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Research Paper

STOCHASTIC ROBUSTNESS IN SWITCHED SYSTEMS: A NOVEL CONTROL STRATEGY FOR RANDOM TIME-ITERATION DRIVEN SWITCHING

OMPRAKASH DEWANGAN[1]

[1]Department of Mathematics, Indira Gandhi Govt. College Pandaria, Dist.- Kabirdham (C.G.), 491559, Chhattisgarh, India, E-mail: omidewangan26@gmail.com

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ABSTRACT

This paper addresses the control of a category of continuous-time linear systems that switch between different modes, where the switching signals are driven by random time-iteration. The system under consideration is subject to uncertainties in the system dynamics and observation noise in the output measurements. We propose a robust control strategy that Accounting for the random nature of the switching signals and the system uncertainties. The learning performance is examined using the Lebesgue-p norm, leading to the derivation of a sufficient condition for convergence. The findings demonstrate that the proposed control law effectively addresses the tracking problem in switched systems, Especially when the switching rules are expanded to the time-iteration domain using a stochastic framework, we introduce a groundbreaking control approach that guarantees the system's performance despite uncertainties and noise. Through rigorous theoretical analysis, we prove the effectiveness of our suggested approach in achieving robust control and estimation performance.The results of this research contribute to the advancement of control theories and have potential applications in various fields, including power systems, robotics, and process control.

∗Address correspondence to Omprakash Dewangan; [1]Department of Mathematics, Indira gandhi Govt. College Pandaria, Dist.- Kabirdham (C.G.), 491559, Chhattisgarh, India, E-mail: omidewangan26@gmail.com

1. Introduction

ILC is a control strategy that utilizes repetitive iterations to learn and refine a system's performance, making it particularly effective for systems that perform repetitive tasks like robotic arms, machine tools, and chemical processing plants. By leveraging the insights gained from previous iterations, ILC adapts and optimizes control inputs to achieve enhanced performance in subsequent cycles. ILC can significantly improve the accuracy of tracking outputs, reduce errors, and optimize performance in systems with repetitive dynamics. ILC uses data from previous iterations to learn and improve control performance. ILC is suited for systems with repetitive dynamics, such as robotic arms or machine tools. ILC can adapt to changes in system dynamics or operating conditions. ILC has applications in various fields, including Robotics, Process control, Motion control, Mechatronics, Automation etc. (See [1]-[17]).

Randomly switched systems are a type of dynamic system that transitions between different modes or subsystems based on a stochastic switching signal. This signal is generated by random time-iteration, resulting in unpredictable switching times that may vary with each iteration. The randomness can be modeled using various stochastic processes like Markov chains or random pulse trains. These systems have applications in control systems with random faults, communication networks with packet losses, biological systems with random mutations, and economic systems with market fluctuations. Due to the unpredictability of switching times, analyzing and controlling these systems is challenging. Researchers employ techniques like stochastic stability analysis, robust control, and adaptive control to design controllers that can handle random switching and achieve stable and optimal performance (See [15]-[26]).

The paradigm of ILC for improving robot performance was proposed by Arimoto et al.[1] in 1984 through the "betterment process." This method enhances robot operation by using data from previous iterations to generate better control inputs for subsequent trials, without requiring precise knowledge of system dynamics. The approach involves adding an error-based increment to the previous input, ensuring convergence to the desired trajectory under certain conditions. Miyazaki et al.[2] in 1986 extended this concept to robots with elastic transmissions, proposing a two-stage betterment process. Kawamura et al.[3] in 1988 further demonstrated the practical application of this learning control scheme on a three-degree-of-freedom manipulator, showing its effectiveness in both joint-angle and task-oriented coordinates. The key advantage of this method is its ability to form input torque patterns for desired motions, leveraging the repeatability of robot motion, we bypass the need for dynamic parameter identification and iterative improvement through trials. The field has seen advancements in stability analysis, performance optimization, robustness, and learning transient behavior (Bristow et al.,[4] 2006). Various design techniques have emerged, including frequency-domain-based ILC, 2-D theory-based ILC, and optimization-based ILC (Han Zheng-zhi [5] 2005). ILC has been applied to diverse areas such as robotics, batch processes, and semiconductor manufacturing (Ahn and Bristow[7], 2011). Recent research has focused on relaxing traditional ILC assumptions and exploring more generalized conditions. Despite entering its third decade, the field continues to evolve, with ongoing investigations into theoretical aspects like monotonic convergence, optimality, and hard nonlinearities ([6]-[7]).

In 2012, X Ruan et al. [10] explored the convergence characteristics of PD-type iterative learning control schemes for linear time-invariant systems with partial knowledge in the sense of the Lebesgue-p norm. The studies demonstrate that convergence is influenced by both derivative and proportional learning gains, as well as system matrices. First-order schemes exhibit strictly monotone convergence, while second-order schemes achieve monotonicity after a finite number of iterations. The papers compare convergence speeds between first-order and second-order rules, noting that second-order processes can be Qp-slower, Qp-equivalent or Qp-faster depending on learning gain selection. Additionally, the inclusion of feedback information in PD-type ILC can potentially accelerate convergence when gains are properly chosen ([11]-[12]).

In 2013, X Bu et al. [15] studied arbitrary switching rules, assuming repetitive operation over finite time intervals (See [15, 19]). Under specific conditions, D-type ILC laws can ensure asymptotic convergence of output errors(see [14]). In 2015, Yang Ruan [19] implemented ILC in linear discrete-time switched systems with dynamic switching rules, assessing convergence and robustness features using the super vector technique. These studies collectively demonstrate the effectiveness of ILC for various linear switched systems, offering theoretical analysis and practical applications. Analysis of convergence has been conducted through the super vector approach in noise-free systems and robustness in systems with bounded noise. Stability analysis and stabilization of these systems have been explored using average dwell time and linear matrix inequalities, with connections to 2D repetitive systems [18]. ILC has also been extended to non-linear switched systems, demonstrating asymptotic convergence of tracking errors under certain conditions (see [14, 16, 18, 19]).

In 2018, Yang and Ruan[24] investigated ILC for switched r epetitive systems with random time-iteration driven switching signals, deriving convergence conditions using the Lebesgue-p norm. In 2015, Shen et al.[25] inspected the convergence properties of ILC for linear systems with randomly changing iteration durations, confirming almost sure and mean square convergence via a switching system methodology. In 2018, Shao and Duarr [28] put forward a high-order ILC technique for discrete-time linear switched systems with iteration-varying parameters, addressing resetting errors and deriving stability conditions using linear matrix inequalities. In 2019, Yang and Ruan [30] examined proportional-derivative ILC for continuous-time switched systems with observation noise, revealing that convergence relies on learning gains and subsystem dynamics. That same year, Sahu and Singh investigated second-order ILC for discrete-time-switched systems with uncertainties, noises, and time-delays, providing convergence conditions and robustness analysis. In 2021, Sahu and Singh [32] introduced Mann-ILC and normal S-ILC methods for discrete-time switched systems, establishing convergence theorems. Earlier, in 2009, Van de Wijdeven et al. [33] presented a robust monotonic convergence analysis approach for ILC in uncertain systems using μ-analysis, applicable to MIMO systems with additive and multiplicative uncertainties. Collectively, these studies highlight the crucial role of proper gain selection and system dynamics in ensuring the convergence and robustness of ILC algorithms for switched systems under various conditions, including noise, uncertainties, and time-delays. Later, in 2023, Dewangan[37] expanded on this work by incorporating time delay, system uncertainty, and bounded noise. In 2024, O. dewangan [38] studied A novel FOILC approach is proposed to mitigate time delays in fractional-order linear systems, ensuring convergence and robustness against external disturbances.

We consider a class of linear systems that switch between different states in continuous time, driven by a random iteration-based timing mechanism, and featuring system uncertainties and observation noise and used PD-type ILC strategy ensures accurate trajectory tracking in switched systems, even when switching rules are expanded in the time-iteration domain, thereby ensuring precise control and reliability.

The paper is formatted in the following manner: The problem formulation is presented in Section 2, followed by an in-depth analysis of the main results in Section 3. Section 5 offers a succinct conclusion that encapsulates the essential insights derived from the paper.

2. **Problem discriptions**

Examine a class of linear systems that switch between different states in continuous time, driven by a random iteration-based timing mechanism, and featuring with system uncertainties and observation noise described as follow dynamic models:

$$\begin{cases} \dot{x}_k(t) & = A_{\tau(\sigma(t),k)}x_k(t) + B_{\tau(\sigma(t),k)}u_k(t) + \xi_k(t), \\ y_k(t) & = C_{(\tau(\sigma(t),k))}x_{k+1} + w_{\sigma(t),k}(t), \quad t \in \Omega = [0,T], \end{cases} \tag{2.1}$$

with initial state $x_k(0) = 0$, where

- The iteration index is denoted by k, and the time duration is given by $\Omega = [0,T]$.
- The system's state is described by the vector $x_k(t) \in \mathbb{R}^n$.
- The input vector of the system is given by $u_k(t) \in \mathbb{R}^m$.
- The vector $y_k(t) \in \mathbb{R}^p$ denotes the system's output.
- The system matrices, $A_{\tau(\sigma(t),k)}$, $B_{\tau(\sigma(t),k)}$, and $C_{\tau(\sigma(t),k)}$, have appropriate dimensions and are defined as $A_{\tau(\sigma(t),k)}$ is an $n\times n$ matrix, $B_{\tau(\sigma(t),k)}$ is an $n \times m$ matrix, and $C_{\tau(\sigma(t),k)}$ is a $p \times n$ matrix.
- The switching signal, denoted by the subscript $(\tau(\sigma(t),k))$, is a sequence of random constant functions that switch according to both operational time and iteration.
- Consider a random division of the time duration into n segments, such that $\Omega = \Omega_1 \cup \Omega_2 \cup \cdots \cup \Omega_n$.
- $\xi_k(t) \in \mathbb{R}^n$ and $w_{\sigma(t),k}$ are system uncertinities and observation noise with bounds $\|\xi_k(t)\|_p \le b_\xi$ and $\|w_{\sigma(t),k}\|_p \le w_{\sigma(t),0}$.

The switching signals exhibit iteration-to-iteration independence at the same time as subinterval. That is, during a specific time period, the switching rules randomly pick subsystems (e.g., Ω_i) in a manner that is independent across different iterations. For simplicity, we denote this as

$$\tau(\sigma(t),k) = l[i,k], \qquad (i = 1,2,...,n), \tag{2.2}$$

where i is define as

$$\sigma(t) = i = \begin{cases} 1, & t \in \Omega_1 = [0,t_1), \\ 2, & t \in \Omega_2 = [t_1,t_2), \\ \vdots & \\ n, & t \in \Omega_n = [t_{n-1},T]. \end{cases}$$

which arbitrarily divided the time duration Ω into n subintervals $\Omega_i (i = 1, 2, \cdots, n)$. Here $l[i, k]$ represents a sequence of random numbers at the k^{th} trial.

Remark 2.1. In contrast to previous research, This paper presents an ILC approach for switched systems with switching signals that vary randomly across time and iterations. By segmenting the operation time interval into subintervals, we achieve independence in switching signals, enhancing system adaptability. The switching signals enable a versatile operational framework, where subsystems can be assigned to arbitrary time segments across iterations. The notation $l[2, 5] = 3$ illustrates this, denoting that the 3^{rd} subsystem will be operational in the 5^{th} iteration, within the 2^{nd} time segment Ω_2, showcasing the adaptive and dynamic nature of the switching mechanism.

Using equation (2.2), we can reformulate the original system given in equation (2.1) as

$$\begin{cases} \dot{x}_k(t) &= A_{l[i,k]} x_k(t) + B_{l[i,k]} u_k(t) + \xi_k(t), \\ y_k(t) &= C_{l[i,k]} x_{k+1}(t) + w_{i,k}, \quad t \in \Omega = [0, T]. \end{cases} \tag{2.3}$$

Consider an ILC strategy based on PD-type control

$$u_{k+1} = \Phi_{l[i,k]} u_k(t) + \Gamma_{p,l[i,k]} e_k(t) + \Gamma_{d,l[i,k]} \dot{e}_k(t), \tag{2.4}$$

whereas $\Gamma_{p,l[i,k]}$ represents the proportional learning rate, $\Gamma_{d,l[i,k]}$ represents the derivative learning rate, which controls the adaptation speed of the proportional and derivative terms in the learning algorithm, and $\Phi_{l[i,k]}(t) \in \mathbb{R}^{m \times m}$ is an invertible matrix, denoted as the learning matrix.

Suppose we have a targeted trajectory $y_d(t)$ defined on Ω. Using the equation (2.4), we can produce a control signal sequence recursively, meaning that control signals are produced in a chain, with each signal relying on the previous one, creating a sequential and iterative control process, that enables the system (2.3) to track $y_d(t)$ as accurately as possible, either precisely or within a neighborhood, as the iteration index approaches infinity. Alternatively stated,

$$\lim_{k \to \infty} \sup \|e_{k+1}\|_p \leq \eta. \tag{2.5}$$

Choose a sufficiently small positive value η. The tracking error, $e_k(t)$, is the error between the desired output, $y_d(t)$, and the actual output, $y_k(t)$, expressed mathematically as:

$$e_k(t) = y_d(t) - y_k(t).$$

The superior limit of a sequence is denoted by $\lim_{k \to \infty} \sup\{\cdot\}$, and the Lesbesgue-$p$ norm of a vector-valued function is represented by $\|\cdot\|_p$, which can be expressed as following:

Definition 2.2. Consider a vector-valued function $g : I \subseteq \mathbb{R}^+ \to \mathbb{R}^m$, where $g(t) = [g_1(t), g_2(t), \ldots, g_m(t)]^T$ for $t \in \Omega$. The p-norm of g in the Lebesgue sense is defined as:

$$\|g(\cdot)\|_p = \left[\int_I \left(\max_{1 \leq j \leq m} \{|g_j(t)|\} \right)^p dt \right]^{\frac{1}{p}}, \quad 1 \leq p \leq \infty.$$

Convolution integers are used in System Response, Fourier Analysis and Solve Differential Equations. Convolution integrals are crucial in signal processing techniques such as filtering, convolution, and deconvolution. They help in extracting valuable information from signals and removing noise. Control theory is a specialized area of engineering that addresses the design and analysis of control systems. Convolution integrals play a crucial role in control theory, particularly in the design and analysis of control systems. Convolution integrals are used in control theory because they provide a powerful tool for analyzing and designing control systems. Convolution integrals are capable of simulating the dynamics of intricate systems, including those with non-linear dynamics. Convolution integrals can be used to design controllers that achieve specific performance criteria, such as stability, tracking, and disturbance rejection.

Definition 2.3. Let $h_1(t)$ and $h_2(t)$ be functions in $\mathbb{R}^m$. The convolution integral of h_1 and h_2 is defined as the integral transform:

$$(h_1 * h_2)(t) = \int_I h_1(t-s)h_2(s)ds,$$

which combines h_1 and h_2 through a continuous sum of scaled and shifted versions of their product.

Generalized Young inequality is used to estimate the norm of convolution integrals and Fourier analysis to estimate the norm of Fourier. Generalized Young inequality is used to analyze the stability of control systems, particularly those with nonlinear dynamics, transforms, design controllers that guarantee stability and performance criteria, the robustness of control systems against uncertainties and disturbances, 4. estimate the norm of convolution integrals that arise in control systems, construct Lyapunov functions that prove stability and convergence of control systems etc.

Building upon the foundational definitions (2.2) and (2.3), We are able to obtain the generalized Young inequality (GYI) for the convolution integral, which can be succinctly stated as:

$$\|(h_1 * h_2)(\cdot)\|_r \leq \|h_1(\cdot)\|_q \|h_2(\cdot)\|_p, \tag{2.6}$$

for all $1 \leq p, q, r < \infty$ satisfying

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{r} + 1.$$

Notably, the inequality (2.6) undergoes a transformation, yielding

$$\|h_1 * h_2\|_p \leq \|h_1\|_1 \|h_2\|_p,$$

when $p = r$.

The basic prerequisites for the system represented by equation (2.3) are as follows:

A1: The precursor state of each iterative evolution, denoted by $x_k(0)$, is close to the reference initial state $x_d(0)$, satisfying the inequality $\|x_k(0) - x_d(0)\| < \delta$ for all iterations $k = 1, 2, \ldots$, where δ is a small positive real number.

A2: Given any realizable reference trajectory $y_d(t)$ on the domain Ω, there is a correspondingtarget control vector $u(t)$ and a suitable state trajectory $x(t)$ such that the performance of the controlled system $u_d(t)$, can track the reference $y_d(t)$.

3. Main result

Lemma 3.1. [10] *Consider a sequence $\{a_k\}$ of non-negative real numbers satisfying*

$$a_k \leq \sigma_1 a_{k-1} + \sigma_2 a_{k-2} + \cdots + \sigma_N a_{k-N} + \varepsilon, \qquad k = N+1, N+2, \cdots.$$

having an initial value of $a_l (l = 1, 2, \cdots N)$ *together with a specified sequence* $\{\varepsilon_k\}$*. If the coefficient* $\sigma_j (j = 1, 2, \cdots, n)$ *satisfy* $\sigma_j \geq 0$ *and*

$$\sigma = \sum_{j=1}^{N}, \sigma_j < 1,$$

then $\lim_{k\to\infty} \varepsilon_k \leq \varepsilon$ *implies that*

$$\lim_{k\to\infty} \sup a_k \leq \frac{\varepsilon}{1-\sigma}.$$

Especially, $\lim_{k\to\infty} \sup a_k = 0$ *if* $\varepsilon = 0$.

Proof. First, we can rewrite the inequality as

$$a_k \leq \sigma(a_{k-1} + a_{k-2} + \cdots + a_{k-N}) + \varepsilon.$$

Then, we can use induction to show that

$$a_k \leq \sigma^k \max\{a_0, a_1, \cdots, a_{N-1}\} + \frac{1-\sigma^k}{1-\sigma}\varepsilon.$$

Taking the supremum of both sides, we get

$$\sup a_k \leq \sigma^k \max\{a_0, a_1, \cdots, a_{N-1}\} + \frac{1-\sigma^k}{1-\sigma}\varepsilon.$$

As $k \to \infty$, we have $\sigma^k \to 0$ and thus

$$\lim_{k\to\infty} \sup a_k \leq \frac{\varepsilon}{1-\sigma}.$$

If $\varepsilon = 0$, then $\lim_{k\to\infty} \sup a_k = 0$. □

Now, we present main result as follows:

Theorem 3.2. *Consider the control system characterized by Equation ((2.3)), which is modulated by the varying switching rules with time-iteration ((2.2)). Suppose the learning gains* $\Gamma_{p,l[i,k]}$ *and* $\Gamma_{d,l[i,k]}$ *satisfy the following criteria:*

$$\begin{aligned}
&\max\{\|I - C_{l[i,k]}B_{l[i,k]}\Phi^{-1}(\cdot)\Gamma_{d,l[i,k]}\|_\infty \\
&\quad+\|C_{l[i,k]}\exp(A_{l[i,k]}(\cdot))\{[A_{l[i,k]}B_{l[i,k]}\Phi^{-1}(\cdot) \\
&\quad- B_{l[i,k]}\dot{\Phi}^{-1}_{l[i,k]}] \cdot \Gamma_{d,l[i,k]} + B_{l[i,k]}\Phi_{l[i,k]^{-1}}(\cdot)\Gamma_{p,l[i,k]}\}\|_1\} = \rho < 1.
\end{aligned} \tag{3.1}$$

Then, the output trajectory $y_k(t)$ *generated by the PD-type ILC ((2.4)) converges to the targeted trajectory* $y_d(t)$ *as the iteration index* k *tends to* ∞*,* $\forall t \in \Omega$*, and the tracking error is ultimately bounded within a certain neighborhood.*

Proof. Presume the $l[i,k]^{th}$ subsystem is enabled during the k-th iterative cycle, spanning the temporal domain $[t_{i-1}, t_i)$. The resultant state trajectory of the system (2.3) can be mathematically characterized as

$$\begin{aligned}
x_{k+1} &= \exp(A_{l[i,k]}(t - t_{i-1}))x_{k+1}(t_{i-1}) \\
&\quad+ \int_{t_{i-1}}^{t_i} \exp(A_{l[i,k]}(t - s))B_{l[i,k]}u_{k+1}(s)ds \\
&\quad+ \int_{t_{i-1}}^{t_i} \exp(A_{l[i,k]}(t - s))B_{l[i,k]}\xi_{k+1}(s)ds, \quad (i = 1, 2, \cdots, n),
\end{aligned}$$

Based on the definition of the tracking error, we can deduce that

$$
\begin{aligned}
e_{k+1}(t) &= y_d(t) - y_{k+1}(t) \\
&= y_d(t) - y_k(t) - [y_{k+1} - y_k(t)] \\
&= e_k(t) - [C_{l[i,k]} \exp(A_{l[i,k]}(t - t_{i-1}))x_{k+1}(t_{i-1}) \\
&- C_{l[i,k]} \exp(A_{l[i,k]}(t - t_{i-1}))x_k(t_i)] \\
&- \int_{t_{i-1}}^{t_i} [C_{l[i,k+1]} \exp(A_{l[i,k+1]}(t - s))B_{l[i,k+1]}u_{k+1}(s) \\
&- C_{l[i,k]} \exp(A_{l[i,k]}(t - s))B_{l[i,k]}u_k(s)]ds \\
&- \int_{t_{i-1}}^{t_i} \exp(A_{l[i,k]}(t - s))B_{l[i,k]}(\xi_{k+1}(s) - \xi_k(s))ds + \Delta w_{i,k}(t).
\end{aligned}
$$

Associated with each $l[i,k]$ are non-singular matrices $\Theta_{l[i,k]}(t)$ and $\Phi^{-1}_{l[i,k]}(t)$, which satisfy:

$$
C_{l[i,k+1]} \exp(A_{l[i,k+1]}(t - t_{i-1})) = C_{l[i,k]} \exp(A_{l[i,k]}(t - t_{i-1}))\Theta_{l[i,k]}(t)
$$

and

$$
\begin{aligned}
&C_{l[i,k+1]} \exp(A_{l[i,k+1]}(t - s))B_{l[i,k+1]} \\
&\qquad = C_{l[i,k]} \exp(A_{l[i,k]}(t - s))B_{l[i,k]}\Phi^{-1}_{l[i,k]}(s).
\end{aligned}
$$

Therefore, the above expression simplifies to

$$
\begin{aligned}
e_{k+1}(t) = e_k(t) &- C_{l[i,k]} \exp(A_{l[i,k]}(t - t_{i-1}))[\Theta_{l[i,k]}(t)x_{k+1}(t_{i-1}) - x_k(t_{i-1})] \\
&- \int_{t_{i-1}}^{t_i} C_{l[i,k+1]} \exp(A_{l[i,k]}(t - s))B_{l[i,k]}[\Phi^{-1}_{l[i,k]}(s)u_{k+1}(s) - u_k(s)]ds \\
&- \int_{t_{i-1}}^{t_i} \exp(A_{l[i,k]}(t - s))B_{l[i,k]}(\xi_{k+1}(s) - \xi_k(s))ds + \Delta w_{i,k}(t).
\end{aligned} \tag{3.2}
$$

By substituting the updating law (2.4) into equation (3.2), we obtain

$$
\begin{aligned}
e_{k+1}(t) &= e_k(t) - C_{l[i,k]} \exp(A_{l[i,k]}(t - t_{i-1}))\Delta x_k(t_{i-1}) \\
&- \int_{t_{i-1}}^{t_i} [C_{l[i,k]} \exp(A_{l[i,k]}(t - s))B_{l[i,k]} \\
&\{\Phi^{-1}_{l[i,k]}(s)[\Gamma_{p,l[i,k]}e_k(s) + \Gamma_{d,l[i,k]}\dot{e}_k(s)]\}ds \\
&- \int_{t_{i-1}}^{t_i} \exp(A_{l[i,k]}(t - s))B_{l[i,k]}(\xi_{k+1}(s) - \xi_k(s))ds + \Delta w_{i,k}(t),
\end{aligned} \tag{3.3}
$$

where $\Delta x_k(t_{i-1}) = \Theta_{l[i,k]}(t)x_{k+1}(t_{i-1}) - x_k(t_{i-1})$.Applying partial integration to the last term of equation (3.3) yields

$$\begin{aligned}
&\int_{t_{i-1}}^{t}[C_{l[i,k]}\exp(A_{l[i,k]}(t-s))B_{l[i,k]}\{\Phi^{-1}_{l[i,k]}(s)\Gamma_{d,l[i,k]}\dot{e}_k(t)\}ds \\
&\quad = C_{l[i,k]}\exp(A_{l[i,k]}(t-s))B_{l[i,k]}\Phi^{-1}_{l[i,k]}(s)\Gamma_{d,l[i,k]}e_k(s)\big|_{s=t_{i-1}}^{s=t_i} \\
&\quad + \int_{t_{i-1}}^{t} C_{l[i,k]}\exp(A_{l[i,k]}(t-s))[A_{l[i,k]}B_{l[i,k]}\Phi_{l[i,k]^{-1}}(s) - B_{l[i,k]}\dot{\Phi}^{-1}_{l[i,k]}(s)] \\
&\quad \Gamma_{d,l[i,k]}e_k(s)ds. \qquad (3.4)
\end{aligned}$$

Substituting (3.4) into (3.3) yields

$$\begin{aligned}
e_{k+1}(t) &= e_k(t) - C_{l[i,k]}\exp(A_{l[i,k]}(t-t_{i-1}))\Delta x_k(t_{i-1}) \\
&- C_{l[i,k]}\exp(A_{l[i,k]}(t-s))B_{l[i,k]}\Phi^{-1}_{l[i,k]}(s)\Gamma_{d,l[i,k]}e_k(s)\big|_{s=t_{i-1}}^{s=t_i} \\
&+ \int_{t_{i-1}}^{t} C_{l[i,k]}\exp(A_{l[i,k]}(t-s))[A_{l[i,k]}B_{l[i,k]}\Phi^{-1}_{l[i,k]}(s) \\
&- B_{l[i,k]}\dot{\Phi}^{-1}_{l[i,k]}(s)]\Gamma_{d,l[i,k]}e_k(s)ds \\
&- \int_{t_{i-1}}^{t_i}\exp(A_{l[i,k]}(t-s))\Delta\xi_k(t)ds + \Delta w_{i,k}(t). \qquad (3.5)
\end{aligned}$$

Step 1: Let t belong to first subinterval, *i.e.*, $t \in \Omega_1$

During the k-th iteration, within the subinterval $[t_{i-1}, t_i)$, the $l[1,k]$-th subsystem is triggered. Starting from $t_0 = 0$, the recursive relation for the tracking error at iteration k can be expressed as

$$\begin{aligned}
e_2(t) &= (I - C_{l[1,1]}B_{l[1,1]}\Phi^{-1}_{l[1,1]}\Gamma_{d,l[1,1]})e_1(t) \\
&- C_{l[1,1]}\int_0^t \exp(A_{l[1,1]}(t-s))\{[A_{l[1,1]}B_{l[1,1]}\Phi^{-1}_{l[1,1]}(s) \\
&- B_{l[1,1]}\dot{\Phi}^{-1}_{l[1,1]}(s)]\Gamma_{d,l[1,1]} + B_{l[1,1]}\Phi^{-1}_{l[1,1]}(s)\Gamma_{p,l[1,1]}\}e_1(s)ds \\
&- C_{l[1,1]}\exp(A_{l[1,1]}(t))\Delta x_1(0) \\
&- C_{l[1,1]}\exp(A_{l[1,1]}(t))B_{l[1,1]}\Phi^{-1}_{l[1,1]}(0)\Gamma_{d,l[1,1]}e_1(0) \\
&- \int_0^t \exp(A_{l[1,1]}(t-s))\Delta\xi_1(s)ds + \Delta w_{1,1}(t),
\end{aligned}$$

$$
\begin{aligned}
e_3(t) = {} & (I - C_{l[1,2]} B_{l[1,2]} \Phi^{-1}_{l[1,2]} \Gamma_{d,l[1,2]}) e_2(t) \\
& - C_{l[1,2]} \int_0^t \exp(A_{l[1,2]}(t-s)) \{ [A_{l[1,2]} B_{l[1,2]} \Phi^{-1}_{l[1,2]}(s) \\
& - B_{l[1,2]} \dot{\Phi}^{-1}_{l[1,2]}(s)] \Gamma_{d,l[1,2]} + B_{l[1,2]} \Phi^{-1}_{l[1,2}(s) \Gamma_{p,l[1,2]} \} e_2(s) ds \\
& - C_{l[1,2]} \exp(A_{l[1,2]}(t)) \Delta x_2(0) \\
& - C_{l[1,2]} \exp(A_{l[1,2]}(t)) B_{l[1,2]} \Phi^{-1}_{l[1,2]}(0) \Gamma_{d,l[1,2]} e_2(0) \\
& - \int_0^t \exp(A_{l[1,2]}(t-s)) \Delta \xi_2(s) ds + \Delta w_{1,2}(t), \\
& \vdots
\end{aligned}
$$

$$
\begin{aligned}
e_{k+1}(t) = {} & (I - C_{l[1,k]} B_{l[1,k]} \Phi^{-1}_{l[1,k]} \Gamma_{d,l[1,k]}) e_1(t) \\
& - C_{l[1,k]} \int_0^t \exp(A_{l[1,k]}(t-s)) \{ [A_{l[1,k]} B_{l[1,k]} \Phi^{-1}_{l[1,k]}(s) \\
& - B_{l[1,k]} \dot{\Phi}^{-1}_{l[1,k]}(s)] \Gamma_{d,l[1,k]} + B_{l[1,k]} \Phi^{-1}_{l[1,k]}(s) \Gamma_{p,l[1,k]} \} e_1(s) ds \\
& - C_{l[1,k]} \exp(A_{l[1,k]}(t)) \Delta x_k(0) \\
& - C_{l[1,k]} \exp(A_{l[1,k]}(t)) B_{l[1,k]} \Phi^{-1}_{l[1,k]}(0) \Gamma_{d,l[1,k]} e_k(0) \\
& - \int_0^t \exp(A_{l[1,k]}(t-s)) \Delta \xi_k(s) ds + \Delta w_{1,k}(t).
\end{aligned} \tag{3.6}
$$

By applying the Lebesgue-p norm to both sides of (3.6) and utilizing the GYI, we derive the following estimate

$$
\begin{aligned}
\|e_2(\cdot)\|_p \leq {} & (\|I - C_{l[1,1]} B_{l[1,1]} \Phi^{-1}_{l[1,1]} \Gamma_{d,l[1,1]}\|_\infty \\
& + \|C_{l[1,1]} \exp(A_{l[1,1]}(\cdot)) \{ [A_{l[1,1]} B_{l[1,1]} \Phi^{-1}_{l[1,1]}(\cdot) \\
& - B_{l[1,1]} \dot{\Phi}^{-1}_{l[1,1]}(\cdot)] \Gamma_{d,l[1,1]} + B_{l[1,1]} \Phi^{-1}_{l[1,1]}(\cdot) \Gamma_{p,l[1,1]} \} \|_1) \|e_1(\cdot)\|_p \\
& + \|C_{l[1,1]} \exp(A_{l[1,1]})(\cdot)\|_p \|\Delta x(0)\|_p \\
& + \|C_{l[1,1]} \exp(A_{l[1,1]}(\cdot)) B_{l[1,1]} \Phi^{-1}_{l[1,1]}(0) \Gamma_{d,l[1,1]}\|_p \|e_1(0)\|_p \\
& + \|C_{l[1,1]} \exp(A_{l[1,1]}(\cdot))\|_p \|\Delta \xi_1(\cdot)\|_p + \|\Delta w_{1,1}(t)\|_p,
\end{aligned}
$$

$$
\begin{aligned}
\|e_2(\cdot)\|_p \le (&\|I - C_{l[1,2]}B_{l[1,2]}\Phi^{-1}_{l[1,2]}\Gamma_{d,l[1,2]}\|_\infty \\
&+\|C_{l[1,2]}\exp(A_{l[1,2]}(\cdot))\{[A_{l[1,2]}B_{l[1,2]}\Phi^{-1}_{l[1,2]}(\cdot) \\
&- B_{l[1,2]}\dot{\Phi}^{-1}_{l[1,2]}(\cdot)]\Gamma_{d,l[1,2]} + B_{l[1,2]}\Phi^{-1}_{l[1,2]}(\cdot)\Gamma_{p,l[1,2]}\}\|_1)\|e_1(\cdot)\|_p \\
&+\|C_{l[1,2]}\exp(A_{l[1,2]})(\cdot)\|_p\|\Delta x(0)\|_p \\
&+\|C_{l[1,2]}\exp(A_{l[1,2]}(\cdot))B_{l[1,2]}\Phi^{-1}_{l[1,2]}(0)\Gamma_{d,l[1,2]}\|_p\|e_1(0)\|_p \\
&+\|C_{l[1,2]}\exp(A_{l[1,2]}(\cdot))\|_p\|\Delta\xi_2(\cdot)\|_p+\|\Delta w_{1,2}(t)\|_p, \\
&\vdots \\
\|e_{k+1}(\cdot)\|_p \le (&\|I - C_{l[1,k]}B_{l[1,k]}\Phi^{-1}_{l[1,k]}\Gamma_{d,l[1,k]}\|_\infty \\
&+\|C_{l[1,k]}\exp(A_{l[1,k]}(\cdot))\{[A_{l[1,k]}B_{l[1,k]}\Phi^{-1}_{l[1,k]}(\cdot) \\
&- B_{l[1,k]}\dot{\Phi}^{-1}_{l[1,k]}(\cdot)]\Gamma_{d,l[1,k]} + B_{l[1,k]}\Phi^{-1}_{l[1,1]}(\cdot)\Gamma_{p,l[1,k]}\}\|_1)\|e_k(\cdot)\|_p \\
&+\|C_{l[1,k]}\exp(A_{l[1,k]})(\cdot)\|_p\|\Delta x(0)\|_p \\
&+\|C_{l[1,k]}\exp(A_{l[1,k]}(\cdot))B_{l[1,k]}\Phi^{-1}_{l[1,k]}(0)\Gamma_{d,l[1,k]}\|_p\|e_k(0)\|_p \\
&+\|C_{l[1,k]}\exp(A_{l[1,k]}(\cdot))\|_p\|\Delta\xi_k(\cdot)\|_p+\|\Delta w_{1,k}(t)\|_p. \qquad (3.7)
\end{aligned}
$$

Notating

$$
\begin{aligned}
\rho_{l[1,k]} =(&\|I - C_{l[1,k]}B_{l[1,k]}\Phi^{-1}_{l[1,k]}\Gamma_{d,l[1,k]}\|_\infty \\
&+\|C_{l[1,k]}\exp(A_{l[1,k]}(\cdot))\{[A_{l[1,k]}B_{l[1,k]}\Phi^{-1}_{l[1,k]}(\cdot) \\
&- B_{l[1,k]}\dot{\Phi}^{-1}_{l[1,k]}(\cdot)]\Gamma_{d,l[1,k]} + B_{l[1,k]}\Phi^{-1}_{l[1,1]}(\cdot)\Gamma_{p,l[1,k]}\}\|_1).
\end{aligned}
$$

Now, we observe that

$$
\begin{aligned}
\|e_{k+1}(\cdot)\|_p \le\ &\rho_{l[1,k]}\|e_k(\cdot)\|_p \\
&+\|C_{l[1,k]}\exp(A_{l[1,k]})(\cdot)\|_p\|\Delta x(0)\|_p \\
&+\|C_{l[1,k]}\exp(A_{l[1,k]}(\cdot))B_{l[1,k]}\Phi^{-1}_{l[1,k]}(0)\Gamma_{d,l[1,k]}\|_p\|e_k(0)\|_p \\
&+\|C_{l[1,k]}\exp(A_{l[1,k]}(\cdot))\|_p\|\Delta\xi_k(\cdot)\|_p+\|\Delta w_{1,k}(t)\|_p.
\end{aligned}
$$

From A1, it follows that the expressions equal

$$\lim_{k\to\infty}\|C_{l[1,k]}\exp(A_{l[1,k]})(\cdot)\|_p\|\Delta x(0)\|_p,$$

and

$$\lim_{k\to\infty}\|C_{l[1,k]}\exp(A_{l[1,k]}(\cdot))B_{l[1,k]}\Phi^{-1}_{l[1,k]}(0)\Gamma_{d,l[1,k]}\|_p\|e_k(0)\|_p.$$

are finite. To simplify the notation, denote

$$\lim_{k\to\infty}\|C_{l[1,k]}\exp(A_{l[1,k]})(\cdot)\|_p\|\Delta x(0)\|_p = \varepsilon_0,$$

and

$$\lim_{k\to\infty}\|C_{l[1,k]}\exp(A_{l[1,k]}(\cdot))B_{l[1,k]}\Phi^{-1}_{l[1,k]}(0)\Gamma_{d,l[1,k]}\|_p\|e_k(0)\|_p = \omega_0.$$

Also, we observe that

$$
\begin{aligned}
&\|C_{l[1,k]}\exp(A_{l[1,k]}(\cdot))\|_p\|\Delta\xi_k(\cdot)\|_p+\|\Delta w_{1,k}\|_p \\
&\qquad\qquad \le\|C_1\exp(A_1(\cdot))\|_p\|b_\xi + w_{1,0} = \beta_0
\end{aligned}
$$

Hence, by amalgamating the assumption (3.1) and the Lemma, we derive $\rho_{l[i,k]} \leq \rho < 1$ and

$$\lim_{k\to\infty} \sup\|e_{k+1}(\cdot)\|_p \leq \frac{\varepsilon_0 + \omega_0 + \beta_0}{1-\rho_1}. \tag{3.8}$$

Step 2: Now t beloge to second subinterval, *i.e.*, $t \in \Omega_2$.

Within the subinterval $[t_{i-1}, t_i)$, during the k-th iteration, the $l[2,k]$-th subsystem is triggered and the tracking error (3.5) becomes

$$\begin{aligned}
e_2(t) &= (I - C_{l[2,1]} B_{l[2,1]} \Phi^{-1}_{l[2,1]} \Gamma_{d,l[2,1]}) e_1(t) \\
&\quad - C_{l[2,1]} \int_0^t \exp(A_{l[2,1]}(t-s))\{[A_{l[2,1]} B_{l[2,1]} \Phi^{-1}_{l[2,1]}(s) \\
&\quad - B_{l[2,1]} \dot{\Phi}^{-1}_{l[2,1]}(s)]\Gamma_{d,l[2,1]} + B_{l[2,1]} \Phi^{-1}_{l[2,1]}(s)\Gamma_{p,l[2,1]}\} e_1(s) ds \\
&\quad - C_{l[2,1]} \exp(A_{l[2,1]}(t))\Delta x_1(t_1) \\
&\quad - C_{l[2,1]} \exp(A_{l[2,1]}(t)) B_{l[2,1]} \Phi^{-1}_{l[2,1]}(t_1)\Gamma_{d,l[2,1]} e_1(t_1) \\
&\quad - \int_0^t \exp(A_{l[21]}(t-s))\Delta\xi_1(s) ds + \Delta w_{2,1}(t), \\
e_3(t) &= (I - C_{l[2,2]} B_{l[2,2]} \Phi^{-1}_{l[2,2]} \Gamma_{d,l[2,2]}) e_2(t) \\
&\quad - C_{l[2,2]} \int_0^t \exp(A_{l[2,2]}(t-s))\{[A_{l[2,2]} B_{l[2,2]} \Phi^{-1}_{l[2,2]}(s) \\
&\quad - B_{l[2,2]} \dot{\Phi}^{-1}_{l[2,2]}(s)]\Gamma_{d,l[2,2]} + B_{l[2,2]} \Phi^{-1}_{l[2,2}(s)\Gamma_{p,l[2,2]}\} e_2(s) ds \\
&\quad - C_{l[2,2]} \exp(A_{l[2,2]}(t))\Delta x_2(t_1) \\
&\quad - C_{l[2,2]} \exp(A_{l[2,2]}(t)) B_{l[2,2]} \Phi^{-1}_{l[2,2]}(t_1)\Gamma_{d,l[2,2]} e_2(t_1) \\
&\quad - \int_0^t \exp(A_{l[2,2]}(t-s))\Delta\xi_2(s) ds + \Delta w_{2,2}(t), \\
&\vdots \\
e_{k+1}(t) &= (I - C_{l[2,k]} B_{l[2,k]} \Phi^{-1}_{l[2,k]} \Gamma_{d,l[2,k]}) e_2(t) \\
&\quad - C_{l[2,k]} \int_0^t \exp(A_{l[2,k]}(t-s))\{[A_{l[2,k]} B_{l[2,k]} \Phi^{-1}_{l[2,k]}(s) \\
&\quad - B_{l[2,k]} \dot{\Phi}^{-1}_{l[2,k]}(s)]\Gamma_{d,l[2,k]} + B_{l[2,k]} \Phi^{-1}_{l[2,k]}(s)\Gamma_{p,l[2,k]}\} e_2(s) ds \\
&\quad - C_{l[2,k]} \exp(A_{l[2,k]}(t))\Delta x_k(t_1) \\
&\quad - C_{l[2,k]} \exp(A_{l[2,k]}(t)) B_{l[2,k]} \Phi^{-1}_{l[2,k]}(t_1)\Gamma_{d,l[2,k]} e_k(t_1) \\
&\quad - \int_0^t \exp(A_{l[2,1]}(t-s))\Delta\xi_k(s) ds + \Delta w_{2,k}(t).
\end{aligned} \tag{3.9}$$

Operating with the Lebesgue-p norm on both sides of (3.9) and leveraging the Generalized Young's Inequality (GYI), we have

$$\begin{aligned}\|e_2(\cdot)\|_p \leq (&\|I - C_{l[2,1]}B_{l[2,1]}\Phi^{-1}_{l[2,1]}\Gamma_{d,l[2,1]}\|_\infty \\ &+\|C_{l[2,1]}\exp(A_{l[2,1]}(\cdot))\{[A_{l[2,1]}B_{l[2,1]}\Phi^{-1}_{l[2,1]}(\cdot) \\ &- B_{l[2,1]}\dot{\Phi}^{-1}_{l[2,1]}(\cdot)]\Gamma_{d,l[2,1]} + B_{l[2,1]}\Phi^{-1}_{l[2,1]}(\cdot)\Gamma_{p,l[2,1]}\}\|_1)\|e_1(\cdot)\|_p \\ &+\|C_{l[2,1]}\exp(A_{l[2,1]})(\cdot)\|_p\|\Delta x(t_1)\|_p \\ &+\|C_{l[2,1]}\exp(A_{l[2,1]}(\cdot))B_{l[2,1]}\Phi^{-1}_{l[2,1]}(t_1)\Gamma_{d,l[2,1]}\|_p\|e_1(t_1)\|_p \\ &+\|C_{l[1,1]}\exp(A_{l[1,1]}(\cdot))\|_p\|\Delta\xi_1(\cdot)\|_p+\|\Delta w_{1,1}(t)\|_p,\end{aligned}$$

$$\begin{aligned}\|e_2(\cdot)\|_p \leq (&\|I - C_{l[2,2]}B_{l[2,2]}\Phi^{-1}_{l[2,2]}\Gamma_{d,l[2,2]}\|_\infty \\ &+\|C_{l[2,2]}\exp(A_{l[2,2]}(\cdot))\{[A_{l[2,2]}B_{l[2,2]}\Phi^{-1}_{l[2,2]}(\cdot) \\ &- B_{l[2,2]}\dot{\Phi}^{-1}_{l[2,2]}(\cdot)]\Gamma_{d,l[2,2]} + B_{l[2,2]}\Phi^{-1}_{l[2,2]}(\cdot)\Gamma_{p,l[2,2]}\}\|_1)\|e_1(\cdot)\|_p \\ &+\|C_{l[2,2]}exp(A_{l[2,2]})(\cdot)\|_p\|\Delta x(t_1)\|_p \\ &+\|C_{l[2,2]}\exp(A_{l[2,2]}(\cdot))B_{l[2,2]}\Phi^{-1}_{l[2,2]}(t_1)\Gamma_{d,l[2,2]}\|_p\|e_1(t_1)\|_p \\ &+\|C_{l[1,2]}\exp(A_{l[1,2]}(\cdot))\|_p\|\Delta\xi_2(\cdot)\|_p+\|\Delta w_{2,2}(t)\|_p,\end{aligned}$$

$$\vdots$$

$$\begin{aligned}\|e_{k+1}(\cdot)\|_p \leq (&\|I - C_{l[2,k]}B_{l[2,k]}\Phi^{-1}_{l[2,k]}\Gamma_{d,l[2,k]}\|_\infty \\ &+\|C_{l[2,k]}\exp(A_{l[2,k]}(\cdot))\{[A_{l[2,k]}B_{l[2,k]}\Phi^{-1}_{l[2,k]}(\cdot) \\ &- B_{l[2,k]}\dot{\Phi}^{-1}_{l[2,k]}(\cdot)]\Gamma_{d,l[2,k]} + B_{l[2,k]}\Phi^{-1}_{l[2,1]}(\cdot)\Gamma_{p,l[2,k]}\}\|_1)\|e_k(\cdot)\|_p \\ &+\|C_{l[2,k]}\exp(A_{l[2,k]})(\cdot)\|_p\|\Delta x(t_1)\|_p \\ &+\|C_{l[2,k]}\exp(A_{l[2,k]}(\cdot))B_{l[2,k]}\Phi^{-1}_{l[2,k]}(t_1)\Gamma_{d,l[2,k]}\|_p\|e_k(t_1)\|_p \\ &+\|C_{l[2,k]}\exp(A_{l[2,k]}(\cdot))\|_p\|\Delta\xi_k(\cdot)\|_p+\|\Delta w_{2,k}(t)\|_p.\end{aligned} \tag{3.10}$$

Evidently, the limit $\lim_{k\to\infty}\sup\|x_d(t_1) - x_{k+1}(t_1)\|_p$ is infinitesimally small at time t_1 owing to the proof on Ω_1. This leads to
$\lim_{k\to\infty}\sup\|\Delta x_k(t_1)\|_p$ being sufficiently small. Hence,

$$\lim_{k\to\infty}\sup\|C_{l[2,k]}\exp(A_2(t-t_1))\|_p\|\Delta x_k(t_1)\|_p$$

is a finite positive quantity. Furthermore, (3.8) implies that

$$\lim_{k\to\infty}\sup\|e_k(t_1)\|_p$$

is finite.

Denoting that

$$\lim_{k\to\infty}\|C_{l[2,k]}\exp(A_{l[2,k]})(t-t_1)\|_p\|\Delta x(t_1)\|_p = \varepsilon_1,$$
$$\lim_{k\to\infty}\|C_{l[2,k]}\exp(A_{l[2,k]}(\cdot))B_{l[2,k]}\Phi^{-1}_{l[2,k]}(t_1)\Gamma_{d,l[2,k]}\|_p\|e_k(t_1)\|_p = \omega_1,$$

Also observe that

$$\begin{aligned}\|C_{l[2,k]}\exp(A_{l[2,k]}(\cdot))\|_p\|\Delta\xi_k(\cdot)\|_p+&\|\Delta w_{2,k}\|_p\\ &\leq\|C_{l[2,k]}\exp(A_{l[2,k]}(\cdot))\|_p b_\xi+w_{1,0}=\beta_1\end{aligned}$$

and synthesizing the condition (3.1) and utilizing Lemma 3.1, we derive

$$\lim_{k\to\infty}\sup\|e_{k+1}(\cdot)\|_p\leq\frac{\varepsilon_1+\omega_1+\beta_1}{1-\rho_2}.$$

The preceding analysis extends to the interval Ω_2, and by iteratively applying the same proof methodology across the successive intervals Ω_i for $l=2,3,...,n$, we deduce

$$\lim_{k\to\infty}\sup\|e_{k+1}(\cdot)\|_p\leq\frac{\varepsilon_{j-1}+\omega_{j-1}+\beta_{j-1}}{1-\rho_j},$$

The derived inequality holds true on the interval Ω_{j-1} for $j=1,2,3,\cdots,n$, encompassing the intervals $\Omega_1,\Omega_2,\cdots,\Omega_n$.

Let

$$M=\max\left\{\frac{\varepsilon_{j-1}+\omega_{j-1}+\beta_{j-1}}{1-\rho_j}\right\},(j=1,2,\cdots,n).$$

Consequently, we can assert that

$$\lim\sup\|e_{k+1}(\cdot)\|_p\leq M,$$

holds true for all time $t\in\Omega=[0,T]$, where $\Omega=\Omega_1\cup\Omega_2\cup\ldots\cup\Omega_n$ satisfying the criterion (2.5). Thus, the proof is complete □

Remark 3.3. If $w_{i,k}(t)=0$ and $,\xi_k(t)=0\forall k\in\mathbb{N}$, Consequently, the outcome coincides with the findings reported in [24].

Remark 3.4. ILC is adept at addressing system uncertainty, a vital consideration in linear switched systems. By iteratively refining its knowledge, ILC compensates for uncertainties in system dynamics. Furthermore, ILC can skillfully accommodate random time-iteration driven switching, a hallmark of the specified system. Its iterative nature enables ILC to adjust to evolving switching patterns. ILC also excels at rejecting noise and disturbances, ensuring accurate tracking performance in the presence of noise. Ultimately, ILC showcases robustness against system uncertainty, noise, and disturbances, ensuring consistent reliability.

This theorem establishes the convergence and ultimate boundedness of the tracking error for a PD-type ILC system with time-iteration varying switching rules. The result shows that the output trajectory approaches the desired trajectory with increasing iteration index, ensuring convergence, and the tracking error remains within a certain bound. This theorem has significant implications for applications in control systems, robotics, and automation, where precise tracking and convergence are crucial.

4. Applications:

(1) Simulation of power grid behavior with switching between various generation and consumption profiles, considering random iteration-based timing and system uncertainties.
(2) Robotic arm or autonomous vehicle control with task switching, accounting for system dynamics uncertainties and observation noise.

(3) Packet switching management in communication networks with random iteration-based timing and system uncertainties.
(4) Modeling of population dynamics or gene regulatory networks with state switching, subject to random iteration-based timing and system uncertainties.
(5) Macroeconomic dynamics modeling with regime switching, considering random iteration-based timing and system uncertainties.
(6) Mechanical system control with gear or mode switching, accounting for system dynamics uncertainties and observation noise.
(7) Aircraft or spacecraft dynamics modeling and control with flight mode switching, subject to random iteration-based timing and system uncertainties.
(8) Smart grid energy distribution and consumption management with switching between generation and consumption patterns, considering random iteration-based timing and system uncertainties.
(9) Traffic flow modeling and control with traffic light phase switching, subject to random iteration-based timing and system uncertainties.

5. Conclusion

This study conclusively demonstrated the effectiveness of a conventional PD-type ILC approach in addressing system uncertainties and bounded observation noise in switched repetitive systems. By leveraging the Lebesgue-2 norm and GYI, a rigorous analysis established a sufficient condition for convergence. The results unequivocally showed that The proposed PD-type ILC strategy ensures accurate trajectory tracking in switched systems, even with expanded switching rules in the time-iteration domain, resulting in reliable and precise control. These findings have significant implications for the control of complex systems, highlighting the potential of PD-type ILC as a robust and reliable control strategy. Future research can build upon these results to explore further applications and extensions of this control approach.

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References

[1] S. Arimoto, S. Kawamura and F. Miyazaki, *Bettering operation of robots by learning*, J Robotic Syst., **1(2)** (1984), 123-140.
[2] F. Miyazaki, S. Kawamura, M. Matsumori and S. Arimoto, *Learning control scheme for a class of robot systems with elasticity.* In 1986 25th IEEE Conference on Decision and Control, IEEE, (1986), 74-79.
[3] S. Kawamura, F. Miyazaki, and S. Arimoto, *Realization of robot motion based on a learning method.* IEEE Trans Syst Man Cybern., **18(1)** (1988), 126-134.
[4] D.A. Bristow, M. Tharayil, and A. G. Alleyne, *A survey of iterative learning control*, IEEE Contr Syst Mag, **26(3)** (2006), 96-114.
[5] R.J. Li and Z. Z. Han. *Survey of iterative learning control*, Control Decis., **20(9)** (2005), 961.

[6] H.S. Ahn, Y.Q. Chen, and K. L. Moore, *Iterative learning control: Brief survey and categorization*, IEEE Trans Syst Man Cybern C Appl Rev., **37(6)** (2007), 1099-1121.

[7] H.S. Ahn and D. Bristow, *Special issue on "iterative learning control*, Asian J Control, **13(1)** (2011), 1-2.

[8] Y. G. Sun, L. Wang, G. Xie, *Exponential stability of switched systems with interval time-varying delay*, IET Control Theory Appl., **3(8)** (2009), 1033-1040.

[9] A. Yu. Aleksandrov, Y. Chen, A. V. Platonov, and L. Zhang. *Stability analysis for a class of switched nonlinear systems*, Automatica, **47(10)** (2011), 2286-2291.

[10] X. Ruan, Z. Bien, *Convergence Properties of Iterative Learning Control Processes in the Sense of the Lebesgue - p Norm*, Asian J Control, **14(4)** (2012), 1095-1107.

[11] X. Ruan, J.B. Lian, and H. Z. Wu, *Convergence of iterative learning control with feedback information in the sense of Lebesgue-p norm.* Acta Autom Sin., **37(4)** (2011), 513-516.

[12] R. Xiaoe, C. Fengmin, and W. Jianguo, *Convergence analysis in the sense of lebesgue-p norm for open-closed-loop iterative learning control* . In 2007 Chinese Control Conference, IEEE, (2007), 511-514.

[13] J. Lian, and Y. Ge. "*Robust H_∞ output tracking control for switched systems under asynchronous switching*, Nonlinear Anal Hybrid Syst., **8** (2013), 57-68.

[14] X. Bu, F. Yu, and Z. Hou. *Iterative learning control for linear switched systems with arbitrary switched rules* . In Proceedings of the 10th world congress on intelligent control and automation, IEEE, (2012), 1182-1187.

[15] X. Bu, F. Yu, et al., *Iterative learning control for a class of linear discrete-time switched systems*, Acta Autom Sin., **39(9)** (2013), 1564-1569.

[16] X. Bu, Z. Hou, *Iterative learning control for a class of non-linear switched systems*, IET Control Theory Appl., **7(3)**, (2013), 470-481.

[17] X. Bu, F. Yu F, et al., *Stability analysis of high-order iterative learning control for a class of nonlinear switched systems*, Abstr Appl Anal., **1** (2013), 1-13, 684642 .

[18] O.H. Eddine, D. Jamel, B.A. Selma, and S. Salah. *Iterative learning control for Linear Discrete Time Switched Systems*, Proceedings-Copyright IPCO (2014), 310-321.

[19] X. Yang and X. Ruan, *Analysis of Iterative Learning Control for a Class of Linear Discrete-Time Switched Systems*, Abstr Appl Anal., (2015), 1-8, 464175.

[20] W. Cao, M. Sum, *Iterative learning control for discrete-time switched systems with attenuation factor*, J. Vib Control, **22(12)**, (2016), 2898-2906.

[21] J. Wang, Luo and Y. Hu, *Monotonically convergent hybrid ILC for uncertain discrete-time switched systems with state delay*, Trans Inst Meas Control, **39(7)** (2017), 1047-1058.

[22] Z. Shao and Z. Xiang, *Design of an Iterative Learning Control Law for a Class of Switched Repetitive Systems, Circuits*, J Syst Signal Process, **36(2)** (2017), 845-866.

[23] H. Ouerfelli, S. Attia and S. Salhi, *Monotonic Switching Iterative Learning Control Method for a Class of Discrete Time Switched System*, Int J Autom Smart Technol, **7(4)** (2017), 179-186.

[24] X. Yang and X. Ruan, *ILC for a Kind of Linear Switched Systems Specified by Random Time-Iteration Driven Switching Signals*. In 2018 IEEE 7th Data Driven Control and Learning Systems Conference (DDCLS), IEEE, (2018), 555-559.

[25] D. Shen, W. Zhang, Y. Wang, and C. J. Chien, *Almost sure and mean square convergence of ILC for linear systems with randomly varying iteration lengths*. In The 27th Chinese Control and Decision Conference (2015 CCDC), IEEE, (2015), 4546-4551.

[26] X. Bu, W. Yu, Q. Yu, Z. Hou, and J. Yang, *Event-triggered model-free adaptive iterative learning control for a class of nonlinear systems over fading channels*, IEEE Trans Cybern., **52(9)** (2021), 9597-9608.

[27] X. Zhou, H. Wang, Y. Tian, and X. Dai, *Consensus tracking via quantized iterative learning control for singular nonlinear multi-agent systems with state time-delay and initial state error*, Nonlinear Dyn., **103** (2021), 2701-2719.

[28] Z. Shao and Z. Duarr, *A high-order iterative learning control for discrete-time linear switched systems*. In 2018 57th Annual Conference of the Society of Instrument and Control Engineers of Japan (SICE), IEEE, (2018), 354-361.

[29] M. Ayatinia, M. Forouzanfar, and A. Ramezani. *An LMI approach to robust iterative learning control with initial state learning*, Int J Syst Sci., **53(12)** (2022), 2664-2678.

[30] X. Yang and X. Ruan. *Iterative learning control for linear continuous-time switched systems with observation noise*, Trans Inst Meas Control, **41(4)** (2019), 1178-1185.

[31] D. R. Sahu, and N.K. Singh. *Second-order iterative learning control for a class of switched discrete-time systems with model uncertainties, external noises and time-delay*, J Differ Equ Appl., **29(4)** (2023), 432-454.

[32] D.R. Sahu, and N. K. Singh, *Convergence of novel iterative learning control methods for a class of linear discrete-time switched systems*. In Innovations in Electrical and Electronic Engineering: Proceedings of ICEEE 2021, Springer Singapore, (2021), 441-457.

[33] J. V. D. Wijdeven, T. Donkers, and O. Bosgra. *Iterative learning control for uncertain systems: Robust monotonic convergence analysis*. Automatica, **45(10)** (2009), 2383-2391.

[34] X. Xu, J. Lu, and J. Chen. *Convergence Analysis of Iterative Learning Control for Initialized Fractional Order Systems*, Fractal and Fractional, **8(3)** (2024), 168.

[35] D. R. Sahu, and N.K. Singh. *p-Accelerated normal S-iterative learning control algorithm for linear discrete singular time-delay systems*, Int J Gen Syst., **53(1)** (2024), 16-40.

[36] D. R. Sahu, and N.K. Singh. *Robust tracking of discrete-time linear switched systems with disturbance via second-order ILC with data loss*. J Control Decis., (2024), 1-12.

[37] O. Dewangan. *Convergence analysis of proportional-derivative -type ILC for linear continuous constant time delay switched systems with observation noise and state uncertainties*, J Hyperstruct., **12(2)** (2023), 351-365. doi: 10.22098/jhs.2023.2808.

[38] O. Dewangan. *Fractional order PD^{α}-type ILC for linear continuous time-delay switched system with disturbance measurement and uncertainties noise*, J Hyperstruct., **13(2)** (2024), 305-320. doi: 10.22098/jhs.2024.15622.1035.

Omprakash Dewangan

Department of Mathematics, Indira Gandhi Govt. College Pandaria, Dist.- Kabirdham (C.G.), India.

Affiliated to Hemchand Yadaw Vishwavidyalaya Durg, (C.G.), India.

Email: omidewangan26@gmail.com