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Research Paper

AN APPLICATION OF FUZZY TIME SERIES WITH DIFFERENT UNIVERSAL DISCOURSE INTERVAL LENGTHS FOR RICE PRODUCTION IN INDIA

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ABSTRACT

In this paper, we offered a FTS-based tutorial on rice farming in India. The relevant literature is reviewed, which serves as a basis for the main concepts and models based on different forms of FTS forecasts. In an effort to inspire readers to contribute to this field of study, we also highlight the challenges and recent work that aims to fill in some of these knowledge gaps. Finally, time series forecasting is a useful tool for organizing and making decisions. An increasing number of methods, ranging from traditional statistical models to soft computing and artificial intelligence approaches, have been developed to generate increasingly accurate forecasts. PyFTS is an open-source, free Python library created by the Laboratory of Machine Intelligence and Data Science that implements a number of FTS models that have been published in the literature. In order to determine the interval in the fuzzy time series, Chen's method of FTS, comparing numerous values of n (Number of Interval) is used in this paper. We are interested to minimizing the MSE in the forecasting using PyFTS.

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1. Introduction

A fuzzy set is a class of items having varying degrees of membership. Such a set is defined by a membership (characteristic) function that assigns a grade of membership to each object between zero and one [1]. The concepts of inclusion, union, intersection, complement, relation, convexity, and so on are extended to fuzzy sets, and various features of these concepts are established. In particular, a separation theorem for convex fuzzy sets is demonstrated without needing the fuzzy sets to be disjoint. Many strategies have been developed for projecting enrolments using fuzzy time series [2]. The existing methods, however, do not provide adequate predicting accuracy. In this study, we describe a new approach for forecasting enrolments using fuzzy time series. The proposed method is a first-order, time-variant algorithm. The suggested method's forecasting process is illustrated using historical enrolment data from the University of Alabama. The proposed method can forecast enrolments more accurately than existing methods. It forecasts the daily TAIEX using the weights of the statistics of the fuzzy variations that appear in the fuzzy variation groups and FLRGs. Because the suggested method combines both fuzzy variation groups and FLRGs to thoroughly analyze the historical training data in [3], it achieves greater predicting accuracy rates for the TAIEX than existing methods. The results show that the Fuzzy Time Series Method outperforms the Heuristic model in terms of short-term forecasting. The ARIMA model produces reduced forecasting errors over a longer experiment period. However, adding increment information does not always improve the predicting performance of fuzzy time series. As a result of [4], the fuzzy time series method is more useful in situations where information is limited and decisions must be made quickly. [5] Extended the sample period in these models, the ARIMA models showed smaller than projected error and a closer predicted path to the realistic trend than the Fuzzy models, resulting in a more accurate forecast of the export amount than the Autoregressive Integrated Moving Average models. In economic terms, the amount of Taiwan exports is primarily due to external sources. This impact, however, decreases with time and export amount in the time series analysis. When all values or data are available, ARIMA models can be used to reliably estimate export value. The advantages of the fuzzy forecasting method are that it uses particle swarm optimization techniques to get the optimal partition of the intervals in the universe of discourse and uses the K-means clustering algorithm to cluster the subscripts of the fuzzy sets of the current states of the fuzzy logical relationships to get the cluster centre of each cluster and to divide the constructed fuzzy logical relationships into fuzzy logical relationship groups [6]. In terms of accuracy, the FTS forecasting method beats existing methods for forecasting the TAIEX and NTD/USD exchange rates. [7] Given beneficial technique to cope with forecasting issues and improve forecasting accuracy. The results of [8] demonstrated the statistical superiority of the FTSF-SVM approach and suggested an improved average-based method based on the well-known Friedman and Nemenyi hypothesis test. It is also discovered that the proposed FTSF approaches perform statistically better than their crisp TSF counterparts. [9] Performance in five additional structures. These findings indicate the robustness and accuracy of our concept when compared to typical adaptive neuro-fuzzy inference system models, as well as the various predictive techniques used in multiple pieces of literature. The FTNS model is the most accurate and efficient model for estimating the number of new cases of Covid-19. Conclusion: To the best of our knowledge, this is the first comparative analysis of three models for predicting Covid-19 new cases in Algeria. [10] Demonstrates that ARIMA models with appropriately selected covariates are effective tools for monitoring and predicting COVID-19 case trends in Algeria. Furthermore, this projection will assist health authorities in better preparing to combat the outbreak by using their healthcare facilities. [11] Examines the compatibility of a developed model for forecasting crop production using historical data from farms in India. The predicted values are compared to the computational method (CM) using various mean techniques. [12] Developed time series forecasting model for oil production based on an upgraded version of the adaptive neuro-fuzzy inference system (ANFIS). This model is enhanced using an optimization process known as the slime mould algorithm (SMA). The SMA is a new algorithm used to solve various optimization challenges. However, its search mechanism has drawbacks, such as trapping at local optima. The model was chosen using four performance criteria: statistical results, maximum likelihood, and standard error. These model, ARIMA (1, 1, 1), was tested with additional historical demand data under identical conditions. The findings show that the model can accurately evaluate and forecast future demand for petrol bookings in an online retail context [13]. These findings will provide reliable direction to the company's management during decision-making. The present work of this paper, we investigate that how predicting to rice production in India using fuzzy time series with different universal discourse interval lengths via Chen's Method.

2. Basic concepts of fuzzy time series

Definition 2.1. Let $U = \{u_i\}_{i=1}^{n}$ be the universe of discourse. A FSA_i in the universe of discourse U is defined as follows: $A_i = \sum_{i=1}^{n} \frac{\mu_{A_i}(u_i)}{u_i}$, Where μ_{A_i} is the MF of the FSA_i, $\mu_{A_i} : U \to [0,1], \mu_{A_i}(u_j)$is the degree of membership of u_j in A_i, $\mu_{A_i}(u_j) \in [0,1]$ and $1 \leq j \leq n$.
Let $U = u_{i(}i = 1)^n$ be the universe of discourse in which $FSA_i(t)(i = 1, 2......)$ are defined in the universe of discourse $U(t)$. Assume that $F(t)$is a collection of $A_i(t)$, then $F(t)$is called a FTS of $U(t)$.

Definition 2.2. Assume that there is a fuzzy relationship (FR)$FR(t-1,t)$, such that $F(t) = F(t-1) \circ FR(t-1,t)$, where the symbol $\circ$ represents the max-min composition operator, then $F(t)$ is called caused by $F(t-1)$. The relation FR is called first order model of $F(t)$. Further, if fuzzy relation $\mathrm{FR(t, t-1)}$ of F(t) is independent of time t , that is to say for different times $\mathrm{t_1}$ and $\mathrm{t_2}, \mathrm{FR}\left(\mathrm{t_1}, \mathrm{t_1}-1\right)=\mathrm{FR}\left(\mathrm{t_2}, \mathrm{t_2}-1\right)$, then F(t) is called a time invariant FTS .

Definition 2.3. Let $F(t-1) = \mathrm{A}_i$ and $F(t) = \mathrm{A}_j$, where A_i and A_j are fuzzy sets, then the FLR between $F(t-1)$ and $F(t)$ can be denoted by $A_i \to A_j$, where A_i and A_j are called the left-hand side(LHS) and the right hand side (RHS) of the FLR.

3. Fuzzy Time Series Chen's Method

The Chen's method Fuzzy Time Series (FTS) for the rice production is given blelow. The algorithm is now presented as following steps:
i. Select the minimum value in $\mathrm{U} = \{\mathrm{u_1}, \mathrm{u_2}, \ldots, \mathrm{u_n}\}$ ($U_{\min}$) and the maximum value ($U_{\max}$), define the universe discourse as $U = [U_{\min} - U_1, U_{\max} + U_2]$, where U_1 and U_2 are two suitable positive real values.
ii. Partition the universe of discourse U into n intervals $\mathrm{A}_1, A_2, \ldots, A_n$ of equal length.

$$A_1 = \frac{a_{11}}{\mu_1} + \frac{a_{12}}{\mu_2} + \ldots + \frac{a_{1m}}{\mu_m}$$
$$A_2 = \frac{a_{21}}{\mu_1} + \frac{a_{22}}{\mu_2} + \ldots + \frac{a_{2m}}{\mu_m}$$
$$\vdots$$
$$A_n = \frac{a_{n1}}{\mu_1} + \frac{a_{n2}}{\mu_2} + \ldots + \frac{a_{nm}}{\mu_n}$$

where $a_{11} \in [0,1]$. The value of a_{ij} indicates the grade of membership of u_j in the fuzzy set $\mathrm{A}_i(\mathrm{i} = 1, 2, \ldots, \mathrm{n})$[18].

The length of interval (L) using the following formula.

iii. L $= \frac{U_{\max} - U_{\min}}{n}, n =$ Number of the interval.

iv. Fuzzy Logical Relationships (FLRs) are derived based on the fuzzified historical data $U = \{u_1, u_2, \ldots, u_n\}$. where FLR $A_i \to A_k$ denoted as "if the data of the year i is A_i, then that of year $i+1$ is A_k, where A_i is called the current state of the data, and A_k is called the next state of the data".

v. Partition the FLRS in to group based on the current states of the data of FLRs. Thus based on below table, we can obtain FLRGs

FLRs	FLRG
$A_i \to A_{j1}$	
$A_i \to A_{j2}$	$A_i \to A_{j1}, A_{j2}, \ldots, A_{jk}$
$\ldots$	
$A_i \to A_{jk}$	

vi. Calculate the forecasted outputs. The calculations are carried out by the following principles.

If the fuzzified data of year i is $\mathrm{A_i}$, and there is only one FLRs in the FLRGs derived in above steps in which the current state of the data is A_k. Which is shown as follows:

$$\mathrm{A_i} \to \mathrm{A_k},$$

Where A_i and A_k are fuzzy sets and the maximum membership value of A_k occurs at interval u_k, and the mid-point of u_k is $m_{k;}$ then the forecasted data of year $\mathrm{i}+1$ is $\mathrm{m_k}$.

If the fuzzified data of year i is A_i, and there are the following FLRs in the FLRGs derived in above steps in which the current state of the FLRs are $\mathrm{A_j}$ respectively. which is shown as follows:

$$\mathrm{A_i} \to \mathrm{A_{j1}},\ \mathrm{A_i} \to \mathrm{A_{j2}}, \ldots,\ \mathrm{A_i} \to \mathrm{A_{jk}}$$

Where $\mathrm{A_j}, \mathrm{A_{j1}}, \mathrm{A_{j2}}, \ldots, \mathrm{A_{jk}}$ are fuzzy sets and the maximum membership values of $\mathrm{A_{j1}}, \mathrm{A_{j2}}, \ldots, \mathrm{A_{jk}}$ occurs at intervals $\mathrm{u_1}, \mathrm{u_2}, \ldots, \mathrm{u_k}$ respectively, and the mid-point of $u_1, u_2, \ldots, u_k$ is $m_1, m_2, \ldots, m_k$ respectively, then the forecasted data of year $i+1$ is $(m_1 + m_2 + \ldots + m_k)/k$.

vii. If the fuzzified data of year i is $\mathrm{A_i}$, and there do not exist any FLRGs whose current state of the data is A_i. where the maximum membership value of A_i occurs at interval u_i, and the mid-point of u_i is m_i, then the forecasted data of year $i+1$ is $\mathrm{m_i}$.

Using above algorithm in Python Fuzzy Time Series (pyFTS) with different interval lengths for data of rice production of farm (India).

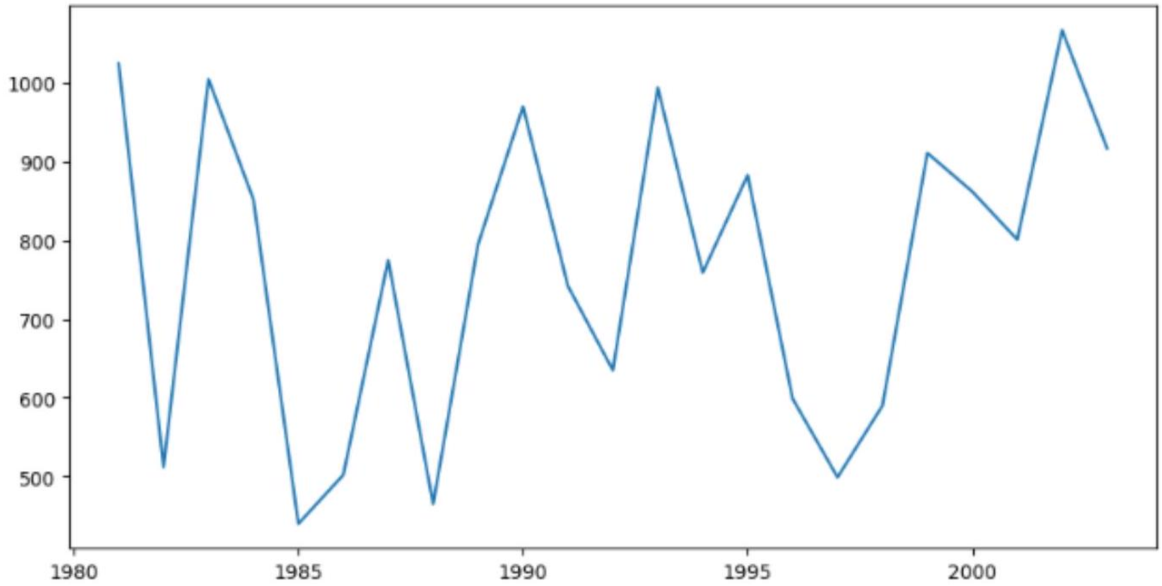

FIGURE 1. The historical time series data of rice production of farm (India)

4. Analysis of Data

Rice production data used yearly data in kg/ha, period of 1981 to 2003in India [11].
Table 1.The historical time series data of rice production of farm (India)

S.No	Year	AV	S.No	Year	AV	S.No	Year	AV
1	1981	1025	9	1989	795	17	1997	499
2	1982	512	10	1990	970	18	1998	590
3	1983	1005	11	1991	742	19	1999	911
4	1984	852	12	1992	635	20	2000	862
5	1985	440	13	1993	994	21	2001	801
6	1986	502	14	1994	759	22	2002	1067
7	1987	775	15	1995	883	23	2003	917
8	1988	465	16	1996	599			

AV - Actual Value (Production) (kg/ha)
Plotting the time series data for rice output in India from 1981 to 2003 is the first step in forecasting this fuzzy time series, as Figure 1 below illustrates.

This section describes a two-part pyFTS. Figure 1 depicts the initial attempts to offer data on rice production along with step-by-step prediction tools. It compares the results and demonstrates how to use different values of Chen's model. This data will be used to forecast India's average annual rice output index. To accomplish this, the data will be divided into a single variable time series as well as training and testing groups.

5. pyFTS Library and Prediction Process

Partitioning the universe of discourse and generating fuzzy sets. Grid partitioning will be used to partition U, however the pyFTS.partitioners package also has additional techniques.

The minimal number of fuzzy sets or partitions (n-part), the membership function(MF) (triangular by default), and the time series data on rice production. The various membership functions are available in pyFTS.common. Membership and disclosure are necessary if the series uses any transformations.

6. Forecasting Procedures

Based on the effective interval that was achieved, linguistic values related to the number of intervals formed can be computed at the fuzziness stage. The fuzzy time series F will be created in this stage by converting the numerical values of the time series Y into fuzzy values of the linguistic variable $A(t)$. Tables 1 through 7 display the linguistic numbers obtained from the fuzzification of the Chen Model.The fuzzy sets generated for the variable average are displayed in Figure 2.

Table 1. Interval length = 300 with average based fuzzy time series

S. No	Actual Value	Fuzzified Linguistic Value (FLV)	Mid-value
1	396	$\mathrm{A}_0 = [300, 600]$	450
2	785	$\mathrm{A}_1 = [600, 900]$	750
3	1174	$\mathrm{A}_2 = [900, 1200]$	1050

Table 2. Interval length = 225 with average based fuzzy time series

S. No	Actual Value	Fuzzified Linguistic Value (FLV)	Mid-value
1	396	$\mathrm{A}_0 = [300, 525]$	412.5
2	655	$\mathrm{A}_1 = [525, 750]$	637.5
3	914	$\mathrm{A}_2 = [750, 975]$	862.5
4	1174	$\mathrm{A}_3 = [975, 1200]$	1087.5

Table 3. Interval length = 180 with average based fuzzy time series

S. No	Actual Value	Fuzzified Linguistic Value (FLV)	Mid-value
1	396	$\mathrm{A}_0 = [300, 480]$	390
2	655	$\mathrm{A}_1 = [480, 660]$	570
3	914	$\mathrm{A}_2 = [660, 840]$	750
4	1174	$\mathrm{A}_3 = [840, 1020]$	930
5	1174	$\mathrm{A}_3 = [1020, 1200]$	1110

Table 4. Interval length = 150 with average based fuzzy time series

S. No	Actual Value	Fuzzified Linguistic Value (FLV)	Mid-value
1	396	$\mathrm{A}_0 = [300, 450]$	375
2	552	$\mathrm{A}_1 = [450, 600]$	525
3	707	$\mathrm{A}_2 = [600, 750]$	675
4	863	$\mathrm{A}_3 = [750, 900]$	825
5	1018	$\mathrm{A}_4 = [900, 1050]$	975
6	1174	$\mathrm{A}_5 = [1050, 1200]$	1125

Table 5. Interval length = 100 with average based fuzzy time series

S. No	Actual Value	Fuzzified Linguistic Value (FLV)	Mid-value
1	396	$A_0 = [300, 400]$	350
2	493	$A_1 = [400, 500]$	450
3	590	$A_2 = [500, 600]$	550
4	688	$A_3 = [600, 700]$	650
5	785	$A_4 = [700, 800]$	750
6	882	$A_5 = [800, 900]$	850
7	979	$A_6 = [900, 1000]$	950
8	1076	$A_7 = [1000, 1100]$	1050
9	1174	$A_8 = [1100, 1200]$	1100

Table 6. Interval length = 90 with average based fuzzy time series

S. No	Actual Value	Fuzzified Linguistic Value (FLV)	Mid-value
1	396	$A_0 = [300, 390]$	345
2	482	$A_1 = [390, 480]$	435
3	569	$A_2 = [480, 570]$	525
4	655	$A_3 = [570, 660]$	615
5	742	$A_4 = [660, 750]$	705
6	828	$A_5 = [750, 840]$	795
7	914	$A_6 = [840, 930]$	885
8	1001	$A_7 = [930, 1020]$	975
9	1087	$A_8 = [1020, 1110]$	1065
10	1174	$A_9 = [1110, 1200]$	1155

Table 7. Interval length = 90 with average based fuzzy time series

S. No	Actual Value	Fuzzified Linguistic Value (FLV)	Mid-value
1	396	$A_0 = [300, 375]$	337.5
2	467	$A_1 = [375, 450]$	412.5
3	537	$A_2 = [450, 525]$	487.5
4	608	$A_3 = [525, 600]$	562.5
5	679	$A_4 = [600, 675]$	637.5
6	750	$A_5 = [675, 750]$	712.5
7	820	$A_6 = [750, 825]$	787.5
8	891	$A_7 = [825, 900]$	862.5
9	962	$A_8 = [900, 975]$	937.5
10	1032	$A_9 = [975, 1050]$	1012.5
11	1103	$A_{10} = [1050, 1125]$	1087.5
12	1174	$A_{11} = [1125, 1200]$	1162.5

Figure 2. Fuzzified Linguistic Value (FLV) of rice production for $n = 3, 4, 5, 6, 9, 10, 12$.

7. Generation of fuzzy logical rules (FLRs)

The FLRs will be grouped in order to create the FTS model. The models used in pyFTS can be found in the pyFTS.models package. This instance will employ the Conventional FTS model, which was released in [13].

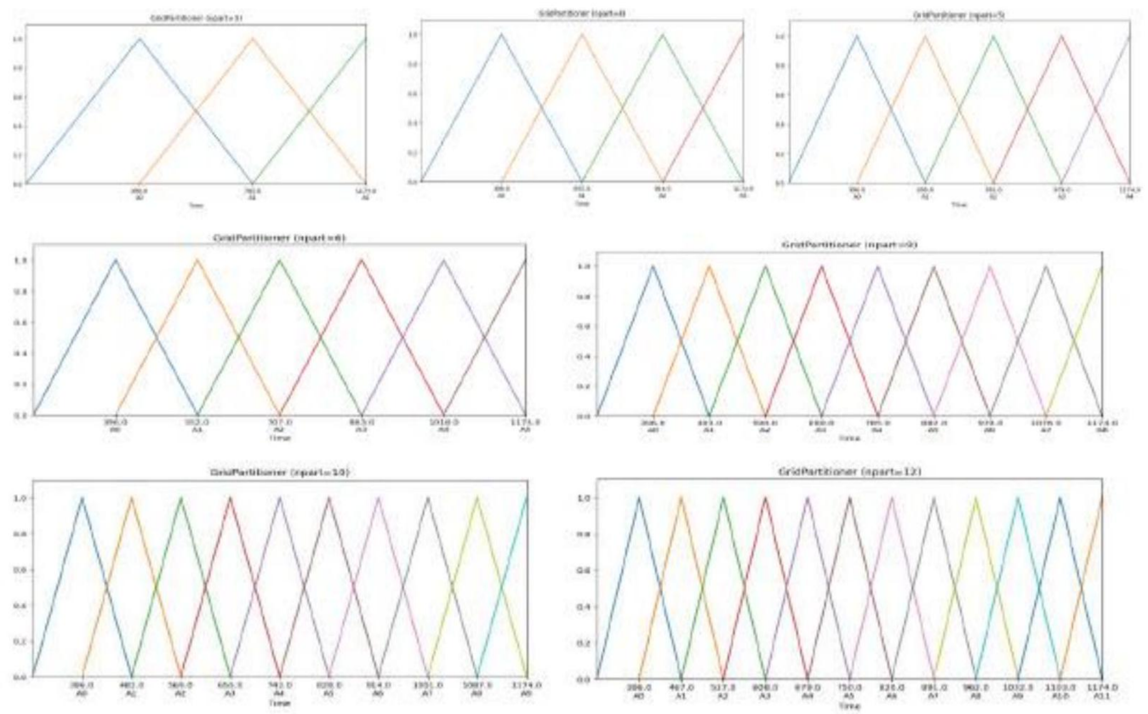

FIGURE 2. The historical time series data of rice production of farm (India)

fuzzyfied value for npart=3:
A2', 'A0', 'A2', 'A1', 'A0', 'A0', 'A1', 'A0', 'A1', 'A1', 'A1', 'A1', 'A2','A1', 'A1', 'A1', 'A0', 'A0', 'A1', 'A1', 'A1', 'A2', 'A1'

Rules for npart=3:

Conventional FTS:

A2 -> A0,A1

A0 > A0,A1,A2

A1 > A0,A1,A2

MSE for npart=3: 60599,60869565218

fuzzyfied value for npart=4:
A2', 'A0', 'A2', 'A2', 'A0', 'A0', 'A1', 'A0', 'A2', 'A2', 'A1', 'A1', 'A2', 'A1', 'A2','A1', 'A0', 'A1', 'A2', 'A2', 'A2', 'A3', 'A2'

Rules for npart=4:

Conventional FTS:

A0 -> A0, A1,A2

A1 » A0,A1,A2

A2 -> A0,A1,A2,A3

A3 -> A2

NSE for npart=4: 19680.472355072463

fuzzyfied value for npart=5:
A3', 'A1', 'A3', 'A2', 'AQ', 'A1', 'A2', 'A0', 'A2', 'A3', 'A2', 'A1', 'A3', 'A2','A3', 'A1', 'A1', 'A1', 'A3', 'A2', 'A2', 'A3', 'A3'

Rules for npart=5:

Conventional FTS:

A2 -> A0,A1,A2,A3

A0 -> A1,A2

A3 -> A1,A2,A3

A1 -> A1,A2,A3

MSE for npart=5: 36794.43359374999

fuzzyfied value for npart=6:
A4’, ’A1’, ’A4’, ’A3’, ’AQ’, ’A1’, ’A2’, ’AQ’, ’A3’, ’A4’, ’A2’, ’A2’, ’A4’, ’A2’,’A3’, ’A1’, ’A1’, ’A1’, ’A3’, ’A3’, ’A3’, ’A4’, ’A3’

Rules for npart=6:

Conventional FTS:

A4 -> A1,A2,A3

A3 » A0,A1,A3,A4

A0 -> A1,A3

A2 -> A0,A2,A3,A4

A1 » A1,A2,A3,A4

MSE for npart=6: 47865.95465217394

fuzzyfied value for npart=9:
A6’, ’A1’, ’A6’, ’A5’, ’A0’, ’A1’, ’A4’, ’A1’, ’A4’, ’A6, ’A4’, ’A2’, ’A6’, ’A4’, ’A5’, ’A2’, ’A1’, ’A2’, ’A5’, ’A5’, ’A4’, ’A7’, ’A5’

Rules for npart=9:

Conventional FTS:

A0 -> A1

A5 » A0, A2,A4,A5

A6 -> A1,A4,A5

A1 -> A2,A4,A6

A2 -> A1,A5,A6

A4 -> A1,A2,A5,A6, A7

A7 -> A5

MSE for npart=9: 45133.96236904627

fuzzyfied value for npart=10:
A7’, ’A1’, ’A7’, ’A5’, ’A1’, ’A1’, ’A4’, ’A1’, ’A5’, ’A7’, ’A4’, ’A3’, ’A7’, ’A4’, ’A6’, ’A2’, ’A1’, ’A2’, ’A6’, ’A5’, ’A5’, ’A8’, ’A6’

Rules for npart=10:

Conventional FTS:

A7 -> A1,A4,A5

A4 -> A1,A3,A6

A6 -> A2,A5

A2 -> A1,A6

A1 -> A1,A2,A4,A5,A7

A5 -> A1,A5,A7,A8

A8 -> A6

A3 -> A7

fuzzyfied value for npart=12:
A9’, ’A2’, ’A9’, ’A6’, ’A1’, ’A1’, ’A5’, ’A1’, ’A6’, ’A8’, ’A5’, ’A3’, ’A8’, ’A5’, ’A7’, ’A3’, ’A1’, ’A3’, ’A7’, ’A7’, ’A6’, ’A9’, ’A7’

Rules for npart=12:

Conventional FTS:

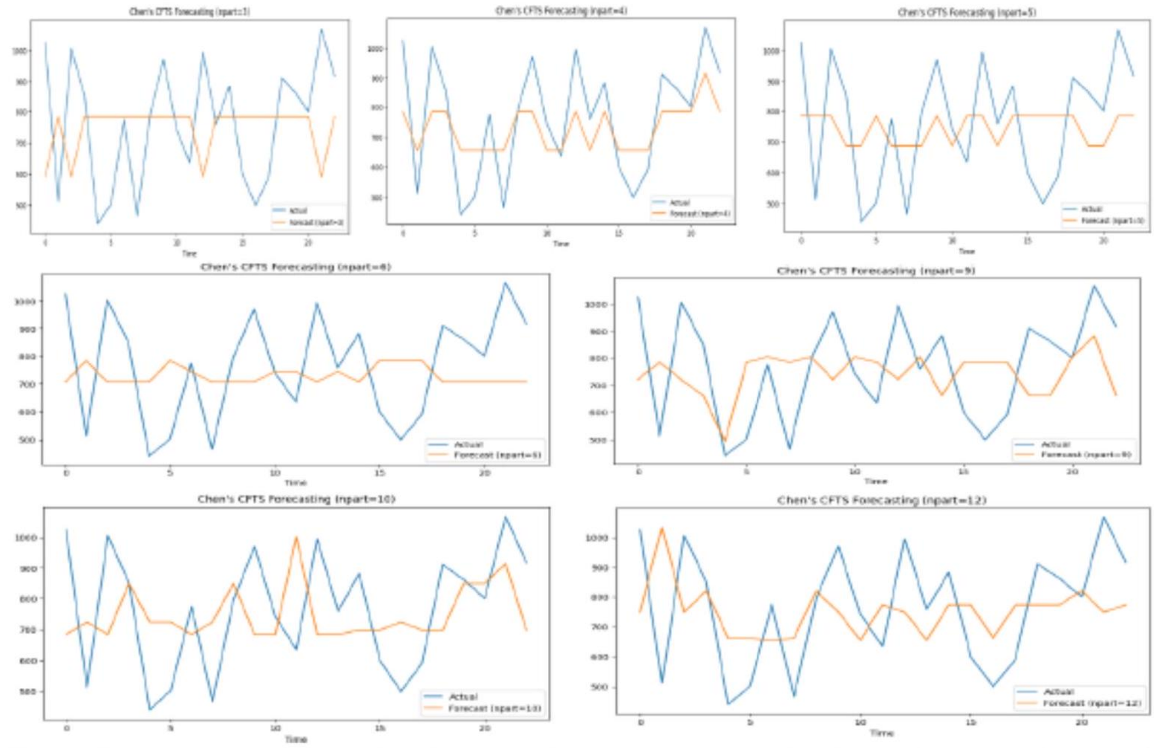

Figure 3. The historical time series data of rice production of farm (India)

A5 -> A1,A3,A7
A1 -> A1,A3,A5,A6
A9 -> A2,A6,A7
A8 -> A5
A3 -> A1,A7,A8
A7 -» A3,A6,A7
A6 -» A1,A8,A9
A2 -> A9
MSE for npart=12: 40753.597838164256

8. Defuzzification

$f(t+1)$ must be converted to a numerical value. The predict technique makes predictions by utilizing the training model. Examine the model being used; not all of them are compatible with all forms of forecasts. Lastly, the forecast horizon, or the number of steps ahead you wish to forecast, is indicated by the steps ahead parameter.Then, the comparison of actual data and forecasting data of rice production for different interval lengths using pyFTS.models package.

Figure 3.Comparison of actual and forecasting data of rice production with for $n = 3, 4, 5, 6,\ 9,\ 10,\ 12$.

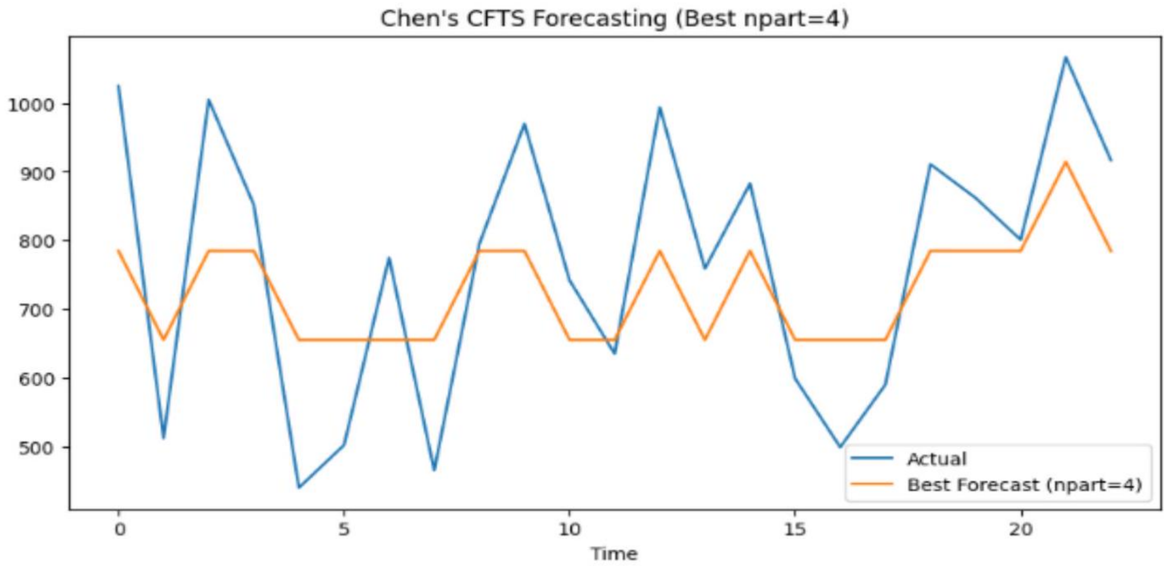

FIGURE 4. The historical time series data of rice production of farm (India)

We demonstrated how to put these ideas into reality by predicting the average daily index of rice production using the one variable Chen approach for a range of partition number values.

n-value	3	**4**	5	6	9	10	12
MSE	60599.81	**19680.47**	36794.43	47665.95	45133.96	44310.73	41847.77

The MSE value is low 19680.47 for n = 4 in Chen's Method of Fuzzy Time Series (FTS).

```
Best npart: 4 (lowest MSE: 19680.472355072463)
Best Model Rules:
Conventional FTS:
A0 -> A0,A1,A2
A1 -> A0,A1,A2
A2 ->A0,A1,A2,A3
A3 -> A2
```

Figure 5.Comparison of actual data and forecasting data for n = 4

9. Conclusion

An overview of the pertinent literature is conducted, providing a foundation for the key ideas and FTS-based models for various time series and forecast types. By outlining the difficulties and current studies meant to close some of these gaps, we also encourage readers to make contributions to this field of study. Finally, using the pyFTS library, we demonstrated how to put these ideas into effect by predicting the average yearly index of rice production in India using one variable FTS Chen's approaches for varying universe discourse lengths. We attain the best results for MSE of forecasting data of each length of interval, then best MSE for n = 4.

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Appendix

from pyFTS.partitioners import Grid from pyFTS.models import chen from pyFTS.common import FLR import matplotlib.pyplot as plt from sklearn.metrics import $mean_{s}quared_{e}rror$

Load training and testing datasets #train = $Enrollments.get_{d}ata()$ #test = $Enrollments.get_{d}ata()$

Define a list of npart values
$\#npart_{v}alues = [3, 4, 5, 6, 9, 10, 12]$

Get user input for the range of npart values
$lower_{b}ound$ = int(input("Enter the lower bound of npart range: "))
$upper_{b}ound$ = int(input("Enter the upper bound of npart range: "))

Define the range of npart values
$npart_{r}ange$ = range(lower$_{b}ound, upper_{b}ound + 1)$

for npart in npart$_{r}ange$:
$\#UniverseofDiscoursePartitioner$

$partitioner = Grid.GridPartitioner(data = train, npart = npart)$

Plot GridPartitioner
fig, ax = plt.subplots(figsize=(10, 5)) partitioner.plot(ax=ax)
Pass the 'ax' argument

plt.title(f'GridPartitioner (npart=npart)')
plt.xlabel('Time')
plt.show()

fuzzyfied = partitioner.fuzzyfy(data, method='maximum', mode='sets')
print(f"fuzzyfied value for npart=npart:")
print(fuzzyfied)

# Create an empty model using the Chen(1996) method model =
chen.ConventionalFTS(partitioner=partitioner)

# The training procedure is performed by the method fit
model.fit(train)

# Print rules
print(f"Rules for npart=npart:")
print(model)

# The forecasting procedure is performed by the method predict
forecasts = model.predict(test)

# Calculate Mean Squared Error
mse = mean$_{s}quared_{e}rror(test, forecasts)$

# Print MSE for the current npart
print(f"MSE for npart=npart: mse")

# Plotting
plt.figure(figsize=(10, 5))
plt.plot(test, label='Actual')
plt.plot(forecasts, label=f'Forecast (npart=npart)')
plt.title(f'Chenś CFTS Forecasting (npart=npart)')
plt.legend()
plt.xlabel('Time')
plt.show()
# Print a separator for better readability
print("" + "=" * 50 + "")
from sklearn.metrics import mean$_{s}quared_{e}rror$

$best_npart = None$
$best_model = None$
$best_mse = float('inf')$

for npart in npart$_range$:
$\#UniverseofDiscoursePartitioner$
$partitioner = Grid.GridPartitioner(data = train, npart = npart)$

Create an empty model using the Chen(1996) method model = chen.ConventionalFTS(partitioner=partitioner)

The training procedure is performed by the method fit model.fit(train)

The forecasting procedure is performed by the method predict forecasts = model.predict(test) # Calculate Mean Squared Error mse = mean$_squared_error(test, forecasts)\#CheckifcurrentnparthasthebestMSE$
$ifmse < best_mse :$
$best_mse = mse$
$best_npart = npart$
$best_model = model$
$\#Printaseparatorforbetterreadability$
$print("" + " = " * 50 + "")$
$\#Printthebestnpart, bestmodelrules, andbest$
MSE
$print(f"npart : best_npart(lowestMSE : best_mse)")$
$print(f"ModelRules : ")$
$print(best_model)$
$\#Trainandplotthebestmodel$
$plt.figure(figsize = (10, 5))$
$plt.plot(test, label =' Actual')$
$plt.plot(best_model.predict(test), label = f'BestForecast(npart = best_npart)')$
$plt.title(f'ChenCFTSForecasting(Bestnpart = best_npart)')$
$plt.legend()$
$plt.xlabel('Time')plt.show()$