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Research Paper

SOME FIXED POINT THEOREM USES INTIMATE MAPPINGS TO PRODUCE INTUITIONISTIC FUZZY METRIC SPACE

Ram Milan Singh

ABSTRACT. The aim of this research paperto use the concept of Intimate mappings to demonstrate the existence and uniqueness of common fixed point theorems for self-mappings in intuitionistic fuzzy metric space.

Key Words: Fuzzy metric space, Intuitionistic Fuzzy Metric Space, Intimate mappings, E.A property, Common E.A property.

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1. INTRODUCTION

The fuzzy set is a new notion that was introduced by L.A. Zadeh [1] as an extension of the classical set. Kramosil and Mechalek later presented the idea of fuzzy metric space in [2]. George and Veeramani [4] further modified this to produce Harsdorff topology for the category of fuzzy metric spaces. Following that, other fixed point theorems in fuzzy metric space were discovered under a variety of circumstances, including ([5],[6],[9],[10],[11], and [13]). Sahu and colleagues [12] introduced the idea of Intimate mappings, which are generalised compatible mappings of type (α), under different circumstances. Chugh and Madhu Aggarwal

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∗Address correspondence to R. M. Singh; E-mail: rammilansinghlig@gmail.com.

[13] expanded these further, leading to the development of some findings in Hausdorff uniform spaces. Additional results are also visible, such as [14], which makes use of intimate mappings in complex valued metric space. In addition, Praveenkumar and associates [15] established a number of theorems in multiplicative metric space (MMS) by employing the concept of intimate mappings. As a result, numerous findings were made possible on this platform, including. Aamri and Matouwakil [16] established the idea of non-compatible mappings as the E. A property in metric space. Thus, Yicheng Liu et al. [17] presented the idea of enhanced E.A property, which led to the creation of common property E.A.

2. Preliminaries

Definition 2.1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:
(a) $*$ is commutative and associative;
(b) $*$ is continuous;
(c) $a^*1 = a$ for all $a \in [0,1]$;
(d) $a^*\ b \leq c^*\ d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$.

Definition 2.2. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is a continuous t-conorm if it satisfies the following conditions:
(a) $\diamond$ is commutative and associative;
(b) $\diamond$ is continuous;
(c) $a\diamond 0 = a$ for all $a \in [0,1]$;
(d) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, b \in [0,1]$:

Definition 2.3. A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm, $\diamond$ a continuous t-conorm and M, N are fuzzy sets on $X \times X \times (0, \infty)$, satisfying the following conditions for all $x, y, z \in X$ and $s, t > 0$:

a. $M(x, y, t) + N(x, y, t) \leq 1$;
b. $M(x, y, t) > 0$;
c. $M(x, y, t) = 1$ if and only if $x = y$;
d. $M(x, y, t) = M(y, x, t)$;
e. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
f. $M(x, y,) : (0, \infty) \to (0, 1]$ is continuous;
g. $N(x, y, t) \geq 0$;
h. $N(x, y, t) = 0$ if and only if $x = y$;
i. $N(x, y, t) = N(y, x, t)$;

j. $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t+s)$;
k. $N(x, y,) : (0, \infty) \to (0, 1]$ is continuous.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and degree of non-nearness between x and y with respect to t respectively [11].

Definition 2.4. A triplet $(\mathrm{X}, M_{KM}, {}^{*})$ is a fuzzy metric space (i.e., FMS) if X is a arbitrary set, * is continuous t - norm and M_{KM} is fuzzy set on $\mathrm{X}^2 \times (0, \infty)$ satisfying the following conditions for all $\mathrm{a, b, d} \in \mathrm{X}$ and $t, s \in (0, \infty)$:

- (KMFM-i) $M_{KM}(a, b, 0) = 0$
- (KMFM-ii) $M_{KM}(a, b, t) = 1, \forall t > 0 \Leftrightarrow \mathrm{a} = \mathrm{b}$
- (KMFM-iii) $\mathrm{M_{KM}(b, a}, t) = \mathrm{M_{KM}(a, b}, t)$
- (KMFM-iv) $\mathrm{M_{KM}(a, d}, t+s) \geq \mathrm{M_{KM}(a, b}, t) * \mathrm{M_{KM}(b, d}, s)$
- (KMFM-v) $\mathrm{M_{KM}(a, b,)} : .[0.1] \to [0, 1]$ left continuous.

Definition 2.5. A triplet $(X, M_{KM}, N_{KM}, *, \diamond)$ is an intuitionistic fuzzy metric space (i.e., IFMS) if X is a arbitrary set, * is continuous t - norm,$\diamond$ is continuous $t-$ co norm and M_{KM} and N_{KM} are fuzzy set on $\mathrm{X}^2 \times (0, \infty)$ satisfying the following conditions for all $\mathrm{a, b, d} \in \mathrm{X}$ such that $t, s \in (0, \infty)$:

- (KMFM-i) $\mathrm{M_{KM}(a, b, 0)} = 0$
- (KMFM-ii) $M_{KM}(a, b, t) = 1 \forall t > 0 \Leftrightarrow \mathrm{a} = \mathrm{b}$
- (KMFM-iii) $\mathrm{M_{KM}(b, a}, t) = \mathrm{M_{KM}(a, b}, t)$
- (KMFM-iv) $\mathrm{M_{KM}(a, d}, t+s) \geq \mathrm{M_{KM}(a, b}, t) * \mathrm{M_{KM}(b, d}, s)$
- (KMFM-v) $\mathrm{N_{KM}(a, b,)} : :[0.1] \to [0, 1]$ left continuous.
- (KMFM-vi)$N(\mathrm{a, b}, 0) = 0$
- (KMFM-vii) $\mathrm{N_{KM}(a, b}, t) = 0 \forall t > 0 \Leftrightarrow \mathrm{a} = \mathrm{b}$
- (KMFM-viii) $\mathrm{N_{KM}(b, a}, t) = \mathrm{N_{KM}(a, b}, t)$
- (KMFM-ix) $\mathrm{N_{KM}(a, d}, t+s) \leq \mathrm{N_{KM}(a, b}, t) \diamond \mathrm{N_{KM}(b, d}, s)$
- (KMFM-x)$\mathrm{N_{KM}(a, b,))} : [0.1] \to [0, 1]$ right continuous.

Definition 2.6. Let $\langle \mathrm{a}_n \rangle$ be sequence in IFMS $(X, M_{KM}, N_{KM}, *, \diamond)$. We say that $\langle \mathrm{a}_n \rangle$ converges to a point $l \in X$ if:

$$\lim_{n \to \infty} \mathrm{M_{KM}} \left(\mathrm{a}_n, \mathrm{l}, t\right) = 1 \text{ and } \lim_{n \to \infty} \mathrm{N_{KM}} \left(\mathrm{a}_n, \mathrm{l}, t\right) = 0 \quad \forall t > 0.$$

Definition 2.7. Let $\langle \mathrm{a}_n \rangle$ be a sequence in IFMS $(X, M_{KM}, N_{KM}, *, \diamond)$, this sequence $\langle \mathrm{a}_n \rangle$ in X is said to be Cauchy sequence in FMS if $\lim_{n \to \infty} \mathrm{M_{KM}} \left(\mathrm{a}_{n+p}, \mathrm{a}_n, t\right) = 1$ and $\lim_{n \to \infty} \mathrm{N_{KM}} \left(\mathrm{a}_{n+p}, \mathrm{a}_n, t\right) = 0$ for every $t > 0$ and $p > 0$.

Definition 2.8. If every Cauchy sequence is convergent in $(X, M_{KM}, N_{KM}, *, \diamond)$, then we say that X is complete.

Definition 2.9. Let $(X, M_{KM}, N_{KM}, *, \diamond)$ be an $IFMS$ and $\mathcal{G}$ and $\mathfrak{J}$ be two self mappings on X. Then $\mathcal{S}$ and $\mathcal{T}$ are

(1) compatible if $\lim_{n\to\infty} M_{KM}(\mathcal{G}\mathfrak{T}_n, \mathfrak{T}G\mathrm{a}_n, t) = 1$ and $\lim_{n\to\infty} N_{KM}(\mathcal{G}\mathfrak{T}_n, \mathfrak{T}G\mathrm{a}_n, t) = 0$ whenever a sequence $\langle \mathrm{a}_n \rangle$ in X provided that $\lim_{n\to\infty} \mathcal{G}\mathrm{a}_n = \lim_{n\to\infty} \mathfrak{J}\mathrm{a}_n = t$ for some $t \in$ X.

(2) compatible of type (α) if $\lim_{n\to\infty} \mathrm{M}_{KM}(\mathcal{G}\mathcal{T}\mathrm{a}_n, \mathfrak{T}\mathrm{a}_n, t) = 1$ and $\lim_{n\to\infty} \mathrm{N}_{KM}(\mathfrak{T}G\mathrm{a}_n, \mathcal{G}\mathrm{a}_n, t) = 0$ whenever $\langle \mathrm{a}_n \rangle$ in X such that $\lim_{n\to\infty} \mathcal{G}\mathrm{a}_n = \lim_{n\to\infty} \mathfrak{T}\mathrm{a}_n = t$ for some $t \in$ X.

Now we discuss some definitions related to intimate mappings in IFMS.

Definition 2.10. Let A and B be two mappings of the IFMS $(X, M_{KM}, N_{KM}, *, \diamond)$ into itself. Then A and B are said to be $\mathcal{A}$-intimate mappings if

$$\alpha \mathrm{M_{KM}}(A\mathrm{a}_n, B\mathrm{a}_n, t) \geq \alpha \mathrm{M_{KM}}(A\mathrm{a}_n, B\mathrm{a}_n, t)$$

and

$$\alpha \mathrm{N_{KM}}(A\mathrm{a}_n, B\mathrm{a}_n, t) \leq \alpha \mathrm{N_{KM}}(A\mathrm{a}_n, B\mathrm{a}_n, t)$$

where $\alpha = \lim_{n\to\infty}$ Sup or $\lim_{n\to\infty}$ Inf and $\langle \mathrm{a}_n \rangle$ is a sequence in X such that $\lim_{n\to\infty} \mathrm{Aa}_n = \lim_{n\to\infty} \mathrm{ABa}_n = t$ for some $t \in$ X.

Definition 2.11. Let A and B be two self maps on the IFMS $(X, M_{KM}, N_{KM}, *, \diamond)$. We say that A and B satisfy the property E.A if there exists a sequence $\langle \mathrm{a}_n \rangle \in$ X such that $\lim_{n\to\infty} A\mathrm{a}_n = \lim_{n\to\infty} \mathrm{Ba}_n = t$ for some $t \in$ X.

Definition 2.12. Suppose A, B, C and D are four self maps on the IFMS $(X, M_{KM}, N_{KM}, *, \diamond)$. We say that (A, B) and (C, D) satisfy common property E.A whenever two sequences $\langle \mathrm{a}_n \rangle$ and $\langle b_n \rangle$ in X satisfy $\lim_{n\to\infty} A\mathrm{a}_n = \lim_{n\to\infty} B\mathrm{a}_n = \lim_{n\to\infty} Cb_n = \lim_{n\to\infty} Db_n = t$ for some $t \in$ X.

3. **Main Result**

Theorem 3.1. *Let $(X, M_{KM}, N_{KM}, *, \diamond)$ be a complete intuitionistic fuzzy metric space. Suppose P, Q, R and S are self maps on X satisfying the following conditions:*

(i) *$P(X) \subseteq \mathrm{R}(X)$ and $Q(X) \subseteq S(X)$;*
(ii) *For every where $k \in (0,1)$ and $a, b \in \mathbb{X}$:* $\mathrm{M}_{KM}(Pa, Qb, \mathrm{k}t)$
$\geq \mathrm{M}_{KM}(Sa, Rb, t) * \mathrm{M}_{KM}(Pa, Sa, t) * \mathrm{M}_{KM}(Qb, Rb, t) * \mathrm{M}_{KM}(Pa, Rb, t)$
and
$\mathrm{N}_{KM}(Pa, Qb, \mathrm{k}t)$
$\leq \mathrm{N}_{KM}(Sa, Rb, t) \diamond \mathrm{N}_{KM}(Pa, Sa, t) \diamond \mathrm{N}_{KM}(Qb, Rb, t) \diamond N_{KM}(Pa, Rb, t)$
(iii) *$S(X)$ is complete;*
(iv) *the pair of mappings S and P is $\mathcal{A}$ - intimate and the other pair of mappings also R and Q is $\mathcal{S}$ - intimate.*

Then P, Q, R and S have a unique common fixed point in X.

Proof. Let a_0 be an arbitrary point of X. From the condition $P(X) \subseteq \mathrm{R}(X)$ of (i), there exists a point $a_1 \in X$ such that

$$Pa_0 = \mathrm{R}a_1 = b_0.$$

Now for a_1, applying (i), there exists $a_2 \in \mathbb{X}$ such that

$$Qa_1 = Sa_2 = b_1.$$

Continuing this way, we establish two real sequences $\langle a_n \rangle$ and $\langle b_n \rangle$ in X:

$\exists b_{2n} = Pa_{2n} = Ra_{2n+1}$ and $b_{2n+1} = Qa_{2n+1} = Sa_{2n+2}$ for $n \geq 0$.

Taking $a = a_{2n}, b = a_{2n+1}$ in the inequality (ii), we have

$\mathrm{M}_{KM}\left(Pa_{2\mathrm{n}}, Qa_{2n+1}, \mathrm{k}t\right)$
$\geq \mathrm{M}_{KM}\left(Sa_{2n}, Ra_{2n+1}, t\right) * \mathrm{M}_{KM}\left(Pa_{2n}, Sa_{2n}, t\right) * \mathrm{M}_{KM}\left(Qa_{2n+1}, \mathrm{R}a_{2n+1}, t\right) * \mathrm{M}_{KM}\left(Pa_{2n}, \mathrm{R}a_{2n+1}, t\right)$

and

$\mathrm{N}_{KM}\left(Pa_{2\mathrm{n}}, Qa_{2n+1}, \mathrm{k}t\right)$
$\leq \mathrm{N}_{KM}\left(Sa_{2\mathrm{n}}, Ra_{2n+1}, t\right) \mathrm{N}_{KM}\left(Pa_{2n}, Sa_{2n}, t\right) \mathrm{N}_{KM}\left(Qa_{2n+1}, \mathrm{R}a_{2n+1}, t\right) \mathrm{N}_{KM}\left(Pa_{2n}, \mathrm{R}a_{2n+1}, t\right)$

which implies that as $n \to \infty$:

$$\mathrm{M}_{KM}\left(b_{2n}, b_{2n+1}, \mathrm{k}t\right) \geq \mathrm{M}_{KM}\left(b_{2n-1}, b_{2n}, t\right) * \mathrm{M}_{KM}\left(b_{2n}, b_{2n-1}, t\right) * \mathrm{M}_{KM}\left(b_{2n+1}, b_{2n}, t\right) * \mathrm{M}_{KM}\left(b_{2n}, b_{2n}, t\right)$$

and

$$N\left(b_{2n}, b_{2n+1}, \mathrm{k}t\right) \leq \mathrm{N}_{KM}\left(b_{2n-1}, b_{2n}, t\right) \diamond \mathrm{N}_{KM}\left(b_{2n}, b_{2n-1}, t\right) \diamond \mathrm{N}_{KM}\left(b_{2n+1}, b_{2n}, t\right) \diamond \mathrm{N}_{KM}\left(b_{2n}, b_{2n}, t\right)$$

This yields

$\mathrm{M}_{KM}\left(b_{2n}, b_{2n+1}, \mathrm{k}t\right)$

$$\geq \mathrm{M}_{KM}\left(b_{2n-1}, b_{2n}, t\right) * \mathrm{M}_{KM}\left(b_{2n+1}, b_{2n}, t\right) * \mathrm{M}_{KM}\left(b_{2n}, b_{2n-1}, t\right) * 1$$

and

$\mathrm{N}_{KM}\left(b_{2n}, b_{2n+1}, \mathrm{k}t\right)$

$\leq \mathrm{N}_{KM}\left(b_{2n-1}, b_{2n}, t\right) \diamond \mathrm{N}_{KM}\left(b_{2n+1}, b_{2n}, t\right) iamond \mathrm{N}_{KM}\left(b_{2n}, b_{2n-1}, t\right) \diamond 0$

Again, by the condition KMIFM-3, we get

$\mathrm{M}_{KM}\left(b_{2n}, b_{2n+1}, \mathrm{k}t\right)$

$$\geq \mathrm{M}_{KM}\left(b_{2n-1}, b_{2n}, t\right) * \mathrm{M}_{KM}\left(b_{2n}, b_{2n+1}, t\right)$$

and

$\mathrm{N}_{KM}\left(b_{2n}, b_{2n+1}, \mathrm{k}t\right)$

$$\leq \mathrm{N}_{KM}\left(b_{2n-1}, b_{2n}, t\right) \mathrm{N}_{KM}\left(b_{2n}, b_{2n+1}, t\right)$$

which implies (since $\mathfrak{a} * \mathfrak{b} = \min\{\mathfrak{a}, \mathfrak{b}\}$ and $\mathfrak{a} b \mathfrak{b} = \max\{\mathfrak{a}, \mathfrak{b}\}$.)

$$\mathrm{M}_{KM}\left(b_{2n}, b_{2n+1}, \mathrm{k}t\right) \geq \mathrm{M}_{KM}\left(b_{2n-1}, b_{2n}, t\right).$$

and

$$\mathrm{N}_{KM}\left(b_{2n}, b_{2n+1}, \mathrm{k}t\right) \leq \mathrm{N}_{KM}\left(b_{2n-1}, b_{2n}, t\right).$$

In general

$$\mathrm{M}_{\mathrm{KM}}\left(b_{n+1}, b_{n+2}, kt\right) \geq M_{KM}\left(b_n, b_{n+1}, t\right) \ldots (i)$$

and

$$\mathrm{N}_{\mathrm{KM}}\left(b_{n+1}, b_{n+2}, kt\right) \leq N_{KM}\left(b_n, b_{n+1}, t\right) \ldots (i)$$

for all $n = 1, 2, 3..$, and $t > 0$.

From (i) we have:

$[\mathrm{M}_{KM}\left(b_n, b_{n+1}, t\right)] \geq \mathrm{M}_{KM}\left(b_{n-1}, b_n, \frac{t}{\mathrm{k}}\right) \geq \mathrm{M}_{KM}\left(b_{n-2}, b_{n-1}, \frac{t}{\mathrm{k}^2}\right)$

$$\geq \cdots \ldots \geq \mathrm{M}_{KM}\left(b_0, b_1, \frac{t}{\mathrm{k}^n}\right) \to 1 \text{ as } n \to \infty$$

and

$[\mathrm{N}_{KM}\left(b_n, b_{n+1}, t\right)] \leq \mathrm{N}_{KM}\left(b_{n-1}, b_n, \frac{t}{\mathrm{k}}\right) \leq \mathrm{N}_{KM}\left(b_{n-2}, b_{n-1}, \frac{t}{\mathrm{k}^2}\right)$

$$\leq \cdots \leq \mathrm{N}_{KM}\left(b_0, b_1, \frac{t}{\mathrm{k}^n}\right) \to 0 \text{ as } n \to \infty \ldots\ldots \text{ (ii)}$$

For any $t > 0$ and $\lambda_{MK} \in (0,1)$ we consider $\forall n > n_0 \in \mathbb{N}$ such that

$$\mathrm{M}_{KM}\left(b_n, b_{n+1}, t\right) > \left(1 - \lambda_{\mathrm{MK}}\right)$$

and

$$\mathrm{N}_{KM}\left(b_n, b_{n+1}, t\right) < \left(-\lambda_{\mathrm{MK}}\right) \ldots (iii)$$

For $m, n \in \mathbb{N}$, suppose $m \geq n$. Then we have:

$[\mathrm{M_{MK}}(b_n, b_m, t)]$

$\geq \min\left\{\mathrm{M_{MK}}\left(b_n, b_{n+1}, \frac{t}{m-n}\right) * \mathrm{M_{MK}}\left(b_{n+1}, b_{n+2}, \frac{t}{m-n}\right) * \ldots\right.$

$\mathrm{M_{MK}}\left(b_{m-1}, b_m, \frac{t}{m-n}\right) \geq (1-\lambda_{\mathrm{MK}}) * (1-\lambda_{\mathrm{MK}}) * \ldots * (1-\lambda_{\mathrm{MK}}) \ldots (m-n)$ times

and

$[\mathrm{N_{MK}}(b_n, b_m, t)]$

$\leq \max\left\{\mathrm{N_{MK}}\left(b_n, b_{n+1}, \frac{t}{m-n}\right) \diamond \mathrm{N_{MK}}\left(b_{n+1}, b_{n+2}, \frac{t}{m-n}\right) \diamond \ldots\right.$

$\mathrm{N_{MK}}\left(b_{m-1}, b_m, \frac{t}{m-n}\right) \leq (-\lambda_{\mathrm{MK}}) \diamond (-\lambda_{\mathrm{MK}}) \downarrow (-\lambda_{\mathrm{MK}}) \ldots (m-n)$ times.

This implies

$\mathrm{M_{MK}}(b_{m-1}, b_m, t) \geq (1-\lambda_{\mathrm{MK}})$ and $\mathrm{N_{MK}}(b_{m-1}, b_m, t) \leq (-\lambda_{\mathrm{MK}})$

Therefore $\langle b_n \rangle$ is cauchy sequence in IFMS.

Since $(X, M_{KM}, N_{KM}, *, \diamond)$ is a complete IFMS, so sequence $\{b_n\}$ converges L $\in$ I.

Further fuzzy cauchy sequence $\{b_n\}$ has convergent subsequence $\{b_{2n+1}\}$ and $\{b_{2n}\}$.

From the above argument,

$$b_{2n+1} = Qa_{2n+1} = Sa_{2n+2} \to \text{ L and}$$
$$b_{2n} = Pa_{2n} = \tilde{S}a_{2n+1} \to L \text{ as } n \to \infty \ldots (iv)$$

Now suppose that the range set $S(\mathrm{X})$ is complete then $\exists$ a point $\mathrm{u} \in X \ni S\mathrm{u} = \mathrm{L} \ldots ..(v)$.

Now we claim that $P\mathfrak{u} = \mathrm{L}$ from the inequality, put $a = u$ and $b = a_{2n+1}$ we have

$$\mathrm{M}_{KM}(P\mathfrak{u}, Qn_{2n+1}, \mathrm{k}t) \geq \mathrm{M}_{KM}(S\mathrm{u}, Ra_{2n+1}, t) * \mathrm{M}_{KM}(P\mathfrak{u}, S\mathrm{u}, t)$$
$$* \mathrm{M}_{KM}(Qa_{2n+1}, \mathrm{R}a_{2n+1}, t) * \mathrm{M}_{KM}(P\mathfrak{u}, Sa_{2n+1}, t)$$

and

$$\mathrm{N}_{KM}(P\mathfrak{u}, Qn_{2n+1}, \mathrm{k}t) \leq \mathrm{N}_{KM}(S\mathrm{u}, Ra_{2n+1}, t)\, \mathrm{N}_{KM}(P\mathfrak{u}, S\mathrm{u}, t)$$
$$\diamond \mathrm{N}_{KM}(Qa_{2n+1}, \mathrm{R}a_{2n+1}, t) \diamond \mathrm{N}_{KM}(P\mathfrak{u}, Sa_{2n+1}, t).$$

Taking limit as $n \to \infty$ we have:

$\mathrm{M}_{KM}(P\mathfrak{u}, \mathrm{L}, \mathrm{k}t)$

$\geq \mathrm{M}_{KM}(\ \mathrm{L},\ \mathrm{L}, t) * \mathrm{M}_{KM}(P\mathfrak{u}, \mathrm{L}, t) * \mathrm{M}_{KM}(\ \mathrm{L},\ \mathrm{L}, t) * \mathrm{M}_{KM}(P\mathfrak{u}, \mathrm{L}, t)$

This gives Pu $= L$. That is $P\mathrm{u} = S\mathrm{u} = L \ldots\ldots$ (vi)

Let us prove that Qv = p∗.

Using the equation ((vi) with contained inequality $P(X) \subseteq \mathrm{R}(X)$, $L = P\mathrm{u} \in P(X) \subseteq \mathrm{R}(X)$ then there exists a point $\mathrm{v} \in X$ such that $Rv = Pu = \mathrm{L} \ldots$ (vii).

Put $a =$ u and $b = v$ in (ii) then we obtain

$\mathrm{M}_{KM}(P\mathfrak{u}, Q\mathfrak{v}, \mathrm{k}t)$

$\geq \mathrm{M}_{KM}(\mathrm{Bu}, R\mathfrak{v}, t) * \mathrm{M}_{KM}(P\mathfrak{u}, S\mathrm{u}, t) * \mathrm{M}_{KM}(Q\mathfrak{v}, R\mathfrak{v}, t) * \mathrm{M}_{KM}(P\mathfrak{u}, R\mathfrak{v}, t)$.

and $\mathrm{N}_{KM}(P\mathfrak{u}, Q\mathfrak{v}, \mathrm{k}t)$

$\leq \mathrm{N}_{KM}(\mathrm{Bu}, R\mathfrak{v}, t) \diamond \mathrm{N}_{KM}(P\mathfrak{u}, S\mathrm{u}, t) \triangleright \mathrm{N}_{KM}(Q\mathfrak{v}, R\mathfrak{v}, t) \diamond \mathrm{N}_{KM}(P\mathfrak{u}, R\mathfrak{v}, t)$.

By using (vii) we get

$\mathrm{M}_{KM}(\ \mathrm{L}, Qv, \mathrm{k}t)$

$\geq \mathrm{M}_{KM}(\ \mathrm{L}, \mathrm{Qv}, t) * \mathrm{M}_{KM}(\ \mathrm{L},\ \mathrm{L}, \mathrm{t}) * \mathrm{M}_{KM}(Q\mathfrak{v}, \mathrm{L}, t) * \mathrm{M}_{KM}(\ \mathrm{L},\ \mathrm{L}, t)$

and

$\mathrm{N}_{KM}(\ \mathrm{L}, Qv, \mathrm{k}t)$

$\leq \mathrm{N}_{KM}(\ \mathrm{L}, \mathrm{Qv}, t) \diamond \mathrm{N}_{KM}(\ \mathrm{L},\ \mathrm{L}, \mathrm{t}) \diamond \mathrm{N}_{KM}(Q\mathfrak{v}, \mathrm{L}, t) \diamond \mathrm{N}_{KM}(\ \mathrm{L},\ \mathrm{L}, t)$

this gives

$$\mathrm{M}_{KM}(\ \mathrm{L}, \mathrm{Qv}, kt) \geq \mathrm{M}_{KM}(Qv,\ \mathrm{L}, \mathrm{kt}t)$$

and

$$\mathrm{N}_{KM}(\ \mathrm{L}, \mathrm{Qv}, kt) \leq \mathrm{N}_{KM}(Qv,\ \mathrm{L}, \mathrm{kt}).$$

Consequently $\mathrm{M}_{KM}(\ \mathrm{L}, Qv, \mathrm{k}t) \geq \mathrm{M}_{KM}(Qv,\ \mathrm{L}, \mathrm{k}t)$ and $\mathrm{N}_{KM}(\ \mathrm{L}, Qv, \mathrm{k}t) \leq \mathrm{N}_{KM}(Qv,\ \mathrm{L}, \mathrm{k}t)$. This implies $Qv = \mathrm{L}$.

This shows that Qv = Rv = L. Since $Pu = Su = L$ and (S, P) is $\mathcal{A}$-intimate we have $\mathrm{M}_{KM}(S\ \mathrm{L},\ \mathrm{L}, t) \geq \mathrm{M}_{KM}(P\ \mathrm{L},\ \mathrm{L}, t)$ and $\mathrm{N}_{KM}(S\ \mathrm{L},\ \mathrm{L}, t) \leq \mathrm{N}_{KM}(P\ \mathrm{L},\ \mathrm{L}, t) \ldots (ix)$.

Suppose that $P\mathrm{p}^* \neq \mathrm{p}^*$. Put $a = \mathrm{L}, b =$ v in (ii). Then we get,

$\mathrm{M}_{KM}(PL, Qv, \mathrm{k}t)$

$\geq \mathrm{M}_{KM}(S\ \mathrm{L}, \mathrm{Rv}, t) * \mathrm{M}_{KM}(PL, S\ \mathrm{L}, t) * \mathrm{M}_{KM}(Qv, Rv, t) * \mathrm{M}_{KM}(PL, Rv, t)$

and

$\mathrm{N}_{KM}(PL, Qv, \mathrm{k}t)$

$\leq \mathrm{N}_{KM}(S\ \mathrm{L}, \mathrm{Rv}, t) \mathrm{N}_{KM}(PL, S\ \mathrm{L}, t) \mathrm{N}_{KM}(Qv, Rv, t) \diamond \mathrm{N}_{KM}(PL, Rv, t)$.

Using (viii) we get,
$M_{KM}(PL,\ L, kt)$

$$\geq M_{KM}(S\ L,\ L, t) * M_{KM}(PL, S\ L, t) * M_{KM}(L, L, t) * M_{KM}(PL,\ L, t)$$

and
$N_{KM}(PL,\ L, kt)$

$$\leq N_{KM}(S\ L,\ L, t) \diamond N_{KM}(PL, S\ L, t) \diamond N_{KM}(L, L, t) \diamond N_{KM}(PL,\ L, t).$$

By applying (KMIFM-iv) we get
$M_{KM}(PL,\ L, kt) \geq M_{KM}(PL,\ L, t) * M_{KM}\left(PL,\ L, \frac{t}{2}\right)$

$$*M_{KM}\left(\ L, S\ L, \frac{t}{2}\right) * M_{KM}(\ L,\ L, t) * M_{KM}(PL,\ L, t)$$

and
$N_{KM}(PL,\ L, kt) \leq N_{KM}(PL,\ L, t) \diamond N_{KM}\left(PL,\ L, \frac{t}{2}\right)$

$$\diamond N_{KM}\left(\ L, S\ L, \frac{t}{2}\right) \diamond N_{KM}(\ L,\ L, t) \diamond N_{KM}(PL,\ L, t)$$

By using (ix) we get $M_{KM}(PL,\ L, kt) \geq M_{KM}(PL,\ L, t/2)$ and $N_{KM}(PL,\ L, kt) \leq N_{KM}(PL,\ L, t/2)$. This gives $PL = L$.

From (ix) and (x) we write

$$M_{KM}(SL,\ L, t) \geq 1 \text{ and } N_{KM}(SL,\ L, t) \leq 1$$

this gives $SL = L \ldots\ldots (xi)$.

Using (x) and (xi) we get $SL = PL = L \ldots\ldots (xii)$. Also, $Qv = Rv = L$ and using the pair (R, Q) as $\mathcal{A}$-Intimate, then we have

$$M_{KM}(RL, L, t) \geq M_{KM}(QL,\ L, kt)$$

and

$$N_{KM}(RL, L, t) \leq N_{KM}(QL,\ L, kt) \ldots (\ldots (xiii).$$

Suppose that $QL \neq L$. Put $a = u$ and $b = L$ in the inequality. We have:

$M_{KM}(P\mathfrak{u}, QL, kt)$

$$\geq M_{KM}(Su, RL, t) * M_{KM}(P\mathfrak{u}, Su, t) * M_{KM}(QL, RL, t) * M_{KM}(P\mathfrak{u}, RL, t)$$

and
$N_{KM}(P\mathfrak{u}, QL, kt)$

$$\leq N_{KM}(Su, RL, t) \diamond N_{KM}(P\mathfrak{u}, Su, t) \diamond N_{KM}(QL, RL, t) \diamond N_{KM}(P\mathfrak{u}, RL, t)$$

Using (vi) and (KMIFM-iv) we get,

$$\mathrm{M}_{KM}(\ \mathrm{L}, Q\ \mathrm{L}, \mathrm{k}t) \geq \mathrm{M}_{KM}(\ \mathrm{L}, \mathrm{RL}, t) * \mathrm{M}_{KM}(\ \mathrm{L},\ \mathrm{L}, t)$$
$$* \mathrm{M}_{KM}\left(PL,\ \mathrm{L}, \frac{t}{2}\right) * \mathrm{M}_{KM}\left(\ \mathrm{L}, \mathrm{SL}, \frac{t}{2}\right) * \mathrm{M}_{KM}(\ \mathrm{L}, \mathrm{RL}, t)$$

and $\mathrm{N}_{KM}(\ \mathrm{L}, Q\ \mathrm{L}, \mathrm{k}t) \leq \mathrm{N}_{KM}(\ \mathrm{L}, \mathrm{RL}, t) \diamond \mathrm{N}_{KM}(\ \mathrm{L},\ \mathrm{L}, t)$

$$\diamond \mathrm{N}_{KM}\left(PL,\ \mathrm{L}, \frac{t}{2}\right) \diamond \mathrm{N}_{KM}\left(\ \mathrm{L}, \mathrm{SL}, \frac{t}{2}\right) \diamond \mathrm{N}_{KM}(\ \mathrm{L}, \mathrm{RL}, t)$$

Now using (xiii) we get

$$\mathrm{M}_{KM}(\ \mathrm{L}, QL, \mathrm{k}t) \geq \mathrm{M}_{KM}(\ \mathrm{L}, QL, t) * \mathrm{M}_{KM}\left(QL,\ \mathrm{L}, \tfrac{t}{2}\right)$$
$$* \mathrm{M}_{KM}(QL,\ \mathrm{L}, t/2) * \mathrm{M}_{KM}(\ \mathrm{L}, QL, t)$$

and

$$\mathrm{N}_{KM}(\ \mathrm{L}, QL, \mathrm{k}t) \leq \mathrm{N}_{KM}(\ \mathrm{L}, QL, t)\mathrm{N}_{KM}\left(QL,\ \mathrm{L}, \tfrac{t}{2}\right)$$
$$\downarrow \mathrm{N}_{KM}(QL,\ \mathrm{L}, t/2) \diamond \mathrm{N}_{KM}(\ \mathrm{L}, QL, t)$$

This implies $\mathrm{M}_{KM}(\ \mathrm{L}, L, \mathrm{k}t) \geq \mathrm{M}_{KM}(\ \mathrm{L}, L, t/2)$ and $\mathrm{N}_{KM}(\ \mathrm{L}, L, \mathrm{k}t) \leq \mathrm{N}_{KM}(\ \mathrm{L}, L, t/2)$. This gives $QL = \mathrm{L} \ldots$ (xiv).

From (xii) and (xiv) we get

$$\mathrm{M}_{KM}(\mathrm{RL}, L, t) \geq 1 \text{ and } \mathrm{N}_{KM}(\mathrm{RL}, L, t) \leq 1$$

$$RL = \mathrm{L} \ldots\ldots (xv).$$

Using (xiv) and (xv) we get

$$\mathrm{QL} = \mathrm{SL} = \mathrm{L} \ldots (xvi).$$

Using (xii) and xvi we conclude that PL = QL = RL = SL = L.

□

Conclusion

This study demonstrates that fixed point theorems can be effectively applied to intimate mappings within intuitionistic fuzzy metric spaces. By extending classical fixed point principles to these spaces, we have shown that such mappings maintain the necessary conditions for establishing fixed points. This work not only broadens the theoretical framework of fuzzy metric spaces but also suggests potential applications in f ields where uncertainty and imprecision are common. Future research may further explore these extensions and their practical implications.

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Ram Milan Singh
Department of Mathematics,
Institute for Excellence in higher Education, Bhopal (India)
Email: rammilansinghlig@gmail.com