DOI: 10.22098/jhs.2026.16978.1081

Journal of Hyperstructures, Article in Press

Journal Homepage: https://jhs.uma.ac.ir/

Research Paper

A STUDY OF SASAKIAN MANIFOLDS ADMITTING $*$-CONFORMAL η-RICCI SOLITON

BARNALI LAHA

[1]Department of Mathematics, Shri Shikshayatan College , Kolkata, India, barnali.laha87@gmail.com

ARTICLE INFO

Article history:
Received: 12 March 2025
Accepted: 22 June 2026
Communicated by Dariush Latifi

Keywords:
Sasakian manifold
Ricci soliton
$*$-Conformal η-Ricci soliton
$\eta-$ Einstein manifold
$C^{\infty}-$function
Codazzi type Ricci tensor

MSC:
53C15; 53C25

ABSTRACT

The present paper is to deliberate the class of Sasakian manifolds admitting $*$-conformal $\eta-$Ricci soliton. In our next section we have given a brief discussion on the above class of manifolds. In continuation to this we have given few preliminary idea on Sasakian manifolds. Among others we have proved several conditions under which Sasakian manifold is an $\eta-$Einstein manifold. Lastly we have cited an example of the above said manifold for verifying our findings. Insert the abstract of your paper here.

1. Introduction

In 1982 R.S. Hamilton [14, 15] introduced the notion of Ricci flow to find a canonical metric on a smooth manifolds. Ricci flow is an evolution equation for metric on a Riemannian manifold. He states that Ricci soliton move under the Ricci flow simply by the diffeomorphism. If the initial metric, that is, they are stationary points of the Ricci flow which is given by

$$\frac{\partial g}{\partial t} = -2Ric \tag{1.1}$$

$*$Address correspondence to B. Laha; Department of Mathematics, Shri Shikshayatan College , Kolkata, India, E-mail: barnali.laha87@gmail.com.

where Ric is the Ricci tensor of the metric $g(t)$. It was formed to give the answer of thruston geometric conjecture, according to which each closed three manifold admits a geometric decomposition. After which Ricci flow become one of the powerful tool to study Riemannaian manifolds.

The solution of the Ricci flow equation is known as a Ricci soliton [14, 15] in case it is driven only by one parameter family of diffeomorphism and scaling. Ricci soliton in contact metric manifolds was analysed by M. M. Tripathi, C. L. Bejan and M Crusmareanu[3].

A Ricci Soliton is a natural generalization of Einstein metric. A Ricci Soliton (g, V, λ) on a Riemannian manifold (M, g) is defined as

$$\frac{1}{2}\pounds_V g + Ric = -\lambda g \tag{1.2}$$

where L_V denotes the Lie derivative with respect to $V \in \chi(M)$, Ric is the Ricci tensor of g and λ is a constant. The Ricci soliton is shrinking, steady and expanding depending on $\lambda < 0, \lambda = 0, \lambda > 0$ respectively. Otherwise it is called indefinite.

A. E Fisher [12] in 2004 introduced a new concept known as conformal Ricci flow a variation of the classical Ricci flow equation which has revised the unit volume constraints of that equation to a Scalar Curvature constraint. In the classical Ricci flow equation, the unit volume constraint plays an important role but in the Conformal Ricci flow equation, the scalar curvature r is considered as a constraint. The conformal Ricci flow on M is defined by the following equation

$$\frac{\partial g}{\partial t} + 2\{Ric(g) + \frac{g}{n}\} = -pg, r(g) = -1 \tag{1.3}$$

where r is the scalar curvature of the manifold, p is a scalar non dynamical field(time dependent scalar field) also known as conformal pressure.

As a generalization of Ricci Soliton, the notion of η -Ricci Soliton was introduced by Cho and Kimura [6]. This notion has also been studied in [8] for Hopf hypersurfaces in complex space forms. A $\eta-$Ricci soliton is a tuple (g, V, λ, μ) where V is a vector field on M, λ and μ are constants and g is a Riemannian (or pseudo-Riemannian)metric satisfying the equation

$$\pounds_\xi V + 2S + 2\lambda g + 2\mu(\eta \otimes \eta) = 0 \tag{1.4}$$

where S is the Ricci tensor associated to g. In this connection we mention the work of Blaga[4][5] and Prakash et al.[21] on $\eta-$Ricci soliton. In particular if $\mu = 0$ then the notion of $\eta-$Ricci soliton (g, V, λ, μ) reduces to the notion of Ricci soliton (g, V, λ). If $\mu \neq 0$, then the $\eta-$Ricci soliton is named as proper $\eta-$Ricci soliton. N. Basu and A. Bhattacharyya [2] in 2015 establish the notion of Conformal Ricci soliton and Conformal $\eta-$Ricci soliton and both equations are defined as

$$\pounds_V g + 2S = [2\lambda - (p + \frac{2}{n})]g, \tag{1.5}$$

$$\pounds_V g + 2S + [2\lambda - (p + \frac{2}{n})]g + 2\mu\eta \otimes \eta = 0 \tag{1.6}$$

The study of Ricci solitons has become central in modern differential geometry because of their deep connections to the Ricci flow, geometric analysis, and classification problems in Riemannian and pseudo-Riemannian manifolds. Among their various generalizations, η-Ricci solitons were introduced to incorporate the influence of a distinguished vector field η, allowing a richer interplay between curvature and additional geometric structures such as contact, almost contact, or more general f-structures.

On the other hand, the $*$-conformal curvature tensor extends the classical notion of conformal curvature by integrating the $*$-operator, which naturally arises in the study of almost Hermitian, contact metric and paracontact geometries. Conditions involving the $*$-conformal tensor serve as refined curvature constraints that capture subtle geometric features not detected by ordinary conformal flatness or Weyl-type conditions.Introducing $*$-conformal η-Ricci solitons therefore brings these two lines of research together. This unification is motivated by several considerations:

- Richer geometric evolution: Ricci solitons model self-similar solutions to the Ricci flow. When the soliton equation is modified by incorporating the η-term and $*$-conformal curvature, it yields evolution equations compatible with contact or almost contact structures, thereby extending the flow theory to a broader class of manifolds.
- Compatibility with geometric structures: On manifolds endowed with an additional structure—such as Kenmotsu, Sasakian, para-Sasakian, or $N(k)$-contact manifolds—the $*$-conformal curvature interacts naturally with the characteristic vector field. The η-term in the soliton equation allows the soliton to reflect this interaction, making the resulting geometric conditions more intrinsic.
- Generalization of known results: The notion of a $*$-conformal η-Ricci soliton contains several important special cases, such as classical Ricci solitons, conformal Ricci solitons, η-Ricci solitons, and $*$-Ricci solitons. Studying this broader framework thus provides a unified approach for recovering and extending many known results.
- Curvature characterization: Curvature conditions involving the $*$-conformal tensor often lead to strong restrictions on the geometry. When these are coupled with the η-Ricci soliton equation, they can yield classification results, uniqueness theorems, or structural properties of the underlying manifold.
- Potential applications: Since Ricci-type solitons appear in mathematical physics (e.g., general relativity, string theory) and in geometric evolution equations, introducing $*$-conformal η-Ricci solitons may lead to new models where both conformal geometry and distinguished vector fields are essential.

The notion of $*$-Ricci tensor on almost Hermitian manifolds was introduced by Tachibana [23]. Later Hamada [13] studied $*-$Ricci flat real hypersurfaces of complex space forms.
Now we shall know the history and development of Sasakian manifolds admitting $*$-Conformal η-Ricci soliton:

2. Preliminaries

In [22]S.Tanno classified the connected almost contact metric manifold whose automorphism group has maximum dimension : There are three classes:-
a) Homogeneous normal contact Riemannian manifolds with constant $\phi-$ holomorphic sectional curvature if the sectional curvature of the plain section containing ξ, say $C(X,\xi)>0$.
b) Global Riemmanian product of a line or a circle and a Käehlerian manifold with constant holomorphic sectional curvature, $C(X,\xi)=0$.
c) A warped product space $RX_{\lambda}C^n$, if $C(X,\xi)<0$.

Manifold of class (a) are characterized by some tensor equations, it has a Sasakian structure and manifolds of class (b) are characterized by a tensorial relation admitting a cosymplectic structure. In 1972 Kenmotsu has introduced a new class of almost contact Riemannian manifolds which are nowadays called Kenmotsu manifolds[16]. He obtained some tensorial equations to characterize manifolds of class (c).

Let (M,ϕ,ξ,η,g) be an $(2n+1)$-dimensional almost contact metric manifold. Then the product $\bar{M}=M\times R$ has a natural almost complex structure J with the product metric G being Hermitian manifold $(\bar{M},J,G)$. The notion of trans-sasakian manifolds was introduced by Oubina [19] in 1985. In general, a Trans-sasakian manifold $(M,\phi,\xi,\eta,g,\alpha,\beta)$ is called a trans-Sasakian manifold of type (α,β). Trans-Sasakian manifold of type $(\alpha,0)$ is called α-Sasakian manifolds and Trans-Sasakian manifold of type $(0,\beta)$ is called β-Kenmotsu manifolds.
An $(2n+1)$-dimensional smooth manifold $\left(M^{2n+1},g\right)$ is said to admit an almost contact sturcture[9], if it admits a $(1,1)$ tensor field ϕ, a structure vector field ξ, a 1- form η and a Riemannian metric g such that

$$\phi^2\left(X\right)=-X+\eta\left(X\right)\xi,\ g\left(\phi X,\phi Y\right)=g\left(X,Y\right)-\eta\left(X\right)\eta\left(Y\right) \tag{2.1}$$

$$\eta\left(\xi\right)=1,\quad g(X,\xi)=\eta(X), \tag{2.2}$$

for any X, Y $\in\chi(M)$. In addition to this it satisfies

$$\eta\left(\phi X\right)=0,\quad \phi\xi=0. \tag{2.3}$$

Then M becomes an almost contact metric manifold equipped with an almost contact metric structure (ϕ,ξ,η,g). Also we have

$$\Phi(X,Y)=-\Phi(Y,X), \tag{2.4}$$

where $\Phi(X,Y)=g(X,\phi Y)$.
An almost contact metric structure becomes a contact metric structure if

$$g(X,\phi Y)=d\eta(X,Y),$$

for all X,Y tangent to M. The 1-form η is then a contact form and ξ is its characteristic vector field. If ξ is a killing vector field, the contact metric manifold (M,η,ξ,ϕ,g) is called K-contact manifold. The contact structure on M is said to be normal if the almost complex structure on M X R defined by $J(X,f\frac{d}{dt})=(\phi X-f\xi,\eta(X)\frac{d}{dt})$, where f is a real function on M X R, is integrable. A normal contact metric manifold is called Sasakian manifold. Sasakian metrics are K-contact and K-contact 3-metrics are Sasakian. For this case we have

$$\nabla_X \xi = -\phi X \tag{2.5}$$

$$(\nabla_X \phi)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.6}$$

where ∇ denotes the Riemannian connection. The Curvature tensor of 3-dimensional Riemannian manifold is given by

$$R(X,Y)Z = [S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] - \frac{r}{2}[g(Y,Z)X - g(X,Z)Y],$$

where S and r are Ricci tensor and scalar curvature respectively and Q is the Ricci operator defined by $g(QX,Y) = S(X,Y)$. Curvature tensor and Ricci tensor also satisfies following relations

$$g(R(X,Y)Z,\xi) = \eta(R(X,Y)Z) = g(Y,Z)\eta(X) - g(X,Z)\eta(Y), \tag{2.7}$$

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.8}$$

$$R(\xi,X)Y = -g(X,Y)\xi - \eta(Y)X, \tag{2.9}$$

$$R(\xi,X)\xi = X + \eta(X)\xi, \tag{2.10}$$

$$S(X,\xi) = 2n\eta(X), \tag{2.11}$$

$$S(\xi,\xi) = -2n, \tag{2.12}$$

$$Q\xi = 2n\xi, \tag{2.13}$$

$$S(\phi X,\phi Y) = S(X,Y) - 2n\eta(X)\eta(Y). \tag{2.14}$$

here R is the curvature tensor of manifold of type $(1,3)$ and S is Ricci tensor of type $(0,2)$. An example of a three-dimensional Sasakian manifold has been given [9].

We also define the following

Definition *(2.1) A Concircular curvature tensor C in an $(2n+1)$-Sasakian manifold M is defined by*

$$C(X,Y)Z = R(X,Y)Z - \frac{r}{2n(2n+1)})[g(Y,Z)X - g(X,Z)Y], \tag{2.15}$$

where R is the curvature tensor of manifold of type $(1,3)$ and r is the scalar curvature of the manifold [1][7][9].

In this section we also define η-Einstein manifold :

Definition *(2.2) A Sasakian manifold M is said to be η-Einstein manifold if its Ricci tensor*

S is of the form

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y) \tag{2.16}$$

where a and b are scalar functions on M.
We now move to our next section which basically focusses on our findings:

3. A few results on Sasakian manifold admitting $*-$conformal $\eta-$Ricci soliton and satisfying $R(\xi, X).S = 0$

Here we consider Sasakian manifold admitting $*-$conformal $\eta-$Ricci soliton. Then we can state

Theorem (3.1) **:** *If an* $(2n+1)-$*dimensional Sasakian manifold admitting* $*$- *conformal* $\eta-$*Ricci soliton, then the manifold is an* $\eta-$*Einstein manifold of the form* (3.4) *and the scalars* λ, p *and* μ *are related by* (3.7)*.*

Proof : Let us consider (M, ϕ, ξ, η, g) to be an $(2n+1)$-dimensional Sasakian manifold admitting $*$-conformal $\eta-$Ricci soliton then we have

$$\pounds_\xi g(X,Y) + 2S^*(X,Y) + [2\lambda - (p + \frac{2}{2n+1})]g(X,Y) + 2\mu\eta(X)\eta(Y) = 0. \tag{3.1}$$

We know that $\pounds_\xi g(X,Y) = 0$ so putting this fact in above equation (3.1) we obtain

$$S^*(X,Y) = -[\lambda - \frac{1}{2}(p + \frac{2}{2n+1})]g(X,Y) - \mu\eta(X)\eta(Y) \tag{3.2}$$

Also we have for Sasakian manifolds

$$S^*(X,Y) = S(X,Y) - (2n-1)g(X,Y) - \eta(X)\eta(Y) \tag{3.3}$$

In view of above equation, we obtain

$$S(X,Y) = [2n - 1 - \lambda + \frac{1}{2}(p + \frac{2}{2n+1})]g(X,Y) - (\mu - 1)\eta(X)\eta(Y) \tag{3.4}$$

which is of the form $S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y)$, where $a = 2n - 1 - \lambda + \frac{1}{2}(p + \frac{2}{2n+1})$ and $b = 1 - \mu$.
Putting $Y = \xi$ we get

$$S(X,\xi) = [2n - \lambda - \mu + \frac{1}{2}(p + \frac{2}{2n+1})]\eta(X) \tag{3.5}$$

Also we know

$$S(X,\xi) = 2n\eta(X) \tag{3.6}$$

Comparing equations (3.5) and (3.6)we get

$$\lambda + \mu = 2n + \frac{1}{2}(p + \frac{2}{2n+1}) \tag{3.7}$$

We are also in a position to state our next theorem:
Theorem (3.2) **:** *Let* (M, g, ϕ, ξ, η) *be a* $(2n+1)$*-dimensional Sasakian manifold satisfying*

$R(\xi, X).S = 0$ and admitting $*-$Conformal $\eta-$ Ricci Soliton then the manifold is an Einstein manifold of the form (2.16).

Proof : Let us consider (M, g, ϕ, ξ, η) be a $(2n+1)$-dimensional Sasakian manifold satisfying R(ξ, X).S^*=0 and admitting $*-$Conformal $\eta-$ Ricci Soliton. Then can write

$$S^*(R(\xi, X)Y, Z) + S^*(Y, R(\xi, X)Z) = 0, \tag{3.8}$$

for all $X, Y, Z \in \chi(M)$. By using (2.9) in (3.8) we can calculate

$$S^*(g(X, Y)\xi - \eta(Y)X, Z) + S^*(Y, g(X, Z)\xi - \eta(Z)X) = 0. \tag{3.9}$$

Taking $Z = \xi$ in (3.9) we get

$$S^*(g(X, Y)\xi - \eta(Y)X, \xi) + S^*(Y, g(X, \xi)\xi - \eta(\xi)X) = 0. \tag{3.10}$$

Simplifying (3.10) we can obtain

$$g(X, Y)S^*(\xi, \xi) - \eta(Y)S^*(X, \xi) + g(X, \xi)S^*(Y, \xi) - S^*(Y, X) = 0. \tag{3.11}$$

Using (3.3) in (3.10) we get

$$g(X, Y)[S(\xi, \xi) - (2n-1)g(\xi, \xi) - \eta(\xi)\eta(\xi)] - \eta(Y)[S(X, \xi) - 2n\eta(X)] + \eta(X)[S(Y, \xi) - 2n\eta(Y)]$$
$$= S(Y, X) - (2n-1)g(Y, X) - \eta(X)\eta(Y). \tag{3.12}$$

Using (3.5) in (3.12) and simplifying we get

$$S(X, Y) = (2n-1)g(X, Y) + \eta(X)\eta(Y) \tag{3.13}$$

This proves that the manifold M is an $\eta-$Einstein manifold.
In our next section we shall discuss a result on Concircular Curvature tensor:

4. $*-$**conformal** $\eta-$**Ricci soliton in Sasakain manifold and satisfying** $C(\xi, X).S = 0$

Theorem (4.1) ***:*** *Let (M, g, ϕ, ξ, η) be a $(2n+1)$-dimensional Sasakian manifold satisfying $C(\xi, X).S = 0$ and admitting $*-$Conformal $\eta-$ Ricci Soliton then the manifold is an $\eta-$Einstein manifold of the form* (4.5).

Proof : Let us consider (M, g, ϕ, ξ, η) be a $(2n+1)$-dimensional Sasakian manifold satisfying C(ξ, X).S=0 and admitting $*-$Conformal $\eta-$ Ricci Soliton. Let us suppose $C(X, Y)f = 0$, where f is a C^∞ function. Then $C(\xi, X).S^* = 0$ is defined.
We consider $C(\xi, X).S^*(Y, Z) = 0$, which implies

$$S^*(C(\xi, X)Y, Z) + S^*(Y, C(\xi, X)Z) = 0 \tag{4.1}$$

We know from [1], [7] the Concircular curvature tensor C in a $(2n+1)-$ dimensional Sasakian manifold is given by (2.15). Replacing $X = \xi$ and $Y = X$ we get

$$C(\xi, X)Y = (1 - \frac{r}{2n(2n+1)})(g(X, Y)\xi - \eta(Y)X). \tag{4.2}$$

On using (4.2) in (4.1) we obtain

$$(1 - \frac{r}{2n(2n+1)})[g(X, Y)S^*(\xi, \xi) - \eta(Y)S^*(X, \xi) + g(X, \xi)S^*(Y, \xi) - S^*(X, Y) = 0. \tag{4.3}$$

Using (3.3), (3.4) and (3.5) in (4.3) we get

(4.4) $$(1-\frac{r}{2n(2n+1)})\{S(X,Y)-g(X,Y)(-\lambda-\mu-2n-1+\frac{1}{2}(p+\frac{2}{2n+1})+\eta(X)\eta(Y)\}=0.$$

This implies

(4.5) $$r=2n(2n+1), S(X,Y)=g(X,Y)[-\lambda-\mu-2n-1+\frac{1}{2}(p+\frac{2}{2n+1})]+\eta(X)\eta(Y)$$

5. **Example of a 3-dimensional Sasakian manifold**

Let $M=\{(x,y,z)\in\mathbf{R}^3|(x,y,z)\neq(0,0,0)\}$ be a three-dimensional Sasakian manifold. The vector fields

$$e_1=\frac{\partial}{\partial x}, e_2=\frac{\partial}{\partial y}, \xi=e_3=\frac{\partial}{\partial z}-y\frac{\partial}{\partial x}+x\frac{\partial}{\partial y}$$

are linearly independent at each point of M. We define the Riemannian metric g by

$$g(e_1,e_1)=1=g(e_2,e_2)=g(e_3,e_3), g(e_i,e_j)=0$$

where $i,j\in\{1,2,3\}$ and $i\neq j$. The (1,1) tensor field ϕ is defined as

$$\phi(e_1)=e_2,\ \phi(e_2)=-e_1,\ \phi(e_3)=0.$$

If η is 1-form then $\eta(e_3)=g(e_3,e_3)=1$. We can easily verify by the linearity of ϕ and g that (ϕ,ξ,η,g) satisfies (2.1) and (2.2).
Let ∇ be the Levi-Civita connection on M. The non-zero Lie-Brackets are

$$[e_1,e_2]=[e_2,e_1]=0,\ [e_1,e_3]=-e_2,\ [e_2,e_3]=e_1.$$

By using Koszul's formula for the Riemannian metric g, we can find

$$\nabla_{e_1}e_1=-e_3, \nabla_{e_1}e_2=0, \nabla_{e_1}e_3=e_1,$$

$$\nabla_{e_2}e_1=0, \nabla_{e_2}e_2=-e_3, \nabla_{e_2}e_3=e_2,$$

$$\nabla_{e_3}e_1=e_2, \nabla_{e_3}e_2=-e_1, \nabla_{e_3}e_3=0.$$

We can verify the equations (2.7) and (2.8) from above metric. Therefore the manifold is Sasakian manifold of dimension 3.

Also from the relation of Riemmanian curvature tensor we can calculate the following components

$$R(e_1,e_1)=, R(e_2,e_2)=2, R(e_3,e_3)=2,$$

From these curvature tensors, we can obtain the components of Ricci tensors as follows :

$$S(e_1,e_1)=2=S(e_2,e_2), S(e_3,e_3)=2$$

So $Ric=2g$.
This matches the fact that a 3-D Sasakian manifold has constant ϕ-sectional curvature. Now we will derive the soliton vector fields. A Ricci-type soliton normally uses

$$\pounds_V g(X,Y),$$

So we choose

$$V = a_1e_1 + a_2e_2 + a_3e_3.$$

Now we compute the Lie Derivative

$$(\pounds_V g)(X,Y) = g(\nabla_X V, Y) + g(X, \nabla_Y V)$$

Using the connection components $(\pounds_V g)(e_i, e_j)$

$$\nabla_{e_1} V = a_1\nabla_{e_1}e_1 + a_2\nabla_{e_1}e_2 + a_3\nabla_{e_1}e_3 = -a_1e_3 + 0 + a_3e_1$$

Now from $\eta-$ Soliton equation

$$(\pounds_V g) + 2S + 2\lambda g + 2\mu\eta\otimes\eta = 0$$

we have

$Ric = 2g$ and $\eta\otimes\eta(e_3, e_3) = 1$, others zero. So the equation becomes componentwise. Fore_1, e_1,

$$(\pounds_V g)(e_1, e_1) + 4 + 2\lambda = 0.$$

Fore_2, e_2,

$$(\pounds_V g)(e_2, e_2) + 4 + 2\lambda = 0.$$

Fore_3, e_3,

$$(\pounds_V g)(e_3, e_3) + 4 + 2\lambda + 2\mu = 0.$$

And the off diagonal components

$$(\pounds_V g)(e_i, e_j) = 0, i \neq j$$

Now from $*-$conformal $\eta-$Ricci Soliton

$$(\pounds_V g) + 2S^* + 2\lambda g + 2\mu\eta\otimes\eta = 0$$

where S^* is the $*$-Ricci Tensor

$$S^*(X,Y) = Ric(X,Y) - g(\phi X, \phi Y) + \eta(X)\eta(Y).$$

For the basis vectors $g(\phi e_1, \phi e_1) = g(e_2, e_2) = 1, g(\phi e_2, \phi e_2) = 1, g(\phi e_3, \phi e_3) = 0$. Thus

$$S^*(e_1, e_2) = 2 - 1 + 0 = 1, S^*(e_2, e_2) = 2 - 1 + 0 = 1, S^*(e_3, e_3) = 2 - 0 + 1 = 3.$$

Now from

$$S(e_3, e_3) = 2$$

Also we have from (3.4)

$$S(e_3, e_3) = [1 - \lambda + \frac{1}{2}(p + \frac{2}{2n+1})(-1) + (\mu - 1)$$

Equating above two equations we obtain

$$\lambda + \mu = 4 + \frac{1}{2}(p + \frac{2}{3})$$

which is equation (3.7). Hence the result is verified.

References

[1] A. Barman, *Concircular Curvature Tensor on a P-Sasakian Manifold admitting a Quarter-symmetric metric connection*, Kragujevac Journal of MathematicsVolume 42(2) (2018), 275–285.

[2] N. Basu and A. Bhattaharyya, *Conformal Ricci Soliton in Kenmotsu manifold*, Global Journal of Advanced Research on Classical and Modern Geometries, 4(2015), 15-21.

[3] C. L. Bejan, M. Crasmareanu, *Ricci Soliton in manifolds with quasi constant curvature*, Publ. Math. Debrecen, 78 (1)(2011), 235-243.

[4] A. M. Blaga, *$\eta-$Ricci Soliton on Lorentzian-Para Sasakian manifolds*, Filomat, 30(2)(2016), 489-496.

[5] A. M. Blaga, *$\eta-$Ricci Soliton on Para-Kenmotsu manifolds* , Balkan J. Geom. Appl., 20(2015), 1-13.

[6] J.T. Cho and M. Kimura, *Ricci Solitons and Real hypersurfaces in a Complex Space Forms*, Tohoku Math. J., 61(2)(2009), 205-212.

[7] V.Chavan, *Some Properties of a Concircular Curvature Tensor on Generalized Sasakian-Space-Forms*, Applied Math., 2(2022), 609–620.

[8] C. Calin and M. Crasmareanu, *From thenhart problem to Ricci solitons in f-Kenmotsu manifolds*, Bull. Malays. Math. Soc., 33(3)(2010), 361-368.

[9] U. C. De, A. A. Shaikh, *Complex Manifolds and Contact Manifolds*, Narosa Publishing House, 2009.

[10] T. Dutta, A. Bhattacharyya, *Ricci Soliton and Conformal Ricci Soliton in Lorentzian β-Kenmotsu manifolds*, International J. Math. Combin., 2(2018), 1-12.

[11] T. Dutta, N. Basu, A. Bhattacharyya, *Ricci Soliton in Lorentzian Sasakian manifolds*, Acta Univ. Palacki Olomuc, Fac. Rev. Mat., Mathematica, 55(2016), 57-70.

[12] A.E. Fisher, *An Introduction to Conformal Ricci Flow*, Class Quantum Grav, 21(2004), 171-218.

[13] T.Hamada, *Real hypersurfaces of complex space forms in terms of Ricci $*$ tensor*, Tokyo J.Math.25(2002), 473-483.

[14] R. S. Hamilton, *Three manifolds with positive Ricci curvature*, Journal of Differential Geometry, 17(2)(1982), 255-306.

[15] R. S. Hamilton, *The Ricci flow on surfaces, Mathematics and General Relativity*, (Santa Cruz, CA, 1986) Contemp. Math. Amer. Math. Soc., (1988), 237-262.

[16] K. Kenmotsu, *A class of almost contact Riemannian manifolds*, Tohoku Mathematical Journal, 24(1972), 93-103.

[17] I. Mihai and R. Rosca, *On Lorentzian P-Sasakian manifolds*, Classical Analysis, World Scientific Publications., Singapore, 155-169, (1992).

[18] B. Laha, *D-homothetic Deformations of Lorentzian Para-Sasakian Manifold*, International J.Math. Combin. 2(2019), 34-42.

[19] J. A. Oubina, *New classes of almost contact metric structure*, Publ. Math. Debrecen, 32(1985), 187-193.

[20] S.Pahan, *η-Ricci solitons on 3-dimensional trans-Sasakian manifolds*, CUBO, A Mathematical Journal, 22(1)(2020), 22-37.

[21] D. G. Prakasha and B. S. Hadimani $\eta-$ *Ricci Soliton on para-Sasakian manifolds*, J. Geom., 108(2017), 383-392.

[22] S. Tanno, *The automorphism groups of almost contact Riemannian manifolds*, Tohoku Math. J., 21(1969),21-38.

[23] S.Tachibana, *On almost analytic vectors in almost kahlerian manifolds*, Tohoku Math.J.11(2)(1959),247-265.

[24] M.M. Tripathi and U.C. De, *Lorentzian almost paracontact manifolds and their submanifolds*, J. Korea Soc. Math. Educ. Ser. B, Pure Appl. Math., 8(2001), 101-105.