lambda_A(x)}^{(-)})^f := \sqrt{\lambda_A(f(x))}^{(-)} \geq \sqrt{\lambda_A(f(0))}^{(-)} = (\sqrt{\lambda_A(0)}^{(-)})^f$
$(\sqrt{\lambda_A(x * y)}^{(-)})^f, \sqrt{\lambda_A(x * y)}^{(-)} \geq$
$\min\{\sqrt{\lambda_A((f(x) * f(z))}^{(-)}, \sqrt{\lambda_A(f(z) * f(y))}^{(-)}\} =$
$\min\{\sqrt{\lambda_A(f(x * z))}^{(-)}, \sqrt{\lambda_A(f(z * y))}^{(-)}\} =$
$\min\{(\sqrt{\lambda_A(x * z)}^{(-)})^f, (\sqrt{\lambda_A(z * y)}^{(-)})^f\}$
Hence, $A^f = \{(x, (\mu_A(x))^f, (\lambda_A(x))^f \mid x \in X\}$ is $(\frac{1}{2}, \frac{1}{2})$-fuzzy QS-ideal of X.

Theorem 4.4 Let $f : X \to Y$ be a epimorphism of QS-algebras. If $A = \{(x, \mu_A(x))^f, , \lambda_A(x))^f \mid x \in X\}$ is $(\frac{1}{2}, \frac{1}{2})$-fuzzy QS-ideal of Y, then $A = \{(x, \mu_A(x), \lambda_A(x) \mid x \in X\}$, is $(\frac{1}{2}, \frac{1}{2})$-fuzzy QS-ideal of Y.

Proof. (1) For any $a \in Y$, there exists $x \in X$ such that $f(x) = a$. Then
$\sqrt{\mu_A(a)} = \sqrt{\mu_A(f(x))} = (\sqrt{\mu_A(x)})^f \geq (\sqrt{\mu_A(0)})^f = \sqrt{\mu_A(f(0))} = (\sqrt{\mu_A(0)})^f$,
Let $a, b, c \in Y$. Then $f(x) = a, f(y) = b, f(z) = c$, for some $x, y, z \in X$. It follows that
$\sqrt{\mu_A(a * b)} = \sqrt{\mu_A(f(x) * f(y))} = (\sqrt{\mu_A(x * y)})^f \geq$
$\min\{(\sqrt{\mu_A(x * z)})^f, (\sqrt{\mu_A(z * y)})^f\} = \min\{\sqrt{\mu_(f(x * z))}, \sqrt{\mu_(f(z * y))}\}$
$= \min\{\sqrt{\mu_A(f(x) * f(z))}, \sqrt{\mu_A(f(z) * f(y))}\}$
$= \min\{\sqrt{\mu_A((f(x) * f(y)) * f(z))}, \sqrt{\mu_A(f(z) * f(y))}\}$
$= \min\{\sqrt{\mu_A((a * c))}, \sqrt{\mu_A(c * b)}\}$,
(2) For any $a \in Y$, there exists $x \in X$ such that $f(x) = a$. Then
$\sqrt{\lambda_A(a)} = \sqrt{\lambda_A(f(x))} = (\sqrt{\lambda_A(x)})^f \geq (\sqrt{\lambda_A(0)})^f = \sqrt{\lambda_A(f(0))} = (\sqrt{\lambda_A(0)})^f$,
Let $a, b, c \in Y$. Then $f(x) = a, f(y) = b, f(z) = c$, for some $x, y, z \in X$. It follows that
$\sqrt{\lambda_A(a * b)} = \sqrt{\lambda_A(f(x) * f(y))} = (\sqrt{\lambda_A(x * y)})^f \geq$
$\max\{(\sqrt{\lambda_A(x * z)})^f, (\sqrt{\lambda_A(z * y)})^f\} = \max\{\sqrt{\lambda_A(f((x * z)}, \sqrt{\lambda_A(f(z * y)}\}$
$\max\{\sqrt{\lambda_A(f(x) * f(z))}, \sqrt{\lambda_A(f(z) * f(y))}\} = \max\{\sqrt{\lambda_A((f(x * z))}, \sqrt{\lambda_A(f(z * y))}\}$
$= \max\{\sqrt{\lambda_A(a * c)}, \sqrt{\lambda_a(c * b)}$, hence $A = \{(x, \mu_A(x), \lambda_A(x) \mid x \in X\}$, is $(\frac{1}{2},\frac{1}{2})$-fuzzy QS-ideals of Y. This completes the proof.

5. Product of $(\frac{1}{2},\frac{1}{2})$-fuzzy QS-ideals

Definition 5.1. Let $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x), \lambda_B(x)) : x \in X\}$ be two SSR-fuzzy sets of X, the Cartesian product $A \times B$ is defined by
$(\sqrt{\mu_A} \times \sqrt{\mu_B})(x, y) = \min\{\sqrt{\mu_A(x)}\sqrt{\mu_B(y)}\}$
$(\sqrt{\lambda_A} \times \sqrt{\lambda_B})(x, y) = \max\{\sqrt{\lambda_A(x)}\sqrt{\lambda_B(y)}\}$ for all $x, y \in X$
Remark 5.2. Let X and Y be QS-algebras, we define $*$ on $X \times Y$ by: For every $(x, y) \in X \times Y$, $(x, y) * (u, v) = (x * u, y * v)$. Clearly $X \times Y; *, (0, 0))$ QS-algebra.
Proposition 5.3. Let $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x), \lambda_B(x)) : x \in X\}$ be two $(\frac{1}{2},\frac{1}{2})$-fuzzy QS-ideals of X, then $A \times B$ is $(\frac{1}{2},\frac{1}{2})$-fuzzy QS-ideal of $X \times X$.
Proof.
$(\sqrt{\mu_A} \times \sqrt{\mu_B})(0, 0) = \min\{\sqrt{\mu_A(0)}, \sqrt{\mu_B(0)}\} \geq$
$\min\{\sqrt{\mu_A(x)}, \sqrt{\mu_B(y)}\}\} = (\sqrt{\mu_A} \times \sqrt{\mu_B})(x, x)$
$(\sqrt{\lambda_A} \times \sqrt{\lambda_B})(0, 0) = \max\{\sqrt{\lambda_A(0)}, \sqrt{\lambda_B(0)}\} \geq$
$\max\{\sqrt{\lambda_A(x)}, \sqrt{\lambda_B(y)}\}\} = (\sqrt{\lambda_A} \times \sqrt{\lambda_B})(x, y)$
for all $x, y \in X$. Now let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, then
$\min\{(\sqrt{\mu_A} \times \sqrt{\mu_B})(((x_1, x_2) * (z_1, z_2)), (\sqrt{\mu_A} \times \sqrt{\mu_B})((z_1, z_2) * (y_1, y_2))\}$
$= \min\{(\sqrt{\mu_A} \times \sqrt{\mu_B})((x_1 * z_1, x_2 * z_2)), (\sqrt{\mu_A} \times \sqrt{\mu_B})((z_1 * y_1, z_2 * y_2))\}$
$= \min\{\min\{\sqrt{\mu_A(x_1 * z_1)}, \sqrt{\mu_B(x_2 * z_2)}\}, \min\{\sqrt{\mu_A(z_1 * y_1)}, \sqrt{\mu_B(z_2 * y_2)}\}\}$
$\min\{\min\{\sqrt{\mu_A(x_1 * z_1)}, \sqrt{\mu_A(z_1 * y_1)}\}, \min\{\sqrt{\mu_B(x_2 * z_2)}, \sqrt{\mu_B(z_2 * y_2)}\}$
$\min\{\min\{\sqrt{\mu_A(x_1 * z_1)}, \sqrt{\mu_A(z_1 * y_1)}\}, \min\{\sqrt{\mu_B(x_2 * z_2)}, \sqrt{\mu_B(z_2 * y_2)}\}$
$\leq \min\{\sqrt{\mu_A}, \sqrt{\mu_B}(x_2 * y_2) = (\sqrt{\mu_A} \times \sqrt{\mu_B})(x_1 * y_1, x_2 * y_2)$.
$\max\{(\sqrt{\lambda_A} \times \sqrt{\lambda_B})(((x_1, x_2) * (z_1, z_2)), (\sqrt{\lambda_A} \times \sqrt{\lambda_B})((z_1, z_2) * (y_1, y_2))\}$
$= \max\{(\sqrt{\lambda_A} \times \sqrt{\lambda_B})((x_1 * z_1, x_2 * z_2)), (\sqrt{\lambda_A} \times \sqrt{\lambda_B})((z_1 * y_1, z_2 * y_2))\}$
$= \max\{\max\{\sqrt{\lambda_A(x_1 * z_1)}, \sqrt{\lambda_B(x_2 * z_2)}\}, \max\{\sqrt{\lambda_A(z_1 * y_1)}, \sqrt{\lambda_B(z_2 * y_2)}\}\}$
$\max\{\max\{\sqrt{\lambda_A(x_1 * z_1)}, \sqrt{\lambda_A(z_1 * y_1)}\}, \max\{\sqrt{\lambda_B(x_2 * z_2)}, \sqrt{\lambda_B(z_2 * y_2)}\}$
$\geq \max\{\sqrt{\lambda_A(x_1 * y_1)}, \sqrt{\lambda_B(x_2 * y_2)} = (\sqrt{\lambda_A} \times \sqrt{\lambda_B})(x_1 * y_1, x_2 * y_2)$.
This completes the proof.

6. Correlation coefficient for $(\frac{1}{2},\frac{1}{2})$fuzzy sets

Correlation plays an important role in statistics , engineering sciences and so on see [8, 9 , 33].In this section, we propose some correlation coefficients for any two SSR fuzzy sets. Let $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x), \lambda_B(x)) : x \in X\}$ be two $(\frac{1}{2},\frac{1}{2})$fuzzy sets in the universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$.
We define the following expression:
♢ Informational $(\frac{1}{2},\frac{1}{2})$fuzzy energies of A in the universe of the discourse $X = \{x_1, x_2, \cdots, x_n\}$ for every $x_i \in X$

$$E_{SSR-fuzzy}(A) = \sum_1^n((\sqrt{\mu_A(x_i)})^2 + (\sqrt{\lambda_A(x_i)})^2 = \sum(\mu_A(x_i) + \lambda_A(x_i))$$

♢ Informational SRfuzzy energies of B in the universe of the discourse $X = \{x_1, x_2, \cdots, x_n\}$ for every $x_i \in X$

$$E_{SSR-fuzzy}(B) = \sum_1^n((\sqrt{\mu_B(x_i)})^2 + (\sqrt{\lambda_B(x_i)})^2 = \sum(\mu_B(x_i) + \lambda_B(x_i))$$

♢ The correlation between two $(\frac{1}{2},\frac{1}{2})$fuzzy sets A and B of X

$$C_{SSR-fuzzy}(A, B) = \sum_1^n(\sqrt{\mu_A(x_i)} \times \sqrt{\mu_B(x_i)} + (\sqrt{\lambda_A(x_i)} \times \sqrt{\lambda_B(x_i)})$$

Definition 6.1. Let A and B be two arbitrary $(\frac{1}{2},\frac{1}{2})$-fuzzy sets on X.

The correlation coefficient of A and B is defined as

$$\kappa(A, B) = \frac{C_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy sets}}(A,B)}{\sqrt{E_{(\frac{1}{2},\frac{1}{2})-\text{ fuzzy}}(A).E_{(\frac{1}{2},\frac{1}{2})-\text{ fuzzy sets}}(B)}}$$

$$\frac{\sum_1^n(\sqrt{\mu_A(x_i)}\times\sqrt{\mu_B(x_i)})+(\sqrt{\lambda_A(x_i)}\times\sqrt{\lambda_B(x_i)})}{\sqrt{\sum_1^n((\sqrt{\mu_A(x_i)})^2+(\sqrt{\lambda_A(x_i)}))^2}.\sqrt{\sum_1^n((\sqrt{\mu_B(x_i)})^2+(\sqrt{\lambda_B(x_i)})^2)}}$$

On the other hand, by using idea of Xu et al [32]. we can suggest an alternative form of the correlation coefficient between two $(\frac{1}{2},\frac{1}{2})$ fuzzy sets A and B as follows

$$\kappa(A, B) = \frac{\sum_1^n(\sqrt{\mu_A(x_i)}\times\sqrt{\mu_B(x_i)}+(\sqrt{\lambda_A(x_i)}\times\sqrt{\lambda_B(x_i)})}{\max\{\sqrt{\sum_1^n(\mu_A(x_i)+\lambda_A(x_i))},\sqrt{\sum_1^n(\mu_B(x_i)+\lambda_B(x_i))}\}}$$

It is obvious that the correlation ofx$(\frac{1}{2},\frac{1}{2})$fuzzy sets of X satisfies the following properties:

$C_{(\frac{1}{2},\frac{1}{2})\text{ fuzzy sets}}(A, A) = C_{(\frac{1}{2},\frac{1}{2})\text{fuzzy sets}}(A)$

$C_{(\frac{1}{2},\frac{1}{2})\text{ fuzzy sets}}(A, B) = C_{(\frac{1}{2},\frac{1}{2})\text{fuzzy sets}}(B, A)$

Theorem 6.2. For any $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x), \lambda_B(x)) : x \in X\}$ two $(\frac{1}{2},\frac{1}{2})$ fuzzy sets in the universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$ the correlation coefficient of A and B satisfies the following conditions:

1- $\kappa(A, B) = \kappa(B, A)$
2- $0 \leq \kappa(A, B) \leq 1$
3- $A = B \Rightarrow \kappa(A, B) = 1$

Proof. 1) It is straightforward.
The inequality $\kappa(B, A) \geq 0$ is evident. We will prove $\kappa(A, B) \leq 1$

$$C_{\text{SSR-fuzzy sets}}(A, B) = \sum_1^n(\sqrt{\mu_A(x_i)} \times \sqrt{\mu_B(x_i)} + (\sqrt{\lambda_A(x_i)} \times \sqrt{\lambda_B(x_i)}) =$$

$(\sqrt{\mu_A(x_1)} \times \sqrt{\mu_B(x_1)} + (\sqrt{\lambda_A(x_1)} \times \sqrt{\lambda_B(x_1)} + \cdots\cdots\cdots +$
$(\sqrt{\mu_A(x_n)} \times \sqrt{\mu_B(x_n)} + (\sqrt{\lambda_A(x_n)} \times \sqrt{\lambda_B(x_n)}$
By CauchySchwarz inequality
$(x_1y_1 + x_2y_2 + \cdots + x_ny_n)^2 \leq (x_1^2 + x_2^2 + \cdots + x_n^2) \bullet (y_1^2 + y_2^2 + \cdots + y_n^2)$
$x_1 + x_2 + \cdots + x_n \in \Re^n$ and $y_1 + y_2 + \cdots + y_n \in \Re^n$, we get
$(C_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy ideal}}(A,B))^2 \leq \{\mu_A(x_1) + \lambda_A(x_1) + \cdots + \mu_A(x_n) + \lambda_A(x_n)\}\times$
$\{\mu_B(x_1) + \lambda_B(x_1) + \cdots + \mu_B(x_n) + \lambda_B(x_n)\} = (\sum_1^n \mu_A(x_i) + \lambda_A(x_i)) \times \sum_1^n \mu_B(x_i) + \lambda_B(x_i)) =$
$E_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy}}(A) \times E_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy}}(B)$
Therefore $(C_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy ideal}}(A,B))^2 \leq E_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy}}(A) \times E_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy}}(B)$
Hence, we obtain the property $0 \leq \kappa(A,B) \leq 1$.
As $A = B$ this implies that $\mu_a(x) = \mu_B(x)$ and $\lambda_A(x) = \lambda_B(x), \forall\, x \in X$.
Thus $\kappa(A,B) = 1$

Example 6.3. Let $X = 0,1,2,3$ be a set with a binary operation $*$ define by Example 3.8. with table (3), then $(X,*,0)$ is a QS-algebra.
Define $A = \{(x,\mu_A(x),\lambda_A(x)) : x \in X\}$ and $B = \{(x,\mu_B(x),\lambda_B(x)) : x \in X\}$ as follows:
$\mu_A(0) = 0.08, \mu_A(1) = 0.06, \mu_A(2) = 0.04, \mu_A(3) = 0.02,$
$\sqrt{\mu_A(0)} = 0.282842712474619 - \sqrt{\mu_A(1)} = 0.2449489742783178 - \sqrt{\mu_A(2)} = 0.2 - \sqrt{\mu_A(3)} = 0.1414213562373095,$
$\lambda_A(0) = 0.01, \lambda_A(1) = 0.03, \lambda_A(2) = 0.05, \lambda_A(3) = 0.7,$
$\sqrt{\lambda_A(0)} = 0.1 - \sqrt{\lambda_A(1)} = 0.1732050807568877 - \sqrt{\lambda_A(2)} = 0.223606797749979 - \sqrt{\lambda_A(3)} = 0.2645751311064591,$
$\mu_B(0) = 0.07, \mu_B(1) = 0.05, \mu_B(2)0.03, \mu_B(3) = 0.01,$
$\sqrt{\mu_B(0)} = 0.2645751311064591 - \sqrt{\mu_B(1)} = 223606797749979 - \sqrt{\mu_B(2)} = 0.1732050807568877 - \sqrt{\mu_B(3)} = 0.1$
$\lambda_b(0) = 0.01, \lambda_b(1) = 0.03, \lambda_B(2) = 0.05, \lambda_B(3) = 0.07,$
$\sqrt{\lambda_b(0)} = 0.1 - \sqrt{\lambda_b(1)} = 0.1732050807568877 - \sqrt{\lambda_b(2)} = 223606797749979 - \sqrt{\lambda_b(3)} = 0.2645751311064591$
It is easy to check that $A = \{(x,\mu_A(x),\lambda_A(x)) : x \in X\}$ is $(\frac{1}{2},\frac{1}{2})$-fuzzy sets on universe of discourse $X = \{0,1,2,3,\}$.
The informational energy of A is written as
$E_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy}}(A) = \sum_0^3((\sqrt{\mu_A(x_i)})^2 + (\sqrt{\lambda_A(x_i)})^2 = \sum_0^3(\mu_A(x_i) + \lambda_A(x_i) =$
$(0.9) + (0.9) + (0.9) + (0.9) = (0.9) \times 4 = 0.36$
$\sqrt{E_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy}}(A)} = \sqrt{0.36} = 0.6$
The informational energy of B is written as
$E_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy}}(B) = \sum_0^3((\sqrt{\mu_B(x_i)})^2 + (\sqrt{\lambda_B(x_i)})^2 = \sum_0^3(\mu_B(x_i) + \lambda_B(x_i) =$
$0.8 + 0.8 + 0.8 + 0.8 = 0.8 \times 4 = 0.32$
$\sqrt{E_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy}}(B)} = \sqrt{0.32} = 0.565685424949238$
The correlation between $(\frac{1}{2},\frac{1}{2})$-fuzzy sets A and B is written as
$C_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy sets}}(A,B) = \sum_1^n(\sqrt{\mu_A(x_i)}\times\sqrt{\mu_B(x_i)}+(\sqrt{\lambda_A(x_i)}\times\sqrt{\lambda_B(x_i)}) \approx 0.33838855261110396837$
Hence, the correlation coefficient between $(\frac{1}{2},\frac{1}{2})$-fuzzy sets A and B is given by

$$\kappa(A,B)=\frac{C_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy sets}}(A,B)}{\sqrt{E_{(\frac{1}{2},\frac{1}{2})-\text{ fuzzy}}(A)\times E_{(\frac{1}{2},\frac{1}{2})-\text{ fuzzy}}(B)}}$$

$$\frac{\sum_{1}^{n}(\sqrt{\mu_A(x_i)}\times\sqrt{\mu_B(x_i)})+(\sqrt{\lambda_A(x_i)}\times\sqrt{\lambda_B(x_i)})}{\sqrt{\sum_{1}^{n}((\sqrt{\mu_A(x_i)})^2+(\sqrt{\lambda_A(x_i)}))^2}\times\sqrt{\sum_{1}^{n}((\sqrt{\mu_B(x_i)})^2+(\sqrt{\lambda_B(x_i)})^2)}}$$

$$\frac{\sum_{1}^{n}(\sqrt{\mu_A(x_i)}\times\sqrt{\mu_B(x_i)})+(\sqrt{\lambda_A(x_i)}\times\sqrt{\lambda_B(x_i)})}{\sqrt{\sum_{1}^{n}(\mu_A(x_i)+\lambda_A(x_i))}\times\sqrt{\sum_{1}^{n}(\mu_B(x_i)+\lambda_B(x_i))}}=$$

$$=\frac{0.338388555261110396837}{0.6\times0.565685424949238}=\frac{0.338388555261110396837}{0.339411}=0.9969875902666678\cdots(*)$$

On the alternative form of the correlation cofficient of $(\frac{1}{2},\frac{1}{2})$ fuzzy sets A and B as follows

$$\kappa(A,B)=\frac{C_{(\frac{1}{2},\frac{1}{2})-\text{fuzzy sets}}(A,B)}{\max\{\sqrt{E_{(\frac{1}{2},\frac{1}{2})-\text{ fuzzy}}(A),E_{(\frac{1}{2},\frac{1}{2})-\text{ fuzzy}}(B)}}\}$$

$$\frac{\sum_{1}^{n}(\sqrt{\mu_A(x_i)}\times\sqrt{\mu_B(x_i)})+(\sqrt{\lambda_A(x_i)}\times\sqrt{\lambda_B(x_i)})}{\max\{\sqrt{\sum_{1}^{n}((\sqrt{\mu_A(x_i)})^2+(\sqrt{\lambda_A(x_i)}))^2},\sqrt{\sum_{1}^{n}((\sqrt{\mu_B(x_i)})^2+(\sqrt{\lambda_B(x_i)})^2)}\}}$$

$$\frac{\sum_{1}^{n}(\sqrt{\mu_A(x_i)}\times\sqrt{\mu_B(x_i)})+(\sqrt{\lambda_A(x_i)}\times\sqrt{\lambda_B(x_i)})}{\max\{\sqrt{\sum_{1}^{n}(\mu_A(x_i)+\lambda_A(x_i))},\sqrt{\sum_{1}^{n}(\mu_B(x_i)+\lambda_B(x_i))}\}}$$

$$=\frac{0.338388555261110396837}{\max\{0.6\times0.565685424949238\}}=\frac{0.338388555261110396837}{0.6}=0.563980925435183994728333\cdots(**)$$

From $(*)$ and $(**)$, we see that the $(\frac{1}{2},\frac{1}{2})$-fuzzy sets A and B are correlated.

7. Conclusions

In this work we have studied the notion of $(\frac{1}{2},\frac{1}{2})$-fuzzy set (denoted by SSR fuzzy set) of QS-ideals on a QS-algebra and investigated its properties. Furthermore we have studied the homomorphic image and inverse image of SSR-fuzzy QS-ideals of a QS-algebra under homomorphism of QS-algebras. Moreover, the Cartesian product of SSR-fuzzy QS-ideals in Cartesian product QS-algebras is given. This part is sufficient to study the topological space for SSR-fuzzy QS-ideals of a QS-algebra. Finally, we have studied novel correlation coefficient between two SSR-fuzzy sets. For more applications are needed to be discussed in our further study.

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